Square Roots

When a number is multiplied by itself, you *square* the number.

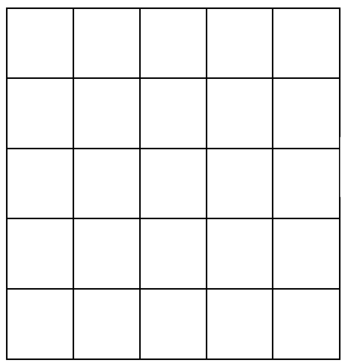
The product is a perfect square.

For example: The square of 5 is $5 \times 5 = 25$

We write: $5^2 = 5 \times 5 = 25$

We say: Five squared is 25.

25 is a **square number**, or a **perfect square**.



5 × 5

25 is a perfect square because one of the quadrilaterals that can be made from 25 tiles is a square!

Practice:

1. List all the ***perfect square values*** from 1 to 150.

2. Determine the value of the square roots.

   $a)\ \sqrt{25}$ $b)\ \sqrt{121}$ $c)\ \sqrt{2500}$ $d)\ \sqrt{640\,000}$

   $e)\ \sqrt{0.36}$ $f)\ \sqrt{0.0081}$ $g)\ \sqrt{1.96}$ $h)\ \sqrt{\frac{25}{100}}$

   $i)\ \sqrt{\frac{64}{36}}$ $j)\ \sqrt{2\frac{1}{4}}$

3. Identify the Perfect Square value for each square root value given.

   a) 17 b) 2.7 c) $\frac{2}{11}$ d) 28.1

4. Identify the two perfect square values the following numbers fall between.

a) 38 b) 92 c) 115

5. Using the information above, estimate the square root values.

a) $\sqrt{38}$ b) $\sqrt{92}$ c) $\sqrt{115}$

Multiplying & Dividing Fractions

To multiply fractions without using a model, multiply the numerators and multiply the denominators.

$$\frac{3}{4}\times\frac{2}{5}=\frac{3\times 2}{4\times 5}=\frac{6}{20}=\frac{3}{10}$$

When finished the multiplying, check if the numerator and the denominator have a common factor. Divide by the common factors to express the fraction in simplest form.

$$\frac{5}{12}\times\frac{8}{15}=\frac{5\times 8}{12\times 15}=\frac{40}{180}=\frac{40\div 20}{180\div 20}=\frac{2}{9}$$

One way to multiply mixed numbers is to first convert to improper fractions.

$$1\frac{1}{5}\times 3\frac{1}{8}=\frac{6}{5}\times\frac{25}{8}$$

$$=\frac{6\times 25}{5\times 8}$$

$$=\frac{150}{40}$$

$$=\frac{150\div 10}{40\div 10}$$

$$=\frac{15}{4}$$

How do we convert a mixed number to an improper fraction?

Remember the checkmark method! $2\frac{5}{6}$

Multiply the denominator by the whole number and add the numerator.

So, $2\frac{5}{6}=\frac{6\times 2+5}{6}=\frac{17}{6}$

To divide fractions, we multiply by the reciprocal. That means we need to change the division sign to a multiplication and flip the second fraction. Multiply the fractions and reduce, if necessary.

$$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{3 \times 5}{4 \times 2} = \frac{15}{8}$$

$$\frac{3}{4} \div \frac{3}{2} = \frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12} = \frac{1}{2}$$

Practice:

7. Multiply or divide the following fractions. Reduce answers to simplest form.

a) $\frac{3}{2} \times \frac{1}{5}$ b) $\frac{4}{5} \times \frac{1}{2}$ c) $\frac{5}{6} \times \frac{2}{3}$ d) $\frac{4}{5} \div \frac{2}{3}$ e) $\frac{2}{5} \div \frac{1}{3}$

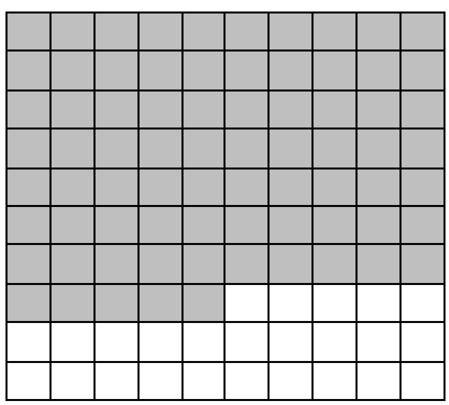
f) $\frac{3}{10} \div \frac{1}{3}$ g) $2\frac{1}{3} \times 1\frac{2}{5}$ h) $1\frac{1}{2} \times \frac{1}{8}$ i) $4\frac{1}{4} \div \frac{5}{6}$ j) $2\frac{1}{2} \div 1\frac{3}{4}$

Percents

You can describe part of a whole in 3 ways:

- As a fraction
- As a decimal
- As a percent

The hundredths grid below can help us demonstrate the three ways to describe part of a whole.



There are 75 squares shaded. There are 100 squares total.

We can easily write this as a fraction because it is $\frac{75}{100}$.

We can reduce this fraction by dividing the numerator and denominator by the same number.

$$\frac{75}{100} = \frac{75 \div 25}{100 \div 25} = \mathbf{\frac{3}{4}}$$

We can change this fraction to a decimal by dividing.

$$\frac{3}{4} = 3 \div 4 = \mathbf{0.75}$$

Percent means "per hundred", so we must change the denominator to 100 if it is not already 100.

$$\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 75\%$$

We could also could find the decimal and then multiply by 100 if a denominator of 100 isn't possible.

$$\frac{3}{4} = 3 \div 4 = 0.75 \times 100 = 75\%$$

<u>Practice:</u>

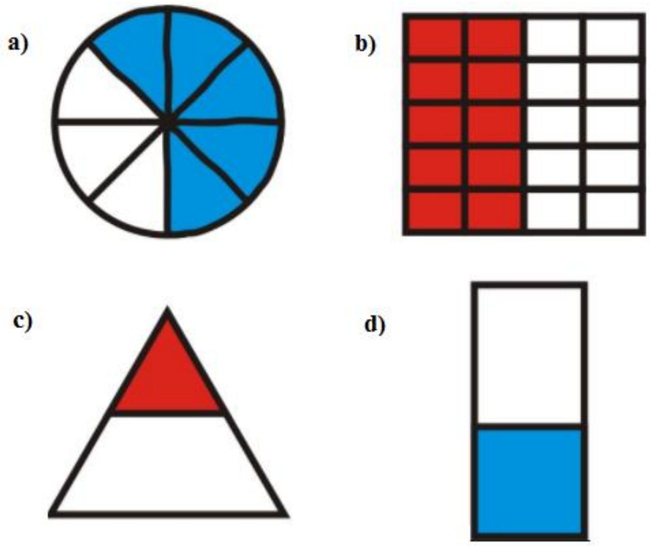
8. Complete the following chart.

Fraction	Decimal	Percent
Example: $\frac{4}{5}$	*0.8*	*80%*
$\frac{3}{8}$		
$\frac{3}{2}$		
	0.75	
	0.19	
		45%
		120%

9. Calculate:

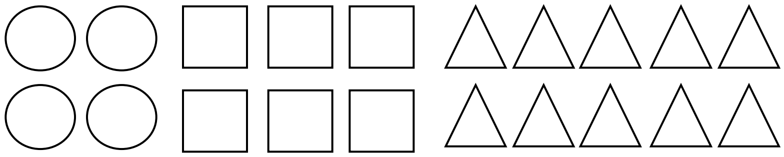
a) 15% of 20 b) 50% of 150 c) 200% of 50

10. Determine the percentage that is shaded in each diagram.



Ratios & Rates

This picture shows 4 circles, 6 squares, and 10 triangles.



Here are some ways you can use ratios, fractions, and percents to compare the shapes.

Example The ratio of *circles* to *all of the shapes* is: 4 to 20 or 4:20

This ratio can be written as the fraction $\frac{4}{20}$ or $\frac{1}{5}$.

It can also be written as a percent. $\frac{4}{20} = \frac{20}{100} = 20\%$ . . . . 20% of the shapes are circles.

Equivalent ratios are very similar to equivalent fractions.

For example, to find an equivalent fraction the numerator and denominator must be multiplied or divided by the same number.

$\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$ or $\frac{6}{10} = \frac{6 \times 2}{10 \times 2} = \frac{12}{20}$ We can conclude that $\frac{6}{10}$, $\frac{3}{5}$, and $\frac{12}{20}$ are all equivalent ratios

A **proportion** is a statement that two ratios are equal.

You can often solve a problem involving ratios by setting up a proportion.

Example

In a box of red and blue marbles, the ratio of red marbles to blue marbles is 3:4.

If there are 48 blue marbles, how many red marbles are there?

Let *r* represent the number of red marbles.

Then: $r : 48 = 3 : 4$

In fraction form: $\frac{r}{48} = \frac{3}{4}$

To find the value of *r*, first isolate *r* by multiplying each side of the proportion by 48.

$$48 \times \frac{r}{48} = \frac{3}{4} \times 48$$

$$r = \frac{144}{4}$$

$r = 36$ There are 36 red marbles.

A **rate** is a comparison of two quantities measured in different units.

Some examples: Apples are on sale for \$1.45 for 10.

There are 500 sheets in one roll of paper towel.

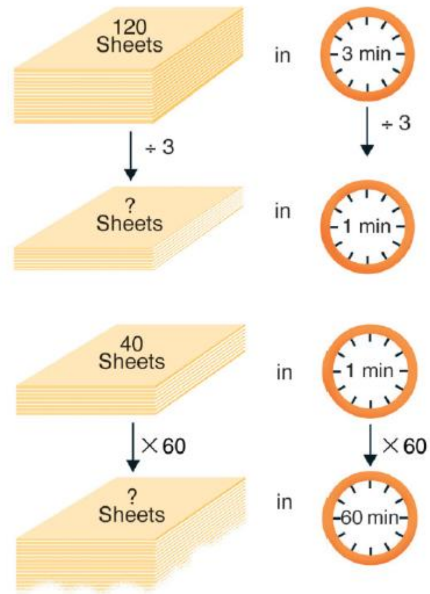
Stephanie earns \$5.00 an hour for baby-sitting.

The last two are **unit rates**. Each rate compares a quantity to 1 unit.

Example A printing press prints 120 sheets in 3 minutes.

a) Express the printing as a unit rate.

b) How many sheets are printed in 1 hour?



a) First write the rate.

(Use the idea of equivalent ratios to help find the unit rate.)

$$12sheets/3\min = \frac{12sheets/3\min}{3} = 40sheets/\min$$

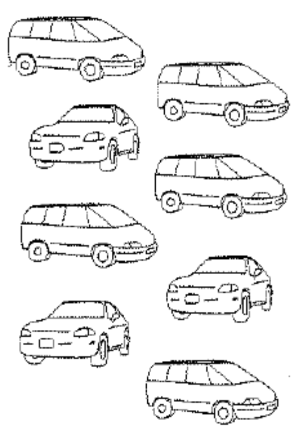
b) We know that there are $4sheets/\min$ printed. There is 60 minutes in 1 hour.

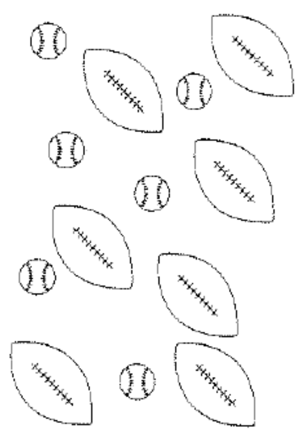
$$4sheets/\min = (4sheets \times 60/\min \times 60) = 2400sheets/hour$$

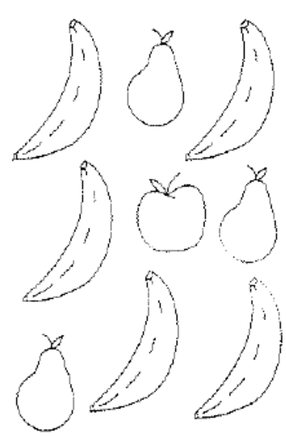
<u>Practice:</u>

11. Write each ratio.

a) cars to vans b) footballs to baseballs c) bananas to fruit







12) Write three equivalent ratios for 4:5.

13) Write 6:9 in simplest form.

14) In a recycle drive last week, Mr. Wiley's class collected bottles and recycled some of them. The ratio of bottles recycled to bottles collected was 3:4. This week, Mr. Wiley's class collected 24 bottles. Mr. Wiley told the students that the ratio of bottles recycled to bottles collected was the same as the previous week. How many bottles were recycled this week?

15) Find the value of the variable: $5 : x = 40 : 56$

16) Express each of the following as a *unit rate.*

a) Leo types 180 words in 3 minutes.

b) Mona can buy 4 packs of gum for $3.56.

Rules for Multiplying Integers

The product of two numbers with the same sign is positive.

The numbers can both be + or they can both be - .

Examples: (5)(2) = 10 (both integers are positive)
(-4)(-3) = 12 (both integers are negative)

The product of two numbers with different signs is negative.

One number will be positive, the other will be negative. It does not matter which one is larger.

Examples: (-8)(2) = – 16 (one positive, one negative)
(9) (-3) = – 27 (one positive, one negative)

When there are more than two numbers, count the negative signs:

- **an odd number of negatives will produce a negative product.**
- **an even number of negatives will produce a positive product.**

Examples: (-3) (-2) (2) (-1) = – 12 (three negatives = odd = –)
(-4) (5) (-2) = 40 (two negative = even = +)

Practice Problems:

17a) (3)(4) =

17b) (-2)(7) =

17c) (-1)(3)(-4) =

17d) (-5)(3) =

17e) (4)(-2)(-2)(-1) =

17f) (-6)(-3) =

17g) (-7)(-2) =

17h) (3)(-4)(0) =

17i) (-2)(-5)(-3) =

÷ Rules for Dividing Integers ÷

**** (Rules for division of signed numbers are the same as for multiplication)**

The quotient of two numbers with the same sign is positive.

The numbers can both be + or they can both be – .

Examples: 12 ÷ 3 = 4 (both positive)
– 24 ÷ (– 8) = 3 (both negative)

The product of two numbers with different signs is negative.

One number will be positive, the other will be negative. It does not matter which one is larger.

Examples: (– 18) ÷ (3) = – 6 (one is positive, one is negative)
(14) ÷ (– 2) = – 7 (one is positive, one is negative)

When there is a mixture of multiplication and division and there are more than two numbers, count the negative signs:

- **an odd number of negatives will produce a negative product.**
- **an even number of negatives will produce a positive product.**

Examples:

$$\frac{(3)(-4)}{2} = -6$$ (one negative = odd = –)

$$\frac{(-2)(-6)}{(-3)} = -4$$ (three negatives = odd = –)

$$\frac{(5)(-4)}{(-2)} = 10$$ (two negatives = even = +)

Practice Problems:

18a) (-4) ÷ (-2) =

18b) (8) ÷ (-2) =

18c) (-24) ÷ (3) =

18d) $\frac{35}{-7} =$

18e) $\frac{(-6)(-3)}{(-9)} =$

18f) $\frac{(2)(-14)}{(-7)} =$

Solve the following word problems using positive and negative numbers.

19) Steve has overdrawn his checking account by $27. His bank charged him $15 for an overdraft fee. Then he quickly deposited $100. What is his current balance?

20) Joe played golf with Sam on a special par 3 course. They played nine holes. The expected number of strokes on each hole was 3. A birdie is 1 below par. An eagle is 2 below par. A bogie is one above par. A double bogie is 2 above par. On nine holes Frank made par on 1 hole, got 2 birdies, one eagle, four bogies, and one double bogie. How many points above or below par was Franks score?

21) Find the difference in height between the top of a hill 973 feet high and a crack caused by an earthquake 79 feet below sea level.

22) In Detroit the high temperatures in degrees Fahrenheit for five days in January were -12°, -8°, -3°, 6°, -15°. What was the average temperature for these five days?

BEDMAS

23a) $18 + (-7) \bullet (32 - 6) =$

23b) $-19 - (-3) + -2(8 + -4) =$

23c) $20 + -4(3^2 - 6) =$

23d) $-3 + 2(-6 \div 3) =$

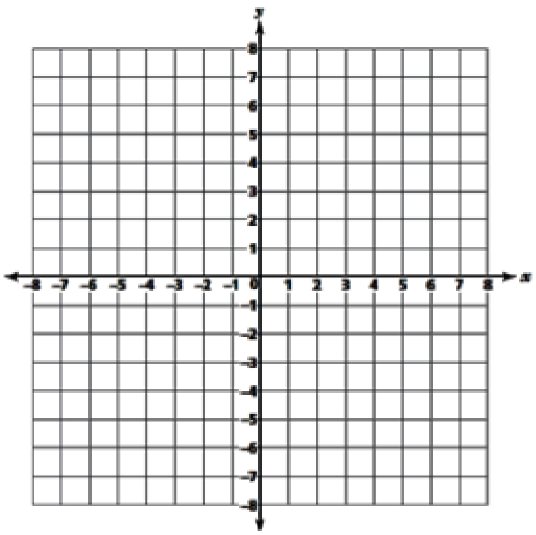
23e) $3 \bullet (-4) + (52 + -4 \bullet 2) - (-9) =$

23f) $2 + (-16) \div 4 \bullet 5 - (-3) =$

Graph and Analyze two-variable Linear Relations

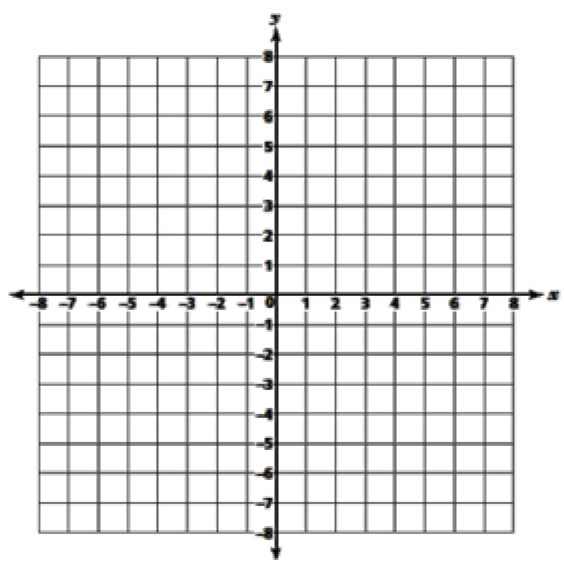
1 Complete the table for $y = x + 3$ and graph the resulting line.

x	y
-5	
0	
4	



2 Complete the table for $y = 3x + 1$ and graph the resulting line.

x	y
-3	
0	
2	



Linear Algebra

Transposing (or rearranging) equations is one of the most common mathematical skills you will use as a mathematician. You can also solve equations with a single variable using identical methods. This review offers a chance to practice these skills.

24) Solve the following equations (try rearranging the equations for x):

a. $5x = 8$

b. $5x + 3 = 8$

c. $\frac{x}{5} = 8$

d. $5x - 3 = -8$

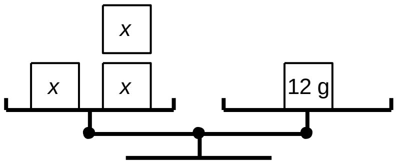
e. $5 - x = 8$

f. $\frac{5x + 3}{2} = 8$

g. $5(x + 2) = 25$

h. $2(2x + 10) = 40$

25) Use this balance-scales model to solve for x.



a. –4 b. 15 c. 9 d. 4

26) Solve this equation. $-15 + 3g = -6$

a. $g = -15$ b. $g = 3$ c. $g = -7$ d. $g = -2$

27) Solve this equation. $\frac{z}{-6} = -13$

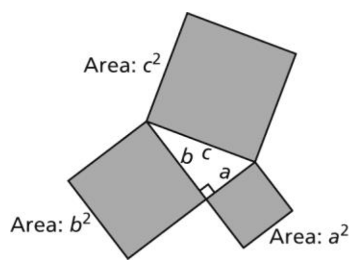
a. $z = -7$ b. $z = -19$ c. $z = 78$ d. $z = -78$

28). Solve this equation. $7 + \frac{d}{3} = 20$

a. 10 b. 39 c. –1 d. 53

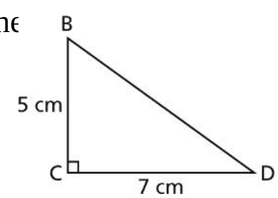
<u>The Pythagorean Theorem</u>

In a right triangle, the Pythagorean Theorem states that the square of the hypotenuse is equal to the sum of the squares of the legs.
The area model as shown is one way to prove this is true.



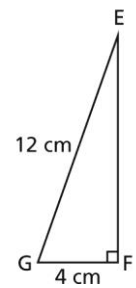
We write: $c^2 = a^2 + b^2$

- To determine the length of the hypotenuse when we know the lengths of the legs
  In ΔBCD, determine the length of BD to the nearest tenth of a centimetre.



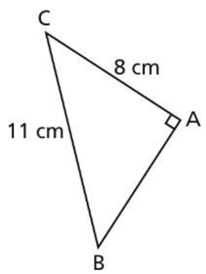
  Use the Pythagorean Theorem.
  $BD^2 = BC^2 + CD^2$ Substitute: BC = 5 and CD = 7
  $BD^2 = 5^2 + 7^2$
  $BD = \sqrt{5^2 + 7^2}$ Use a calculator.
  $= 8.6023\ldots$
  BD is about 8.6 cm long.

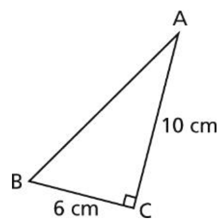
- To determine the length of a leg when we know the lengths of the other leg and the hypotenuse
  In ΔEFG, determine the length of EF to the nearest tenth of a centimetre.

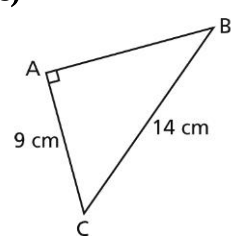


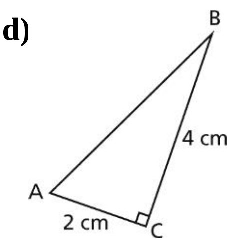
  Use the Pythagorean Theorem.
  $EG^2 = EF^2 + FG^2$ Substitute: EG = 12 and FG = 4
  $12^2 = EF^2 + 4^2$ Solve for EF.
  $EF^2 = 12^2 - 4^2$
  $EF = \sqrt{12^2 - 4^2}$
  $= 11.3137\ldots$
  EF is about 11.3 cm long.

29) Determine the length of each side AB to the nearest tenth of a centimetre.

a)


b)


c)


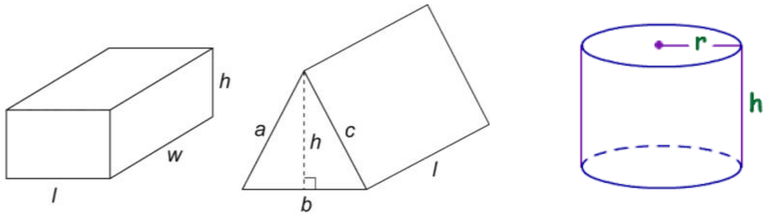
d)


<u>Real World Application: Technology Upgrade</u>

30) Your family wants to purchase a new laptop with a 20" widescreen. Since the 20 inches represents the diagonal measurement of the screen (upper corner to lower corner), you want to find out the actual dimensions of the laptop. When you measured the laptop at the store, the width was 16 inches, but you don't remember the height. Calculate and describe how you could figure out the height of the laptop to the nearest inch.

Surface Area and Volume of Prisms and Cylinders

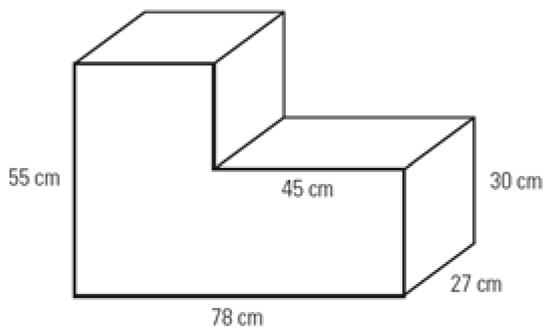
A prism has 2 congruent bases and is named for its bases. A rectangular prism has a rectangle base and a triangular prism has triangle bases at opposite ends. A cylinder has circular bases.



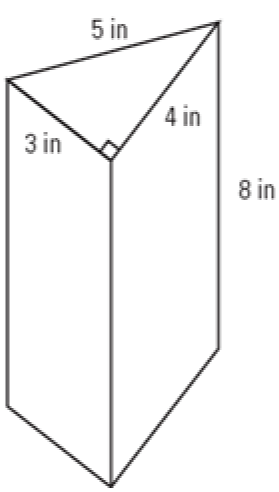
To find the ***surface area*** of a prism or cylinder, you can unfold all the sides to create a net. Find the area of each individual face and add them together to get the surface area of the whole shape.

To find the ***volume*** of a prism or cylinder,
$Volume = area\ of\ the\ base \times height$

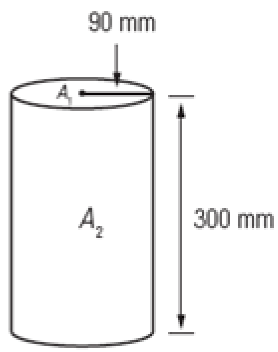
31) Find the Surface Area and of the Following Object



32) Find the Surface Area of the Following Triangular Prism

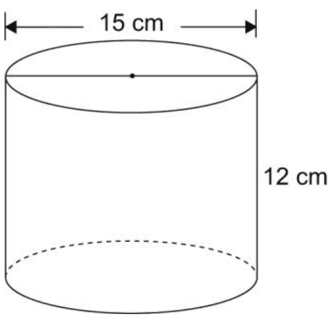


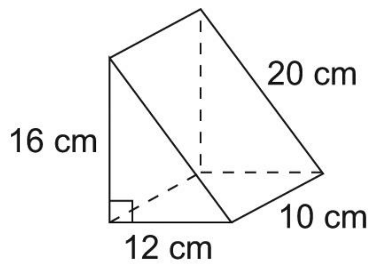
33) Find the Area of the Following Cylinder

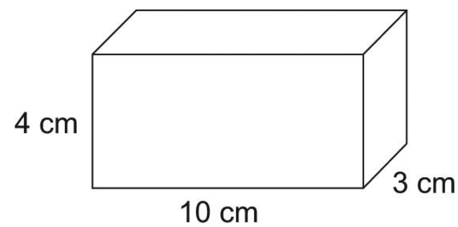


Practice:

34) Calculate the Surface Area and Volume of each shape to the nearest whole number:







Probability of Independent Events

Probability is a way of determining how likely something is to happen.

$$Probability = \frac{Number\ of\ ways\ something\ can\ happen}{total\ number\ of\ outcomes}$$

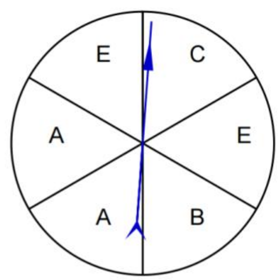
Probability can be written as a fraction, decimal, or a percent.

Two events are **independent events** when one event does not affect the other event. For example, tossing a coin will not affect which side a die lands on. To calculate the probability of two or more independent events, you have to multiply the individual event probabilities. You can use a tree diagram or table to list all the possible outcomes.

Practice:

35a) If you were to roll a 6 sided die one time what is the probability it will land on a 3?

35b) If you were to roll a 6 sided die one time what is the probability it will NOT land on a 2?



35c) If you were to roll a 6 sided die one time, what is the probability of it landing on an even number?

35d) What is the probability of the spinner landing on **C**?

35e) What is the probability of the spinner not landing on **E**?

35f) What is the probability of the die landing on 5 and the spinner landing on **B**?

35g) What is the probability of the die landing on an even number and the spinner landing on **E**?

35h) What is the probability of a green die landing on 4, a white die landing on 2, and the spinner landing on **A**?

Solutions:

1) 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144

2a) 5 2b) 11 2c) 50

2d) 800 2e) 0.62 2f) 0.09

2g) 1.4 2h) $\frac{5}{10} = \frac{1}{2}$ 2i) $\frac{8}{6} = \frac{4}{3}$

2j) $\sqrt{\frac{9}{4}} = \frac{3}{2}$

3a) $17 = \sqrt{289}$ 3b) $2.7 = \sqrt{7.29}$ 3c) $\frac{2}{11} = \sqrt{\frac{4}{121}}$

3d) $28.1 = \sqrt{789.61}$

4a) 36 - 49 4b) 81 – 100 4c) 100 – 121

5a) 6.2 5b) 9.6 5c) 10.7

7a) $\frac{3}{2} \times \frac{1}{5} = \frac{3\times1}{2\times5} = \frac{3}{10}$ 7b) $\frac{4}{5} \times \frac{1}{2} = \frac{4\times1}{5\times2} = \frac{4}{10} = \frac{2}{5}$ 7c) $\frac{5}{6} \times \frac{2}{3} = \frac{5\times2}{6\times3} = \frac{10}{18} = \frac{5}{9}$

7d) $\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2} = \frac{4\times3}{5\times2} = \frac{12}{10} = \frac{6}{5}$ 7e) $\frac{2}{5} \div \frac{1}{3} = \frac{2}{5} \times \frac{3}{1} = \frac{2\times3}{5\times1} = \frac{6}{5}$

7f) $\frac{3}{10} \div \frac{1}{3} = \frac{3}{10} \times \frac{3}{1} = \frac{3\times3}{10\times1} = \frac{9}{10}$ 7g) $2\frac{1}{3} \times 1\frac{2}{5} = \frac{7}{3} \times \frac{7}{5} = \frac{49}{15}$

7h) $1\frac{1}{2} \times \frac{1}{8} = \frac{3}{2} \times \frac{1}{8} = \frac{3}{16}$ 7i) $4\frac{1}{4} \div \frac{5}{6} = \frac{17}{4} \div \frac{5}{6} = \frac{17}{4} \times \frac{6}{5} = \frac{102}{20} = \frac{51}{10}$

7j) $2\frac{1}{2} \div 1\frac{3}{4} = \frac{5}{2} \div \frac{7}{4} = \frac{5}{2} \times \frac{4}{7} = \frac{20}{14} = \frac{10}{7}$

8)

Fraction	Decimal	Percent
Example: $\frac{4}{5}$	*0.8*	*80%*
$\frac{3}{8}$	0.375	37.5%
$\frac{3}{2}$	1.5	150%
$\frac{75}{100} = \frac{3}{4}$	0.75	75%
$\frac{19}{100}$	0.19	19%
$\frac{45}{100} = \frac{9}{20}$	0.45	45%
$\frac{120}{100} = \frac{6}{5}$	1.2	120%

9) a) 15% of 20 = 0.15 x 20 = 3 b) 50% of 150 = 0.5 x 150 =75

c) 200% of 50 = 2.00 x 50 = 100

10a) $\frac{5}{8} = 62.5\%$ 10b) $\frac{10}{20} = 50\%$ 10c) $\frac{1}{4} = 25\%$ 10d) $\frac{1}{2} = 50\%$

11a) 3:5 11b) 7:6 11c) 5:9

12) 4:5 = 8:10 = 12:15 = 16:20 = 20:25

13) 2:3

14) $\frac{3}{4} = \frac{?}{24}$? = 18

15) 5 : x = 40 : 56 $\frac{5}{x} = \frac{40}{56}$ $x = 7$

16a) $\frac{180\ words}{3\ minuted}$ = 180$\div 3 = 60\ words/min$ 16b) \$3.56 ÷ 4 = \$0.89

17a) 12 17b) -14 17c) 12 17d) -15 17e) -16 17f) 18

17g) 14 17h) 0 17i) -30

18a) 2 18b) -4 18c) -8 18d) -5 18e) -2 18f) 4

19) 58

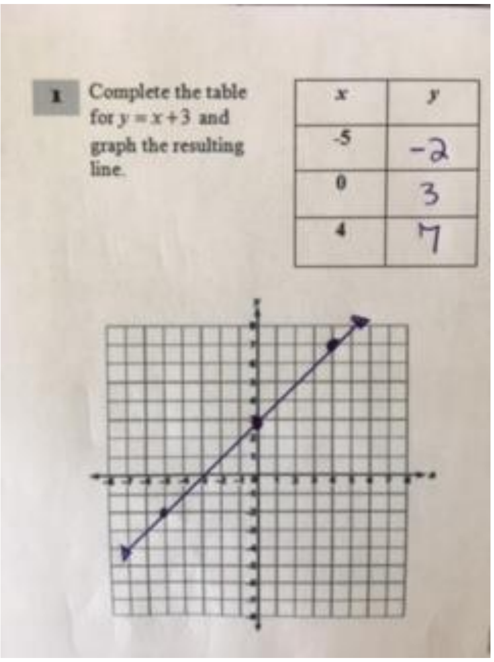
20) 2 above par

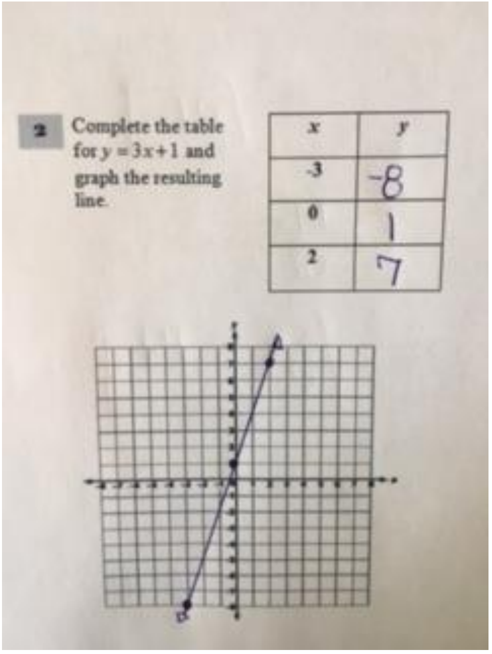
21) 1052 Feet Apart

22) -6.4° average

23a) -164 23b) -24 23c) 8 23d) -7 23e) 41 23f) -15

Graph and Analyze two-variable Linear Relations





24a) $x = \frac{8}{5}$ 24b) $x = 1$ 24c) $x = 40$ 24d) $x = -1$

24e) $x = -3$ 24f) $x = \frac{13}{5}$ 24g) $x = 3$ 24h) $x = 5$

25) d 26) b 27) c 28) b

29a) 7.5 cm 29b) 11.7 cm 29c) 10.7 cm 29d) 4.5 cm

30) 12 inches

31) 13512 cm^2 32) 108 in^2 33) 220540 mm^2

34) Cylinder Surface Area = 919 cm^2 Volume = 2121 cm^3
Triangular Prism Surface Area = 672 cm^2 Volume = 960 cm^3
Right Rectangular Prism Surface Area = 164 cm^2 Volume = 120 cm^3

35a) 1/6 35b) 5/6 35c) 1/2 35d) 1/6

35e) 1/3 35f) 1/36 35g) 1/6 35h) 2/216