Math 20 Review Questions

KEY

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example:

$x^2 + 3x - 5 = 0$ a =1 b = 3 c = -5

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 + 20}}{2}$$

$$x = \frac{-3 \pm \sqrt{29}}{2}$$

1. **Solve by using the quadratic formula.**

$6y^2 - 5 = 3y$ $a=6$ $b=-3$ $c=-5$

$6y^2 - 3y - 5 = 0$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(6)(-5)}}{2(6)}$$

$$x = \frac{3 \pm \sqrt{9+120}}{12}$$

$$\boxed{x = \frac{3 \pm \sqrt{129}}{12}}$$

$3r^2 - r + 2 = 0$ $a=3$ $b=-1$ $c=2$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{1 \pm \sqrt{1-24}}{6}$$

$$x = \frac{1 \pm \sqrt{-23}}{6}$$

no solution, the discriminant is undefined.

2. **Solve by using the quadratic formula.**

$4x^2 + \cancel{x} = \cancel{x} - 5$

$4x^2 + 5 = 0$ $a=4$ $b=0$ $c=5$

$$x = \frac{-(0) \pm \sqrt{0^2 - 4(4)(5)}}{2(4)}$$

$$x = \frac{\pm \sqrt{-80}}{8}$$

no solution, the discriminant is undefined.

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3. **Simplify Fractions in your solutions from the Quadratic Formula and express your answers as exact.**

$$\frac{6\pm2\sqrt{45}}{6}$$

$$\frac{6\pm2\sqrt{9}\sqrt{5}}{6}$$

$$\frac{6\pm2\cdot3\sqrt{5}}{6}$$

$$\frac{\cancel{6}\pm\cancel{6}\sqrt{5}}{\cancel{6}}$$

$$\boxed{1\pm\sqrt{5}}$$

$$\frac{10\pm3\sqrt{12}}{4}$$

$$\frac{10\pm3\sqrt{4}\sqrt{3}}{4}$$

$$\frac{10\pm3\cdot2\sqrt{3}}{4}$$

$$\frac{10\pm6\sqrt{3}}{4}$$

$$\boxed{\frac{5+3\sqrt{3}}{2},\ \frac{5-3\sqrt{3}}{2}}$$

$$\frac{5\pm\sqrt{75}}{10}$$

$$\frac{5\pm\sqrt{25}\sqrt{3}}{10}$$

$$\frac{\cancel{5}\pm\cancel{5}\sqrt{3}}{2\cdot\cancel{5}}$$

$$\boxed{\frac{1+\sqrt{3}}{2},\ \frac{1-\sqrt{3}}{2}}$$

$$\frac{14\pm\sqrt{360}}{8}$$

$$\frac{14\pm\sqrt{36}\sqrt{10}}{8}$$

$$\frac{\cancel{2}\cdot7\pm\cancel{2}\cdot3\sqrt{10}}{\cancel{2}\cdot4}$$

$$\boxed{\frac{7+3\sqrt{10}}{4},\ \frac{7-3\sqrt{10}}{4}}$$

$$\frac{15\pm\sqrt{125}}{5}$$

$$\frac{15\pm\sqrt{25}\sqrt{5}}{5}$$

$$\frac{3\cdot\cancel{5}\pm\cancel{5}\sqrt{5}}{\cancel{5}}$$

$$\boxed{3\pm\sqrt{5}}$$

$$\frac{8\pm\sqrt{100}}{16}$$

$$\frac{8\pm10}{16}$$

$$\frac{\cancel{2}\cdot4\pm\cancel{2}\cdot5}{\cancel{2}\cdot8}$$

$$\frac{4+5}{8} \qquad \frac{4-5}{8}$$

$$\boxed{\frac{9}{8},\ \frac{-1}{8}}$$

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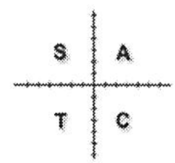
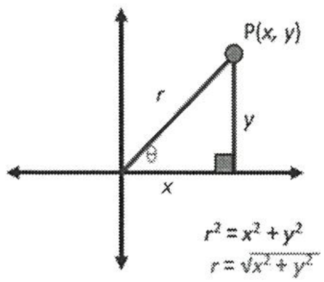
CAST Rule and Trigonometry in the Coordinate Plane

Finding the Trig Ratios of an Angle in Standard Position





$\sin\theta = \dfrac{y}{r}$

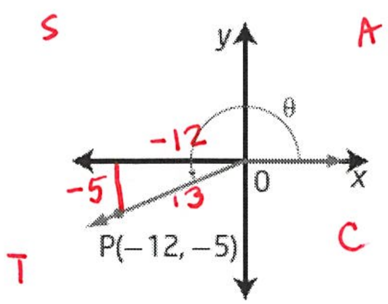
$\cos\theta = \dfrac{x}{r}$

$\tan\theta = \dfrac{y}{x}$

2.2.2

1. The coordinates of a point *P* on the terminal arm of each angle are shown. Write the exact trigonometric ratios sin θ, cos θ, and tan θ for each. **HINT:** Determine the value of ***r*** first.

a)



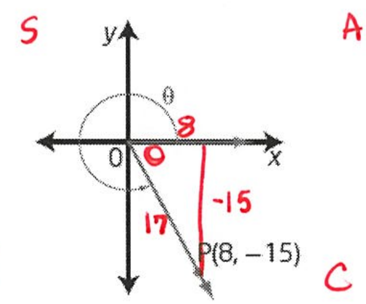
$\sqrt{(-12)^2+(-5)^2}$
$= 13$

$\sin\theta = -\dfrac{5}{13}$

$\cos\theta = -\dfrac{12}{13}$

$\tan\theta = \dfrac{5}{12}$

b)



$\sqrt{17^2+(-15)^2}$
$= 17$

$\sin\theta = -\dfrac{15}{17}$

$\cos\theta = \dfrac{8}{17}$

$\tan\theta = -\dfrac{15}{8}$

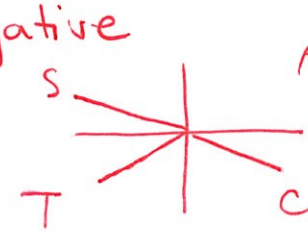
2. Without using a calculator, state whether each ratio is positive or negative. **HINT:** Use the CAST rule.

a) sin 155° positive

b) cos 320° positive

c) tan 120° negative

d) cos 220° negative



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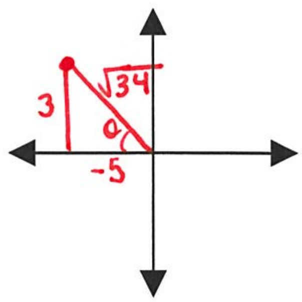
3. Determine the exact values of sin θ, cos θ, and tan θ if the terminal arm of an angle in standard position passes through the given point. **HINT:** Determine the value of ***r*** first.

a) P(−5, 3)

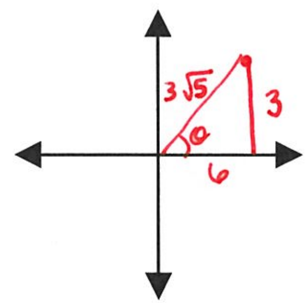


$\sin\theta = \frac{3}{\sqrt{34}}$

$\cos\theta = -\frac{5}{\sqrt{34}}$

$\tan\theta = -\frac{3}{5}$

b) P(6, 3)



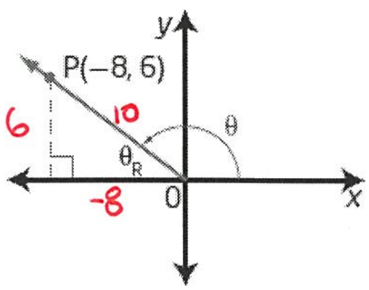
$\sin\theta = \frac{1}{\sqrt{5}}$

$\cos\theta = \frac{2}{\sqrt{5}}$

$\tan\theta = \frac{1}{2}$

4. Determine the values of *x*, *y*, *r*, sin θ, cos θ, and tan θ in each.

a)



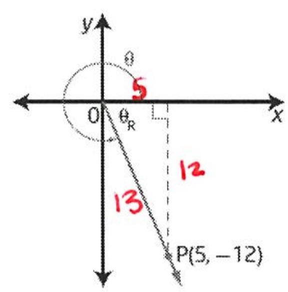
$x = -8,\ y = 6,\ r = 10$

$\sin\theta = \frac{3}{5}$

$\cos\theta = -\frac{4}{5}$

$\tan\theta = -\frac{3}{4}$

b)



$x = 5,\ y = -12,\ r = 13$

$\sin\theta = -\frac{12}{13}$

$\cos\theta = \frac{5}{13}$

$\tan\theta = -\frac{12}{5}$

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5. For each description, in which quadrant does the terminal arm of angle θ lie? **HINT:** Use the CAST rule.

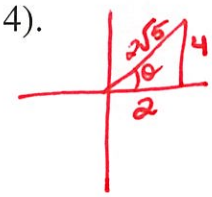
a) $\cos\theta < 0$ and $\sin\theta > 0$ $\boxed{\text{QII}}$

b) $\cos\theta > 0$ and $\tan\theta > 0$ $\boxed{\text{QI}}$

c) $\sin\theta < 0$ and $\cos\theta < 0$ $\boxed{\text{QIII}}$

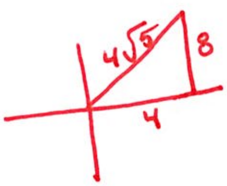
d) $\tan\theta < 0$ and $\cos\theta > 0$ $\boxed{\text{QIV}}$

6. a) Determine sin θ when the terminal arm of an angle in standard position passes through the point P(2, 4).



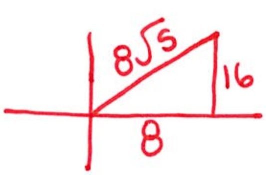
$\sin\theta = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

b) Extend the terminal arm to include the point Q(4, 8). Determine sin θ for the angle in standard position whose terminal arm passes through point Q.



$\sin\theta = \frac{8}{4\sqrt{5}} = \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

c) Extend the terminal arm to include the point R(8, 16). Determine sin θ for the angle in standard position whose terminal arm passes through point R.



$\sin\theta = \frac{16}{8\sqrt{5}} = \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

d) Explain your results from parts a), b), and c). What do you notice? Why does this happen?

Sin θ is the same because side length's can change in ratios and as long as the angle does not chang the trigonometric ratios remain the same.

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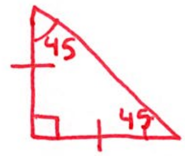
Discovering Properties of Special Right Triangles

There are two types of special right triangles, both defined by their angles. The first is a 45° – 45° – 90° and the second is a 30° – 60° – 90°.

Part A: 45° – 45° – 90° Triangles

Before you begin, how would you classify a 45° – 45° – 90° triangle by its sides?

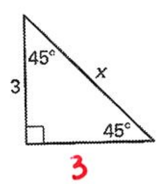
(a) scalene (b) $\boxed{\text{isosceles}}$ (c) equilateral



Now to begin: You're looking for an easy way to find the length of the sides in these triangles. **Your goal is to avoid using the Pythagorean theorem**, but in order to do that you must first use the theorem and see if you notice a pattern.

Find the lengths of the missing sides. Leave all of your answers in reduced radical form. (For example: $\sqrt{20} = \sqrt{4} * \sqrt{5} = 2\sqrt{5}$)

1) Example.



$\sqrt{3^2+3^2}$

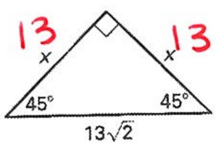
$\sqrt{9+9}$

$\sqrt{18}$

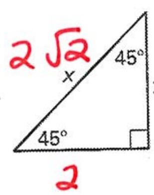
$\sqrt{9}\sqrt{2}$

$\boxed{3\sqrt{2}}$

2)



3)



$\sqrt{2^2+2^2}$

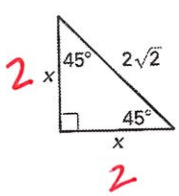
$\sqrt{4+4}$

$\sqrt{8}$

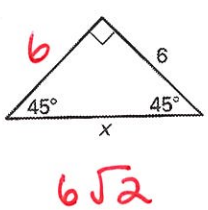
$\sqrt{4}\sqrt{2}$

$\boxed{2\sqrt{2}}$

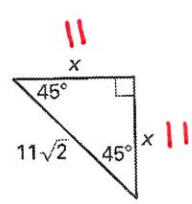
4)



5)



6)



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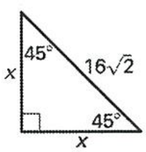
Write a conjecture that describes what you see:

The hypotenuse length will be the side length times the $\sqrt{2}$ → $\boxed{\text{side}\sqrt{2}}$

Test your conjecture on these triangles.

1)

16


16

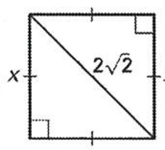
$\sqrt{16^2+16^2}$

$\sqrt{512}$

$\sqrt{256}\sqrt{2}$

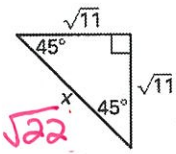
$16\sqrt{2}$ ✓

2)

2


2

3)



$\sqrt{22}$

$\sqrt{\sqrt{11}^2+\sqrt{11}^2}$

$\sqrt{11+11}$

$\sqrt{22} \Rightarrow \sqrt{11}\sqrt{2}$

Math 20 Review Questions

Part B: 30° – 60° – 90° Triangles

This time, you're looking for an easy way to find the length of the sides in 30° – 60° – 90° triangles.

Again, your goal is to avoid using the Pythagorean theorem, but in order to do that you must first use the theorem and see if you notice a pattern.

And again, please leave all of your answers in reduced radical form.

We'll start with equilateral triangles and three questions:

1) What do the angles in an equilateral triangle measure? 60°

2) When you draw the altitude in an equilateral triangle, what does it do to the top angle?

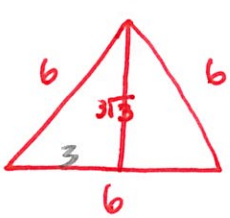
   Bisects it.

3) When you draw the altitude in an equilateral triangle, what does it do to the base?

   Bisects it.

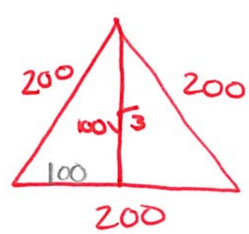
Now to begin: Find the length of the altitude in these equilateral triangles (sketch the triangle if it helps)

1) sides = 6 cm altitude = $3\sqrt{3}$



2) sides = 10 cm

3) sides = 200 cm



How does the altitude compare to half of the base?

The altitude is equal to half of the base times $\sqrt{3}$.

$\boxed{\frac{1}{2} \text{ base } \sqrt{3}}$

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When you draw the altitude in an equilateral triangle, you create two congruent 30° – 60° – 90° triangles. The two legs of each triangle are called "shorter leg" and "longer leg" (doh!).

So using what you determined before, how does the longer leg in a 30° – 60° – 90° triangle compare to the shorter leg?

Length of longer leg = __$\sqrt{3}$__ * Length of shorter leg

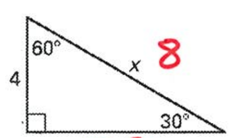
And how does the hypotenuse (the sides of your equilateral triangles) compare to the shorter leg?

Length of hypotenuse = __2__ * Length of shorter leg

Test your results on these triangles.

Find the value of *x*. Write your answer in radical form.

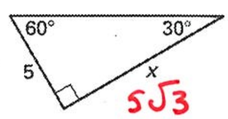
13.



$x = 8$

$4\sqrt{3}$

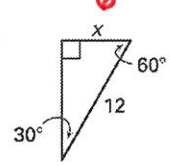
14.



$5\sqrt{3}$

15.

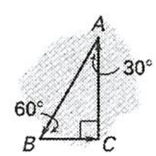
6



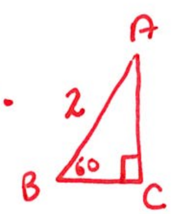
A jogging path starts at point *A*, turns at point *B*, turns at point *C* and stops at point *A*, as shown.

16. If $AB = 2$ miles, find BC and CA. Round your answers to the nearest tenth of a mile.

17. Find the total length of the jogging path. Round your answer to the nearest tenth of a mile.



16.



17. 4.7 miles

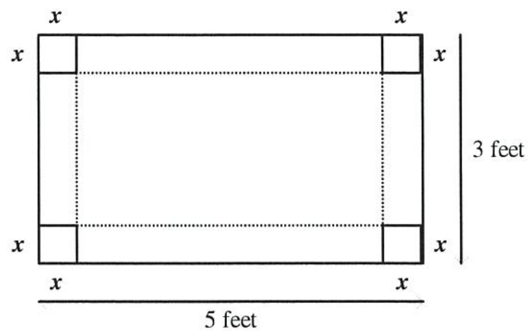
$\overline{BC} = 1$ mile

$\overline{CA} = \sqrt{3}$ miles

$= 1.7$ miles.

Algebraic Solving of Systems of Equations

Four corners are cut from a rectangular piece of cardboard that measures 5 ft by 3 ft. The cuts are x feet from the corners, as shown in the figure below. After the cuts are made, the sides of the rectangle are folded to form an open box. The area of the bottom of the box is 12 ft².



$(5-2x)(3-2x)$

$15-10x-6x+4x^2$

$4x^2-16x+15$

What two equations represent the area, *A*, of the bottom of the box?

A $A = 4x^2 - 16x + 15$
$A = 12$

B $A = 4x^2 + 15$
$A = 12$

C $A = 4x^2 + 15$
$A = 8$

D $A = 4x^2 - 30x + 8$
$A = 8$

$\boxed{\cancel{x=3.8}}$ or $\boxed{x=0.2}$

gives a negative length/width

What are the approximate dimensions of the box? Give your answer to one decimal place.

A width = 2.6 ft
length = 4.6 ft
height = 3.8 ft

B width = 4.0 ft
length = 2.0 ft
height = 0.5 ft

C width = 4.8 ft
length = 2.8 ft
height = 0.1 ft

D width = 4.6 ft
length = 2.6 ft
height = $\boxed{0.2 \text{ ft}}$

$5-2(0.2)=4.6$

$3-2(0.2)=2.6$

What is the approximate volume of the box? Give your answer to one decimal place.

A 1.4 ft³

B 4.0 ft³

C 2.4 ft³

D 45.6 ft³

$4.6 \cdot 2.6 \cdot 0.2 \doteq 2.392$

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Solve each system of equations by using *substitution*. If there is no solution, put Ø. If it is the same line, write {(x,y): equation of the line}

$y = 3x + 4$
$4x + 2y = 18$

$\boxed{(1,7)}$

$4x + 2(3x+4) = 18$
$4x + 6x + 8 = 18$
$10x = 10$
$x = 1$

$y = 3(1)+4$
$y = 7$

$3x - 3y = 15$
$x = 1 + 2y$

$\boxed{(9,4)}$

$3(1+2y) - 3y = 15$
$3 + 6y - 3y = 15$
$\frac{3y}{3} = \frac{12}{3}$
$y = 4$

$x = 1 + 2(4)$
$x = 9$

$5x + 6y = 32$
$12y - 4x = 8$

$\boxed{(4,2)}$

$x = 3y - 2$
$5(3y-2) + 6y = 32$
$15y - 10 + 6y = 32$
$\frac{21y}{21} = \frac{42}{21}$
$y = 2$

$x = 3(2) - 2$
$x = 4$

Use *elimination* to solve the systems of equations. If there is no solution, put Ø. If it is the same line, write {(x,y): equation of the line}

$x - 2y = 4$
$y = x - 2$

$\boxed{(0,-2)}$

$x - 2y - 4 = 0$
$-\ x - y - 2 = 0$
$-y - 2 = 0$
$-y = 2$
$y = -2$

$x - 2(-2) = 4$
$x + 4 = 4$
$-4 \quad -4$
$x = 0$

$x + 3y = 18$
$-x + 2y = 7$

$\boxed{(3,5)}$

$x + 3y = 18$
$-\ x - 2y = -7$
$5y = 25$
$y = 5$

$x + 3(5) = 18$
$x + 15 = 18$
$x = 3$

$4x - 3y = 22$
$2(2x + 8y = 30)$

$\boxed{(7,2)}$

$4x - 3y = 22$
$-\ 4x + 16y = 60$
$-19y = -38$
$y = 2$

$4x - 3(2) = 22$
$4x - 6 = 22$
$\frac{4x}{4} = \frac{28}{4}$
$x = 7$

* Remember there are multiple ways to use substitution or elimination; these are just one option for you.

Math 20 Review Questions

Solve the system of equations by elimination and by substitution.

$y = 2x^2 - 2x - 3$ **and** $y = -x^2 - 2x - 3$

Elimination (0, -3)

$y = 2x^2 - 2x - 3$

$-\ y = -x^2 - 2x - 3$

$0 = 3x^2$

$x = 0$

$y = 2(0)^2 - 2(0) - 3$

$y = -3$

Substitution (0, 3)

$2x^2 - \cancel{2x} - \cancel{3} = -x^2 - \cancel{2x} - \cancel{3}$

$+\cancel{2x} + \cancel{3}$

$2x^2 = -x^2$

$+x^2 \quad +x^2$

$3x^2 = 0$

$x = 0$

What are the coordinates of the point(s) of intersection of the line $y = -7x - 5$ **and the quadratic function** $y = -x^2 - 15x + 4$**? Solve algebraically.**

$\boxed{(1, -12)\ (-9, 58)}$

Elimination

$y = \quad -7x - 5$

$-\ y = -x^2 - 15x + 4$

$0 = x^2 + 8x - 9$

$0 = (x - 1)(x + 9)$

$x = 1, -9$

$y = -7(1) - 5 =$

$y = -7(-9) - 5 =$

Substitution

$-7x - 5 = -x^2 - 15x + 4$

$0 = -x^2 - 15x + 4 + 7x + 5$

$0 = -x^2 - 8x + 9$

$0 = x^2 + 8x - 9$

$0 = (x - 1)(x + 9)$

$x = 1, -9$

← Then solve for y

Math 20 Review Questions

What are the solutions for the following system of equations? Solve algebraically.

$y = 8x + 7$
$y = -x^2 - 5x + 7$

$(0, 7)$ $(-13, -97)$

Elimination

$y = 8x + 7$
$- \quad y = -x^2 - 5x + 7$

$0 = x^2 + 13x$

$0 = x(x + 13)$

$x = 0, -13$

$y = 8(0) + 7 = 7$
$y = 8(-13) + 7 = -97$

Substitution

$8x + 7 = -x^2 - 5x + 7$

$x^2 + 5x - 7 + 8x + 7 = 0$

$x^2 + 13x = 0$

$x(x + 13) = 0$

$x = 0, -13$

*Then solve for y

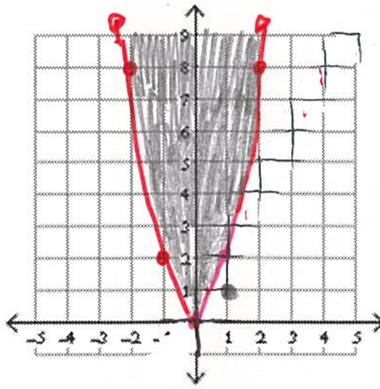
Math 20 Review Questions

Solving Inequalities & Graphing Solution on a Number Line

Sketch the graph of each quadratic inequality. Use a test point to determine shading.

a) $y \geq 2x^2$

Test $(1, 1)$



$1 \geq 2(1)^2$

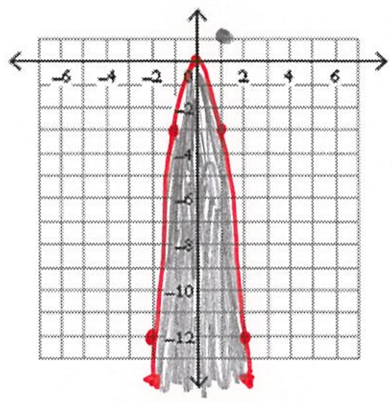
$1 \geq 2$

False

Shade in

b) $y \leq -3x^2$

Test $(1, 1)$



$1 \leq -3(1)^2$

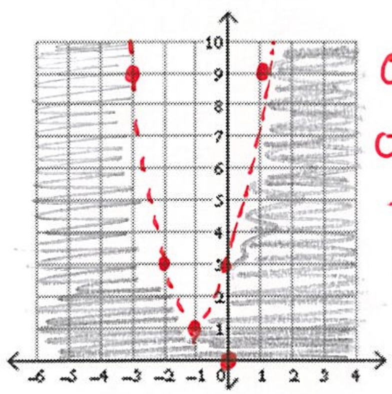
$1 \leq -3$

False

Shade in.

c) $y < 2x^2 + 4x + 3$

Test $(0,0)$



$0 < 2(0)^2 + 4(0) + 3$

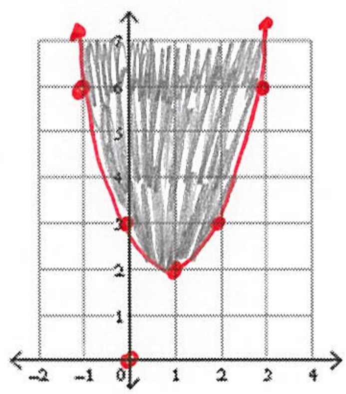
$0 < 3$

True

Shade out.

d) $y \geq x^2 - 2x + 3$

Test $(0,0)$



$0 \geq 0^2 - 2(0) + 3$

$0 \geq 3$

False

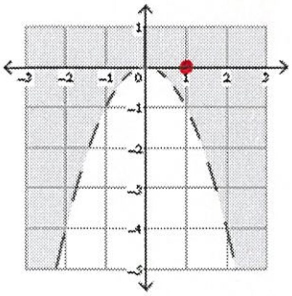
Shade in

* Remember any point will do to test. If you are able to choose $(0,0)$ it will make the test faster.

Math 20 Review Questions

Write an inequality for each graph given the equation for the boundary function. (Change the = sign to >, < ≥, or ≤ to write the inequality. You will want to use a test point to decide which side to shade.

a) $y = -x^2$ $\boxed{y > -x^2}$

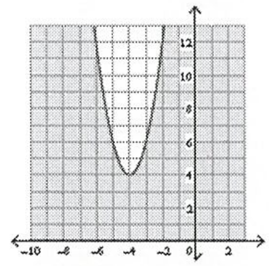


$1 > -1$

True

Shade out

b) $y = 2x^2 + 16x + 36$ $\boxed{y \leq 2x^2 + 16x + 36}$



$0 \leq 36$

True

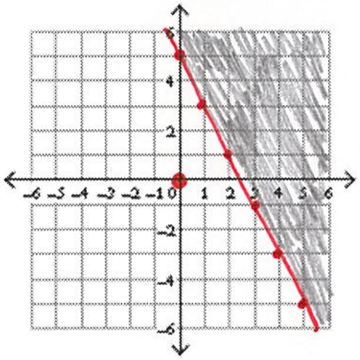
Shade out.

Does the given value of *x* make the inequality a true or a false statement?

$x \geq \frac{5}{2}$, $x = 2$ **TRUE** or **FALSE**

Sketch the graph of each linear inequality. Use a test point to determine shading.

a) $y \geq -2x + 5$

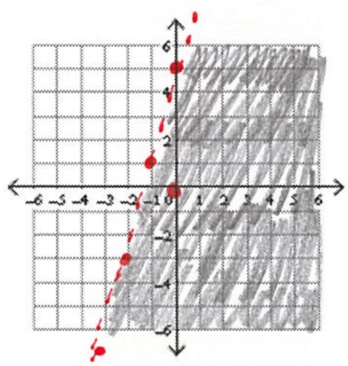


$0 \geq -2(0) + 5$

$0 \geq 5$

False

b) $y < 4x + 5$

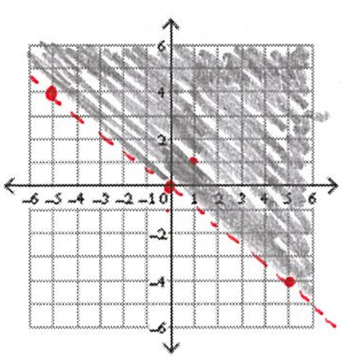


$0 < 4(0) + 5$

$0 < 5$

True

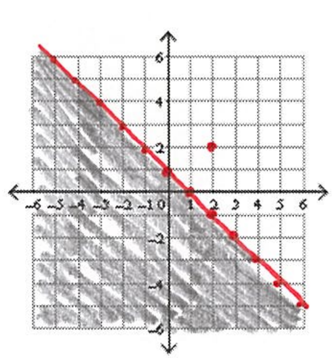
c) $y > -\frac{4}{5}x$



$1 > -\frac{4}{5}(1)$

$1 > -\frac{4}{5}$ True

d) $y \leq -x + 1$



$2 \leq -(2) + 1$

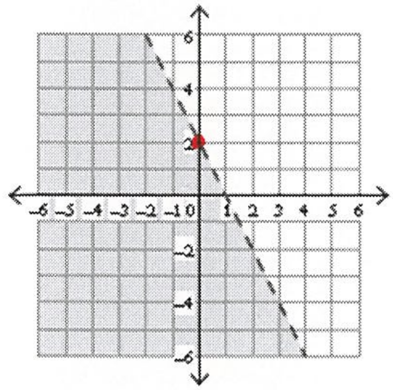
$2 \leq -1$

False

Math 20 Review Questions

Write the inequality for each graph.

a)



$y = -2x + 2$

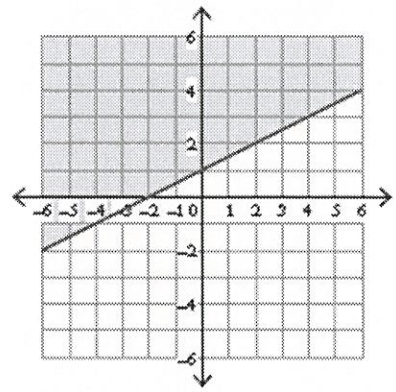
Test $(0,0)$

$0 \square -2(0) + 2$

$0 \boxed{<} 2$

$\boxed{y < -2x + 2}$

b)



$y = \frac{1}{2}x + 1$

Test $(0,2)$

$2 \square \frac{1}{2}(0) + 1$

$2 \boxed{\geq} 1$

$\boxed{y \geq \frac{1}{2}x + 1}$

Math 20 Review Questions

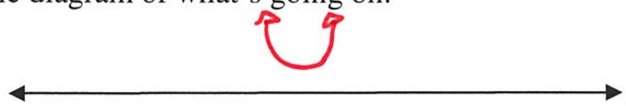
Solve: $x^2 + x + 5 < 0$

(a) How many x-intercepts are there for $y = x^2 + x + 5$? If there are any x-intercepts, where are they?

zero

(b) Does the parabola (when graphed) open: UPWARDS or DOWNWARDS?

(c) Draw a general schematic diagram of what's going on:



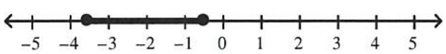
x-axis

(d) Answer the question: $x^2 + x + 5 < 0$

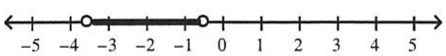
$\{x|$ no solution $, x \in R\}$

Which number line represents the solution set to the inequality $-2x^2 - 7.9x > 3$**?** $-2x^2 - 7.9x - 3 > 0$

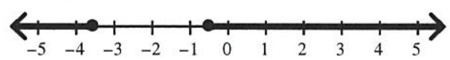
A



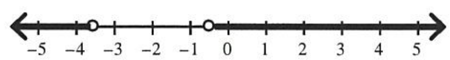
B



C



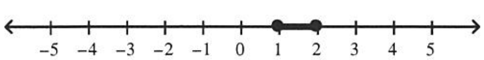
D



Which graph represents the solution to the inequality $2x^2 - 6x + 4 \geq 0$**?**

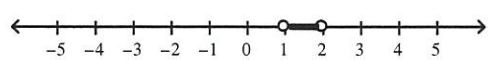
A

B



C

D



The solution set to the inequality $-3x^2 \leq -9x + 6$ **is**

$0 \leq 3x^2 - 9x + 6$

A $\{x|\ 1 \leq x \leq 2, x \in R\}$

B $\{x|-2 \leq x \leq -1, x \in R\}$

C $\{x|\ x \leq -2 \text{ or } x \geq -1, x \in R\}$

D $\{x|\ x \leq 1 \text{ or } x \geq 2, x \in R\}$

$0 \leq 3(x^2 - 3x + 2)$

$0 \leq 3(x-2)(x-1)$

$x = 2,\ x = 1$