Pearson Physics Level 20
Unit IV Oscillatory Motion and Mechanical Waves: Unit IV Review Solutions

Student Book pages 440–443

Vocabulary

1. **amplitude:** maximum displacement of an oscillation
antinodes: points of interaction between waves on a spring or other medium at which only constructive interference occurs; in a standing wave, antinodes occur at intervals of ½λ; in an interference pattern antinodes occur at path difference intervals of whole wavelengths
closed-pipe air column: an air column found in a pipe closed at one end; the longest wavelength that can resonate in a closed pipe is four times the length of the pipe
constructive interference: overlap of pulses to create a pulse of greater amplitude
crest: region where the medium rises above the equilibrium position
destructive interference: overlap of pulses to create a pulse of lesser amplitude
diverging: term applied to a ray that spreads out as it moves away from the origin
Doppler effect: apparent change in frequency and wavelength of a wave that is perceived by an observer moving relative to the source of the wave
equilibrium: term applied to the rest position or the position of a medium from which the amplitude of a wave can be measured
forced frequency: the frequency at which an external force is applied to a resonating object
frequency: the number of oscillations (cycles) per second
fundamental frequency: lowest frequency produced by a particular instrument; corresponds to the standing wave having a single antinode, with a node at each end of the string
Hooke's law: the deformation of an object is proportional to the force causing it
incident wave: wave front moving out from the point of origin toward a barrier
in phase: condition of crests or troughs from two waves occupying the same point in a medium; produces constructive interference
interference: effect of two pulses or waves crossing within a medium; the medium takes on a shape that is different from the shape of either pulse alone
longitudinal wave: wave with the motion of the medium being parallel to the motion of the wave
maximum: (line of antinodes) line of points linking antinodes that occur as the result of constructive interference between waves
mechanical resonance: the increase in amplitude of oscillation of a system as a result of a periodic force whose frequency is equal or very close to the resonant frequency of the system
medium: material, for example air or water through which waves travel; the medium does not travel with the wave
minimum: (nodal line) line of points lining nodes that occur as the result of destructive interference between waves

nodes or nodal points: point on a spring or other medium at which only destructive interference occurs; in a standing wave, nodes occur at intervals of ½λ; in an interference pattern, nodes occur at path difference intervals of ½λ
open-pipe air column: air column present in a pipe opened at both ends; the longest wavelength that can resonate in an open pipe is twice the length of the pipe
oscillation: a repetitive back-and-forth motion
oscillatory motion: motion that repeats at regular intervals
overtone: any frequency of vibration of a string that may exist simultaneously with the fundamental frequency
period: the time required for an object to make one complete oscillation (cycle)
phase shift: result of waves from one source having to travel farther to reach a particular point in the interference pattern than waves from another source
principle of superposition: displacement of the combined pulse at each point of interference is the sum of the displacements of the individual pulses
pulse: disturbance of short duration in a medium; usually seen as the crest or trough of a wave
ray: line that indicates only the direction of motion of the wave front at any point where the ray and the wave front intersect
reflected wave: wave front moving away from a barrier
resonance: increase in the amplitude of a wave due to a transfer of energy in phase with the natural frequency of the wave
resonant frequency: the natural frequency of vibration of an object
restoring force: a force acting opposite to the displacement to move an object back to its equilibrium position
shock wave: strong compression wave produced as an aircraft exceeds the speed of sound
simple harmonic motion: periodic motion where the restoring force is proportional to the displacement
simple harmonic oscillator: an object that moves with simple harmonic motion
sonic boom: the sound like and explosion, experienced by an observer, of the shock wave formed at the front of a plane travelling at or above the speed of sound
sound barrier: term applied to the increase in aerodynamic resistance as an aircraft approaches the speed of sound
spring constant: the stiffness of a spring (k)
standing wave: condition in a spring or other medium in which a wave seems to oscillate around stationary points called nodes; wavelength of a standing wave is the distance between alternate nodes or alternate antinodes
transverse wave: wave with the motion of the medium being perpendicular to the motion of the wave
trough: region where the medium is lower than the equilibrium position
two-point-source interference pattern: a pattern of maxima and minima that results from the interaction of waves emanating from two point sources that are in phase; the pattern results from the overlapping of crests and troughs while the waves move through each other
wave: disturbance that moves outward from its point of origin transferring energy through a medium by means of vibrations

wave front: imaginary line that joins all points reached by the wave at the same instant
wave train: series of waves forming a continuous series of crests and troughs
wave velocity: the rate of change in position of a wave moving outward from its point of origin
wavelength: distance between two points on a wave that have identical status; usually measured from crest to crest or from trough to trough

Knowledge

Chapter 7

2. Frequency and period are the inverse of one another. Frequency is the number of cycles per second, and period is the number of seconds per cycle.

3. Two other accepted units are:
- rpm: revolutions per minute
- cycles/s: cycles (or oscillations) per second

4. (a) The velocity is zero when the object is at its farthest displacement from the equilibrium position.
(b) The restoring force is greatest at its farthest displacement from the equilibrium position.

5. For amplitudes greater than 15°, a pendulum cannot be considered a simple harmonic oscillator because the restoring force no longer varies directly with its displacement.

6. The sign is necessary because the restoring force always acts in the opposite direction to the displacement.

7. To measure the spring constant of the wood, a method similar to extension question 10 in 7.2 Check and Reflect on page 365 of the Student Book could be used. A length of this wood could be fastened to a horizontal surface so it extends beyond the edge of the surface. A weight could be attached to this free end, and its displacement measured.
The equation $\vec{F} = k\vec{x}$ would be manipulated to solve for the spring constant: $k = \frac{\vec{F}}{\vec{x}}$.

8. The equation for the period of oscillation of a pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$. According to this equation, the period varies inversely with the square root of the gravitational field strength. This means that as gravity decreases, the period gets larger. At a higher altitude where gravity is less, the period will be longer. Therefore, the pendulum will oscillate more slowly in Jasper (higher altitude) than it would in Calgary.

9. The sound from a tuning fork is a result of it oscillating at its resonant frequency. If a similar tuning fork with the same resonant frequency were held nearby, the sound waves from the first tuning fork become the forced frequency that induces the second tuning fork to resonate.

10. (a) The length of the pendulum arm affects the resonant frequency of the pendulum.
(b) The acceleration of gravity affects the resonant frequency of a pendulum. Since Earth is not perfectly round, its gravitational field changes with latitude and so does the resonant frequency of a pendulum.
(c) The longitude of the pendulum's position will not affect its resonant frequency because the acceleration of gravity is not affected by longitudinal position.

(d) The elevation of the pendulum does affect its resonant frequency because the acceleration of gravity changes with elevation.

(e) The restoring force is a function of the pendulum bob's position from its equilibrium position. The amplitude of a pendulum's motion does not affect its resonant frequency so neither does the restoring force.

Chapter 8

11. In the upper spring there are 2½ wavelengths shown. In the lower spring there are 3½ wavelengths shown.

12. Reflection of a sound wave from a wall:

(a) does not affect the speed;

(b) does not affect the wavelength;

(c) does not affect the amplitude;

(d) directs the reflected wave away from the wall such that the angle between the incident wave and the wall is equal to the angle between the reflected wave and the wall.

13. In a longitudinal wave, the energy of the wave is stored in the compressions and rarefactions.

14. When a circular wave is reflected from a straight barrier, the shape of the wave is not affected; only its direction of motion changes.

15. The energy stored in a pulse depends on the amplitude of the pulse.

16. If waves move from a region of a given velocity to one where the velocity is reduced, then

(a) the frequency is unchanged,

(b) the wavelength is reduced (by the same factor as the speed) and

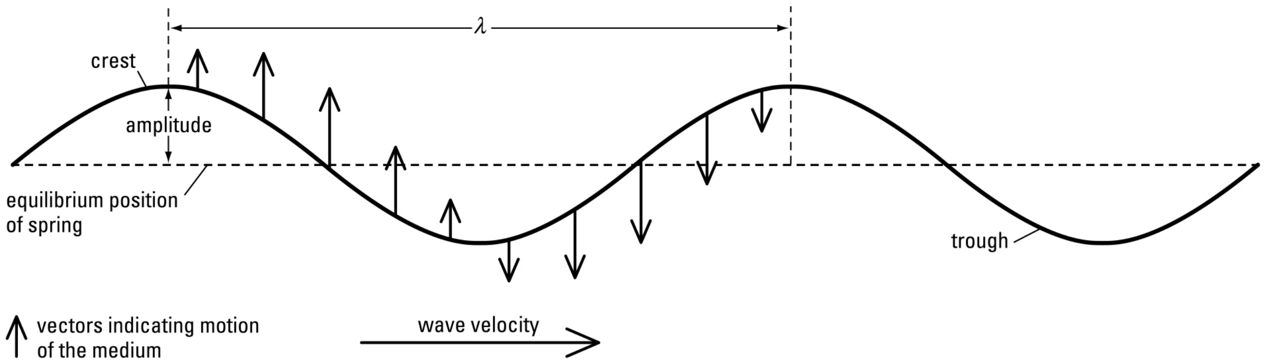
(c) the direction of the wave changes.

17. In the interference pattern created by two in-phase point sources the line along which destructive interference occurs is called a minimum or a nodal line.

18. The speed of a wave in a spring is decided by the tension in the spring and the mass of the spring.

19. When two wave trains of equal wavelength and amplitude move through each other in opposite directions they create a standing wave. Thus if a wave is generated at one end of a spring and reflected from the other end, when the reflected and incident waves pass through each other they will generate a standing wave. The antinodes and nodes of the standing wave are regions where, respectively, constructive and destructive interference occur. These regions occur at fixed points along the spring making it appear that the spring is oscillating back and forth in a wave pattern but no waves actually move along the spring.

20.



21. When you place your finger at various positions along a violin string, you effectively change the length of the string without changing the tension in the string. Thus the length of the longest standing wave that can be created in the string is also changed. Since the speed of the standing wave is the same (the tension of the string is not affected) the universal wave equation states that the fundamental frequency for the string is increased as the string is shortened.

22. If you strike a tuning fork with different forces you will alter the amplitude of the vibrations of its arms. This results in an increase in the volume of the sound produced by the tuning fork.

23. **(a)** The central maximum of an interference pattern created by two in-phase point sources is located at points where the waves have travelled equal distances from their sources. In geometry, the set of points equidistant from two fixed points is the perpendicular bisector of the line segment joining the fixed points. Since the waves started out in-phase and have travelled the same distance from their respective sources, they arrive in-phase, producing a series of points at which only constructive interference occurs.

(b) The second order minimum is produced along the set of points (a line) that are 1½ wavelengths farther from one source than the other. Since the waves began their respective journeys in-phase, the wave from the source that is farther away will arrive at these points 1½ wavelengths after the wave from the closer source. Thus, a crest from one source is always arriving at the same time as a trough from the other source. Hence, along this set of points the waves produce only destructive interference resulting in a line known as a minimum or nodal line.

24. For an open-pipe resonator there must be antinodes at each end of the pipe. The shortest system in which this can occur is when there is a crest (compression) at one end of the pipe and a trough (rarefaction) at the other end. This distance is equal to a ½ wavelength. Thus the longest wavelength that can be generated is equal to twice the length of the pipe.

25. If you were near the intersection, as the car (moving eastward) approached, you would hear the siren at a pitch that is higher than its true pitch, as well as hear the siren get louder. When the car turned north on the street in front of you, the pitch would suddenly get lower and the sound would start to become less loud as the car moved northward away from you. On the other hand, if you were some distance south of the intersection that the car was approaching, even though the siren would get louder as it approached the intersection, the pitch of the siren would appear to be the true pitch of the siren. This is because as the car moves eastward it is moving across your path rather than toward you. When the car turned north, you would hear the same decrease in pitch and loudness of the sound as described above.

26. According to the universal wave equation, the velocity of a wave is the product of its frequency and its wavelength.

27. As a sound moves toward you, the wavelengths are compressed by the motion of the source. That is, the distance between crests is reduced by the distance the source moves during one period of the wave's frequency. The universal wave equation states that if the speed of a wave is constant (as is true for this sound) then the frequency varies inversely as the wavelength. Since the wavelength is decreased by the motion of the source, the frequency is increased by the same factor that the wavelength is decreased.

Applications

28. ***Analysis and Solution***

$$\begin{aligned}\vec{F} &= k\vec{x} \\ &= \left(2.55\frac{\text{N}}{\cancel{\text{m}}}\right)\left(1.20\,\cancel{\text{m}}\right) \\ &= 3.06\,\text{N}\end{aligned}$$

The force necessary to stretch the spring 1.20 m is 3.06 N.

29. ***Given***

$f = 400.0\,\text{Hz}$

$k = 5.0 \times 10^4\ \text{N/m}$

Required

mass of the string (m)

Analysis and Solution

The string behaves as a simple harmonic oscillator, so you can determine its mass by changing its frequency to period, and applying the equation $T = 2\pi\sqrt{\frac{m}{k}}$.

$$\begin{aligned}T &= \frac{1}{f} \\ &= \frac{1}{400.0\,\text{Hz}} \\ &= 0.002\,500\,\text{s}\end{aligned}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\begin{aligned} m &= \frac{T^2 k}{4\pi^2} \\ &= \frac{\left(2.500\times10^{-3}\ \text{s}\right)^2\left(5.0\times10^{4}\ \frac{\text{N}}{\text{m}}\right)}{4\pi^2} \\ &= 7.9\times10^{-3}\ \text{kg} \\ &= 7.9\ \text{g} \end{aligned}$$

Paraphrase

The mass of the guitar string is 7.9 g.

30. ***Given***

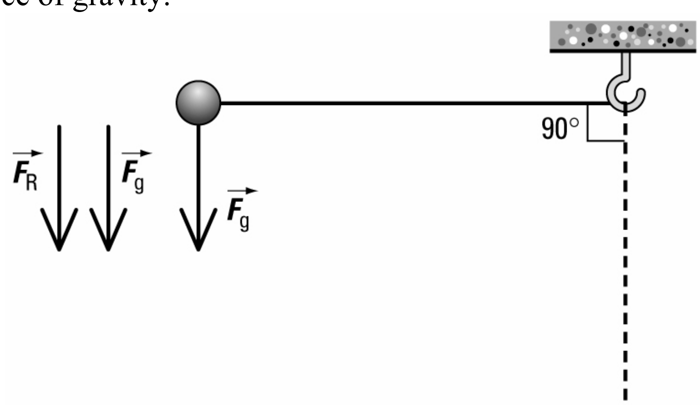
$\theta = 90.0°$

Required

proportion of the force of gravity that is the restoring force

Analysis and Solution

Determine the restoring force for the angle given. Then create a simple ratio with the force of gravity.



$$\begin{aligned} F_{g\perp} &= F_g \sin\theta \\ &= F_g \sin 90° \\ &= F_g (1) \\ &= F_g \end{aligned}$$

This equation shows that the restoring force is equal to the force of gravity.

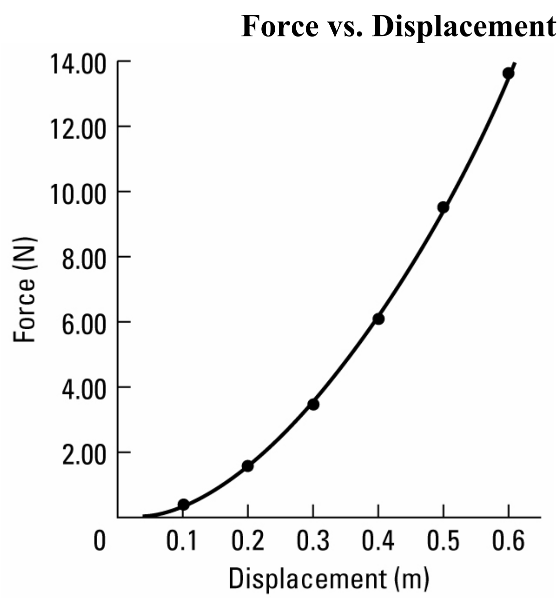
Paraphrase

The entire force of gravity is the restoring force.

31. ***Given***

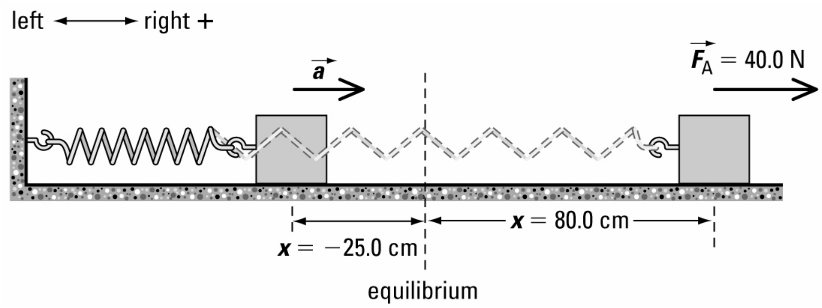
Displacement (m)	Force (N)
0.1	0.38
0.2	1.52
0.3	3.42
0.4	6.08
0.5	9.5
0.6	13.68

Required
spring constant (k)
Analysis and Solution
If the elastic obeys Hooke's law, then a force-displacement graph should be linear, and you can determine the spring constant.



The graph is not linear for the displacement of the elastic band.
Paraphrase
The elastic band does not obey Hooke's law as the force-displacement graph is not linear. You cannot determine a spring constant.

32. ***Given***
$\vec{F} = 40.0$ N
$\vec{x} = 80.0$ cm
Required
acceleration of the mass when the displacement is –25.0 cm (a)
Analysis and Solution
Determine the spring constant first. Then use Hooke's law determine the acceleration at the displacement of –25.0 cm. Remember to convert all measurements to appropriate SI units.



$$\vec{F} = k\vec{x}$$

$$k = \frac{\vec{F}}{\vec{x}} = \frac{40.0\,\text{N}}{0.800\,\text{m}} = 50.0\,\text{N/m}$$

$$\vec{F} = -k\vec{x}$$

$$m\vec{a} = -k\vec{x}$$

$$\vec{a} = \frac{-k\vec{x}}{m} = \frac{-\left(50.0\frac{\text{N}}{\text{m}}\right)(-0.25\,\text{m})}{10.0\,\text{kg}} = +1.25\,\text{m/s}^2$$

Paraphrase

The acceleration of the mass is $+1.25\,\text{m/s}^2$.

33.

Displacement (cm)	Force (mN)
2.5	10.0
5.0	21.0
7.5	31.0
10	39.0
12.5	49.0

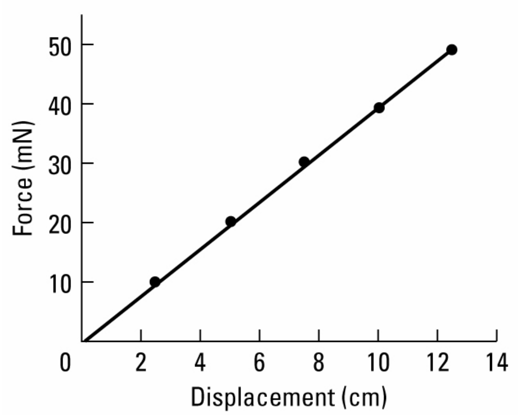
Required

spring constant (k)

Analysis and Solution

If the spring obeys Hooke's law, then a force-displacement graph should be linear, and you can determine the spring constant.

Force vs. Displacement



The graph is linear, so you can determine the slope.

$$
\begin{aligned}
k &= slope \\
&= \frac{\Delta y}{\Delta x} \\
&= \frac{(47.5 - 0.0)\,\text{mN}}{(0.12 - 0.0)\,\text{m}} \\
&= 4.0 \text{ mN/m}
\end{aligned}
$$

Paraphrase

The spring constant of the spring is 4.0 mN/m.

34. (a) ***Given***

$m = 50.0$ g

$k = 25.0$ N/m

$\vec{a} = 50.0 \text{ m/s}^2$

Required

amplitude of vibration (A)

Analysis and Solution

The amplitude is the maximum displacement. At this position, the mass experiences maximum acceleration. Since you know this value, you can determine the amplitude from Hooke's law.

If the acceleration is at its maximum, then the mass is at its maximum displacement.

$\vec{x} = A$

So,

$\vec{F} = -kA$

$m\vec{a} = -kA$

$$A = \frac{m\vec{a}}{-k}$$

$$= \frac{(0.0500\,\text{kg})\left(50.0\,\frac{\text{m}}{\text{s}^2}\right)}{-25.0\,\frac{\text{N}}{\text{m}}}$$

$$= -0.100\ \text{m}$$

Paraphrase

The amplitude refers to the maximum displacement and is a scalar quantity, so it is 0.100 m.

(b) ***Given***

m = 50.0 g

k = 25.0 N/m

$\vec{a}$ = 50.0 m/s^2

Required

maximum velocity of the mass (v_{max})

Analysis and Solution

Determine the maximum velocity using the amplitude you calculated in part (a).

$$v_{\text{max}} = A\sqrt{\frac{k}{m}}$$

$$= 0.100\,\text{m}\sqrt{\frac{25.0\,\frac{\text{N}}{\text{m}}}{0.0500\,\text{kg}}}$$

$$= 2.24\ \text{m/s}$$

Paraphrase

The maximum velocity of the mass is 2.24 m/s.

35. ***Given***

m = 1.0×10^{-5} kg

$T = 4.5 \times 10^{-3}$ s

$A = 1.10\,\text{cm}$

Required

maximum wing speed (v_{max})

Analysis and Solution

Determine the maximum wing speed by first determining its spring constant k. Then apply the equation for maximum velocity. Remember to convert all measurements to appropriate SI units.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\begin{aligned} k &= \frac{4\pi^2 m}{T^2} \\ &= \frac{4\pi^2\left(1.0\times10^{-5}\ \text{kg}\right)}{\left(4.5\times10^{-3}\ \text{s}\right)^2} \\ &= 19.496\ \text{N/m} \end{aligned}$$

$$\begin{aligned} v_{\text{max}} &= A\sqrt{\frac{k}{m}} \\ &= 0.0110\ \text{m}\sqrt{\frac{19.496\dfrac{\text{N}}{\text{m}}}{1.0\times10^{-5}\ \text{kg}}} \\ &= 15.4\ \text{m/s} \end{aligned}$$

Paraphrase

The maximum wing speed of the bee is 15 m/s.

36. ***Given***

m = 10.0 t

A = 1.50 m

$v_{\text{max}} = 1.40\ \text{m/s}$

Required

period of the damper's oscillations (T)

Analysis and Solution

To find the period, determine the spring constant first by using the equation for maximum speed. Then use the spring constant in the period equation.

$$v_{\text{max}} = A\sqrt{\frac{k}{m}}$$

$$\begin{aligned} k &= \frac{v_{\text{max}}^2 m}{A^2} \\ &= \frac{\left(1.40\dfrac{\text{m}}{\text{s}}\right)^2 (1.00\times10^4\ \text{kg})}{(1.50\ \text{m})^2} \\ &= 8711.1\ \text{N/m} \end{aligned}$$

$$\begin{aligned} T &= 2\pi\sqrt{\frac{m}{k}} \\ &= 2\pi\sqrt{\frac{1.0\times10^4\ \text{kg}}{8711.1\dfrac{\text{N}}{\text{m}}}} \\ &= 6.73\ \text{s} \end{aligned}$$

Paraphrase

The period of the damper is 6.73 s.

37. ***Given***

$A = 0.80\text{ m}$

$\vec{x} = 0.60\text{ m}$

$v_{\text{max}} = 1.5\text{ m/s}$

Required

speed of the branch when its displacement is 0.60 m (v)

Analysis and Solution

You can determine the speed of any simple harmonic oscillator using the law of conservation of energy.

The total energy of a simple harmonic oscillator is the sum of the kinetic and potential energy of the oscillator in any position. Therefore, the kinetic energy in any position is the total energy minus the potential energy in that position. Furthermore, the total energy is the same as the maximum potential energy (when the object is at its maximum displacement). Therefore:

$E_{\text{k}} = E_{\text{p}_{\text{max}}} - E_{\text{p}}$

You can use this equality to write an expression that does not require the mass or spring constant to find the speed of the oscillator. Here is the derivation:

$$
\begin{aligned}
E_{\text{k}} &= E_{\text{p}_{\text{max}}} - E_{\text{p}} \\
\frac{mv^2}{\cancel{2}} &= \frac{kA^2}{\cancel{2}} - \frac{kx^2}{\cancel{2}} \\
mv^2 &= k(A^2 - x^2) \\
v^2 &= \frac{k}{m}(A^2 - x^2) \\
&= \frac{k}{m}\left(A^2 - \frac{A^2x^2}{A^2}\right) \\
&= \frac{A^2k}{m}\left(1 - \frac{x^2}{A^2}\right) \quad \text{since } v_{\text{max}}^2 = \frac{A^2k}{m} \\
&= v_{\text{max}}^2\left(1 - \frac{x^2}{A^2}\right) \\
v &= v_{\text{max}}\sqrt{1 - \frac{x^2}{A^2}}
\end{aligned}
$$

Now substitute the values into the equation.

$$
\begin{aligned}
v &= 1.5\ \frac{\text{m}}{\text{s}}\sqrt{1 - \frac{0.60\,\text{m}}{0.80\,\text{m}}} \\
&= 0.75\text{ m/s}
\end{aligned}
$$

Analysis and Solution

The speed of the branch is 0.75 m/s when it is displaced 0.60 m from the equilibrium position.

38. ***Given***

$f = 0.125$ Hz

Required

length of cable in the mass damper to match the resonance of the building (l)

Analysis and Solution

The damper's frequency must match the resonant frequency of the building to dampen it. Convert the frequency to a
period and use that to solve for the length of the cable.

$$\begin{aligned} T &= \frac{1}{f} \\ &= \frac{1}{0.125\,\text{Hz}} \\ &= 8.00\,\text{s} \end{aligned}$$

$$\begin{aligned} T &= 2\pi\sqrt{\frac{l}{g}} \\ l &= \frac{T^2 g}{4\pi^2} \\ &= \frac{\left(8.00\,\cancel{\text{s}}\right)^2\left(9.81\,\frac{\text{m}}{\cancel{\text{s}^2}}\right)}{4\pi^2} \\ &= 15.9\,\text{m} \end{aligned}$$

Paraphrase

The length of the pendulum damper cable must be 15.9 m.

39. When a wave slows down, the property of the wave that is not changed is its frequency. The universal wave equation states that when the frequency is constant the wavelength varies directly as the velocity. Thus the decrease in speed results in a proportionate decrease in wavelength. [**NOTE:** Even though it is not a specified outcome in the curriculum, you might mention to the students that the change in speed also results in a partial reflection of the wave. This means that the amplitude of the wave that continues on at a reduced speed must be less than that of the incident wave since some of its energy has been reflected away. This concept will be dealt with in more detail in the section on light in Physics 30.]

40. Because of the elastic nature of the medium, the wave sets up a series of motions that move, in sequence, through the medium, with each subsequent segment of the medium repeating the motion of the previous segment. The energy is carried through the medium by this sequence of motions. After the energy passes, the segments will be approximately in their original positions. The energy has moved through the medium leaving the medium, for the most part, unchanged.

41. ***Given***

$v = 3.00 \times 10^{8}\ \frac{\text{m}}{\text{s}}$

$\lambda_1 = 4.30 \times 10^{-7}\ \text{m}$

$\lambda_2 = 7.50 \times 10^{-7}\ \text{m}$

Required

frequency range from the given range of wavelengths.

Analysis and Solution

For each wavelength, the frequency can be calculated using the universal wave equation.

$$v = f\lambda$$

$$\begin{aligned} f_1 &= \frac{v}{\lambda_1} \\ &= \frac{3.00 \times 10^{8}\ \frac{\text{m}}{\text{s}}}{4.30 \times 10^{-7}\ \text{m}} \\ &= 6.976 \times 10^{14}\ \frac{1}{\text{s}} \\ &= 6.98 \times 10^{14}\ \text{Hz} \end{aligned}$$

$$\begin{aligned} f_2 &= \frac{v}{\lambda_2} \\ &= \frac{3.00 \times 10^{8}\ \frac{\text{m}}{\text{s}}}{7.50 \times 10^{-7}\ \text{m}} \\ &= 4.00 \times 10^{14}\ \frac{1}{\text{s}} \\ &= 4.00 \times 10^{14}\ \text{Hz} \end{aligned}$$

Paraphrase

For visible light, the range of frequencies is from 4.00×10^{14} Hz to 6.98×10^{14} Hz.

42. ***Given***

$v = 3.00 \times 10^{8}\ \frac{\text{m}}{\text{s}}$

$$\begin{aligned} f &= 1250\ \text{kHz} \\ &= 1.25 \times 10^{6}\ \text{Hz} \end{aligned}$$

Required

wavelength for the radio waves

Analysis and Solution

Use the universal wave equation.

$$v = f\lambda$$

$$\lambda = \frac{v}{f}$$

$$= \frac{3.00 \times 10^8 \ \frac{\text{m}}{\text{s}}}{1.25 \times 10^6 \ \frac{1}{\text{s}}}$$

$$= 2.40 \times 10^2 \ \text{m}$$

Paraphrase

Radio waves with a frequency of 1250 kHz have a wavelength of 240 m.

43. ***Given***

$$T = 0.350 \text{ s}$$

$$v = 0.840 \ \frac{\text{m}}{\text{s}}$$

Required

wavelength of the sine curve that the pendulum traces on the paper.

Analysis and Solution

Calculate the frequency of the pendulum; then use the universal wave equation to calculate the wavelength of the sine curve.

$$f = \frac{1}{T}$$

$$= \frac{1}{0.350 \text{ s}}$$

$$= 2.857 \ \frac{1}{\text{s}}$$

$$= 2.86 \text{ Hz}$$

$$v = f\lambda$$

$$\lambda = \frac{v}{f}$$

$$= \frac{0.840 \ \frac{\text{m}}{\text{s}}}{2.857 \ \frac{1}{\text{s}}}$$

$$= 0.294 \text{ m}$$

A more succinct solution would be:

$$v = f\lambda$$

$$\lambda = \frac{v}{f}$$

$$= \frac{v}{1/T}$$

$$= vT$$

$$= \left(0.840 \ \frac{\text{m}}{\text{s}}\right)(0.350 \text{ s})$$

$$= 0.294 \text{ m}$$

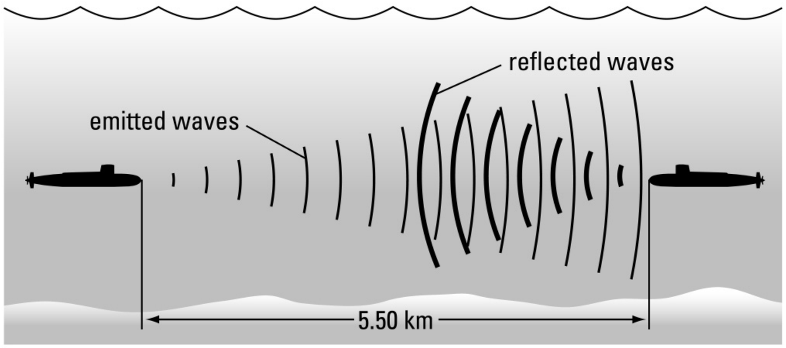
Paraphrase
The wavelength of the sine curve traced by the pendulum is 0.294 m.

44. ***Given***

$f = 545$ Hz

$\lambda = 2.60$ m

$$\begin{aligned}\Delta d &= 2(5.50\,\text{km}) \\ &= 11.0\text{ km}\end{aligned}$$



Required
time required for the sound to travel a distance of 5.50 km and back.

Analysis and Solution
First use the universal wave equation to calculate the speed of the sound under water. Then use the relationship for distance, speed, and time to calculate the time.

$$\begin{aligned}v &= f\lambda \\ &= \left(545\ \frac{1}{\text{s}}\right)(2.60\text{ m}) \\ &= 1.417\times10^3\ \frac{\text{m}}{\text{s}} \\ &= 1.42\times10^3\ \frac{\text{m}}{\text{s}}\end{aligned}$$

$$\begin{aligned}\Delta d &= v\Delta t \\ \Delta t &= \frac{\Delta d}{v} \\ &= \frac{1.10\times10^4\text{ m}}{1.417\times10^3\ \frac{\text{m}}{\text{s}}} \\ &= 7.762\text{ s} \\ &= 7.76\text{ s}\end{aligned}$$

Paraphrase
The sound travels at 1.42×10^3 m/s so that it takes 7.76 s for it to travel 11.0 km.

45. ***Given***

$L = 3.00 \text{ m}$

$f = 480 \text{ Hz}$

$= 480 \ \frac{1}{\text{s}}$

$n = 24$

Required

speed of the wave in the wire

Analysis and Solution

Since each wavelength contains two antinodes, the 24 antinodes mean that there are 12 wavelengths along the wire. Divide the length of the wire by 12 to find the wavelength of the standing wave. Use the wavelength and the frequency to calculate the speed of the wave in the wire.

$$\begin{aligned}\lambda &= 2\frac{L}{n}\\ &= 2\frac{3.00 \text{ m}}{(24)}\\ &= 0.250 \text{ m}\end{aligned}$$

$$\begin{aligned}v &= f\lambda\\ &= \left(480 \ \frac{1}{\text{s}}\right)(0.250 \text{ m})\\ &= 120 \ \frac{\text{m}}{\text{s}}\end{aligned}$$

Paraphrase

The speed at which a wave moves along the wire is 120 m/s.

46. ***Given***

$L = 5.40 \text{ m}$

$v = 3.00 \ \frac{\text{m}}{\text{s}}$

(a) $f = 2.50 \text{ Hz}$

$= 2.50 \ \frac{1}{\text{s}}$

Required

(a) number of antinodes and nodes in the standing wave.

(b) next lower frequency for which a standing wave would exist.

Analysis and Solution

(a) Calculate the wavelength of the standing wave. Divide the length of the spring by the wavelength to calculate the number of wavelengths along the spring. Multiply the number of wavelengths by two to give the number of antinodes. The number of nodes is one greater than the number of antinodes.

$$v = f\lambda$$

$$\lambda = \frac{v}{f}$$

$$= \frac{3.00\ \frac{\text{m}}{\text{s}}}{2.50\ \frac{1}{\text{s}}}$$

$$= 1.20\ \text{m}$$

Number of antinodes:

$$n = 2\frac{L}{\lambda}$$

$$= 2\left(\frac{5.40\ \text{m}}{1.20\ \text{m}}\right)$$

$$= 9$$

Number of nodes:

$n + 1 = 10$

(b) As the frequency increases, the number of antinodes in a standing wave also increases. Thus, the standing wave for the next lower frequency has one less node. Reduce the number of nodes by one; calculate the new wavelength and the new frequency. The speed of the wave in the spring is the same.

Number of nodes at the next lower frequency = 8

$$n = 8$$

$$n = 2\left(\frac{L}{\lambda}\right)$$

$$\lambda = \frac{2L}{n}$$

$$= \frac{2(5.40\ \text{m})}{8}$$

$$= 1.35\ \text{m}$$

$$v = f\lambda$$

$$f = \frac{v}{\lambda}$$

$$= \frac{3.00\ \frac{\text{m}}{\text{s}}}{1.35\ \text{m}}$$

$$= 2.222\ \frac{1}{\text{s}}$$

$$= 2.22\ \text{Hz}$$

Alternative argument:

Since the speed of the wave in the spring is constant, the frequency varies inversely as the wavelength. The wavelength varies directly inversely as the number of nodes. Thus the frequency varies directly as the number of nodes.

Hence,

$$\frac{f_2}{f_1} = \frac{n_2}{n_1}$$

$$f_2 = f_1 \frac{n_2}{n_1}$$
$$= 2.50 \text{ Hz}\left(\frac{8}{9}\right)$$
$$= 2.22 \text{ Hz}$$

Paraphrase

(a) The standing wave contains 9 antinodes and 10 nodes.

(b) To decrease the number of antinodes by one the frequency must decrease to 2.22 Hz.

47. ***Given***

$$f_1 = 293 \text{ Hz}$$
$$= 293 \ \frac{1}{\text{s}}$$
$$L_1 = 33.0 \text{ cm}$$
$$= 0.330 \text{ m}$$
$$L_2 = \tfrac{2}{3} L_1$$

Required

(a) speed of the wave in the string.

(b) frequency when the string is shortened to 2/3 its original length.

Analysis and Solution

(a) For the fundamental frequency the string is ½ wavelength long. Multiply the length of the string by two to give the wavelength. Use the wavelength and the frequency to find the speed using the universal wave equation.

$$\lambda_1 = 2L_1$$
$$= 2(0.330 \text{ m})$$
$$= 0.660 \text{ m}$$
$$v = f_1 \lambda_1$$
$$= \left(293 \ \frac{1}{\text{s}}\right)(0.660 \text{ m})$$
$$= 193.4 \ \frac{\text{m}}{\text{s}}$$
$$= 193 \ \frac{\text{m}}{\text{s}}$$

(b) Calculate the new length of the string, then double that length to give the wavelength. Since the speed of the wave in the spring is unchanged when you press the string at a new position, use the wave speed calculated in (a) to find the new frequency.

$$
\begin{aligned}
L_2 &= \frac{2}{3}L_1 \\
&= \frac{2}{3}(0.330 \text{ m}) \\
&= 0.220 \text{ m} \\
\lambda_2 &= 2L_2 \\
&= 2(0.220 \text{ m}) \\
&= 0.440 \text{ m} \\
v &= f_2\lambda_2 \\
f_2 &= \frac{v}{\lambda_2} \\
&= \frac{193.4 \ \frac{\text{m}}{\text{s}}}{0.440 \text{ m}} \\
&= 439.5 \ \frac{1}{\text{s}} \\
&= 440 \text{ Hz}
\end{aligned}
$$

Paraphrase

(a) The speed of the wave in the string is 193 m/s.

(b) When the string is shortened to 2/3 its original length the frequency increases from 293 Hz to 440 Hz.

48. ***Given***

$$
\begin{aligned}
f &= 154 \text{ Hz} \\
&= 154 \ \frac{1}{\text{s}} \\
v &= 340 \ \frac{\text{m}}{\text{s}}
\end{aligned}
$$

Required

(a) length of the shortest closed pipe for which resonance would occur

(b) length of the next longer closed pipe for which resonance would occur

Analysis and Solution

(a) The shortest closed pipe that produces resonance for a given frequency is ¼ the wavelength. Calculate the wavelength using the universal wave equation then divide the answer by four.

$$
\begin{aligned}
v &= f\lambda \\
\lambda &= \frac{v}{f} \\
&= \frac{340 \ \frac{\text{m}}{\text{s}}}{154 \ \frac{1}{\text{s}}} \\
&= 2.208 \text{ m} \\
&= 2.21 \text{ m}
\end{aligned}
$$

$$\begin{aligned} L_1 &= \frac{1}{4}\lambda \\ &= \frac{1}{4}(2.208\text{ m}) \\ &= 0.5519\text{ m} \\ &= 0.552\text{ m} \end{aligned}$$

(b) The next-longer closed pipe for which resonance occurs is ¾-wavelength long. Multiply the wavelength from part (a) by ¾ to find the pipe length.

$$\begin{aligned} L_2 &= \frac{3}{4}\lambda \\ &= \frac{3}{4}(2.208\text{ m}) \\ &= 1.656\text{ m} \\ &= 1.66\text{ m} \end{aligned}$$

Paraphrase

(a) The shortest closed pipe for which resonance would occur is 0.552 m (55.2 cm) long.

(b) The next-longer closed pipe resonator for this frequency would be 1.66 m long.

49. ***Given***

$f_s = 875\text{ Hz}$

$f_d = 870\text{ Hz}$

$v_w = 1500\ \frac{\text{m}}{\text{s}}$

Required

velocity of the source (submarine)

Analysis and Solution

Since your measurement of the frequency for the submarine's sonar is lower than its true frequency, the submarine must be moving away from you. Using the Doppler effect equation for an object moving away from you, isolate the variable for the speed of the source and solve.

$$f_d = \left(\frac{v_w}{v_w + v_s}\right) f_s$$

$$\begin{aligned} v_s &= \left(\frac{f_s - f_d}{f_d}\right) v_w \\ &= \left(\frac{875\text{ Hz} - 870\text{ Hz}}{870\text{ Hz}}\right) 1500\ \frac{\text{m}}{\text{s}} \\ &= 8.620\ \frac{\text{m}}{\text{s}} \\ &= 8.62\ \frac{\text{m}}{\text{s}} \end{aligned}$$

Paraphrase

The submarine is moving away from you at 8.62 m/s (~31 km/h).

50. ***Given***

$$v_s = 144\ \frac{\text{km}}{\text{h}}$$
$$= \left(1.44\times10^5\ \frac{\text{m}}{\text{h}}\right)\left(\frac{1\ \text{h}}{3.6\times10^3\ \text{s}}\right)$$
$$= 40.0\ \frac{\text{m}}{\text{s}}$$
$$f = 1.12\times10^3\ \text{Hz}$$
$$= 1.12\times10^3\ \frac{1}{\text{s}}$$
$$v_w = 320\ \frac{\text{m}}{\text{s}}$$

Required

(a) apparent frequency of the siren when the car is moving toward you

(b) apparent frequency of the siren when the car is moving away from you.

Analysis and Solution

(a) Use the equation for the Doppler effect for an object moving toward you.

$$f_d = \left(\frac{v_w}{v_w - v_s}\right)f_s$$
$$= \left(\frac{320\ \frac{\text{m}}{\text{s}}}{320\ \frac{\text{m}}{\text{s}} - 40.0\ \frac{\text{m}}{\text{s}}}\right)1.12\times10^3\ \frac{1}{\text{s}}$$
$$= 1.280\times10^3\ \frac{1}{\text{s}}$$
$$= 1.28\times10^3\ \text{Hz}$$

(b) Use the equation for the Doppler effect for an object moving away from you.

$$f_d = \left(\frac{v_w}{v_w + v_s}\right)f_s$$
$$= \left(\frac{320\ \frac{\text{m}}{\text{s}}}{320\ \frac{\text{m}}{\text{s}} + 40.0\ \frac{\text{m}}{\text{s}}}\right)1.12\times10^3\ \frac{1}{\text{s}}$$
$$= 9.955\times10^2\ \frac{1}{\text{s}}$$
$$= 9.96\times10^2\ \text{Hz}$$

Paraphrase

(a) When car is approaching you, you hear its siren at a frequency of 1280 Hz rather than 1120 Hz.

(b) When car is receding from you, you hear its siren at a frequency of 996 Hz rather than 1120 Hz.

Extensions

51. The frequency of a mass-spring system is inversely related to the square root of the mass.

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad T = \frac{1}{f}$$

$$\frac{1}{f} = 2\pi\sqrt{\frac{m}{k}} \qquad \text{or}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \text{or} \quad f \propto \sqrt{\frac{1}{m}}$$

If the mass gets larger, the frequency gets smaller. Conversely, if the mass gets smaller, the frequency gets larger.

52. ***Given***

$f = 3.27$ s

$l = 1.0$ m

Required

gravitational field strength (g)

Analysis and Solution

A pendulum's frequency depends on the gravitational field strength. Determine the gravitational field strength of the planet the alien landed on from the frequency given so you can compare it with the gravitational field strengths of Mercury, Venus, and Earth. You know that Earth's gravitational field strength is 9.81 m/s^2, so you need to determine the gravitational field strength for Mercury and Venus.

Mercury's gravitational field strength can be determined using Newton's equation from chapter 4:

$$\vec{g} = \frac{Gm}{r^2}$$

Mercury's radius, $r_{\text{M}} = 2.44 \times 10^{6}\ \text{m}$

Mercury's mass, $m_{\text{M}} = 3.30 \times 10^{23}\ \text{kg}$

$$\begin{aligned}\vec{g}_{\text{M}} &= \frac{Gm_{\text{M}}}{r^2} \\ &= \frac{\left(6.67 \times 10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)(3.30 \times 10^{23}\ \text{kg})}{(2.44 \times 10^{6}\ \text{m})^2} \\ &= 3.70\ \text{m/s}^2\end{aligned}$$

Venus's radius, $r_{\text{V}} = 6.05 \times 10^{6}\ \text{m}$

Venus's mass, $m_{\text{V}} = 4.87 \times 10^{24}\ \text{kg}$

$$\vec{g}_{\mathrm{V}} = \frac{Gm_{\mathrm{V}}}{r^2}$$

$$= \frac{\left(6.67\times10^{-11}\ \frac{\mathrm{N\bullet m^2}}{\mathrm{kg^2}}\right)(4.87\times10^{24}\ \mathrm{kg})}{(6.05\times10^{6}\ \mathrm{m})^2}$$

$$= 8.87\ \mathrm{m/s^2}$$

A table of the gravitational field strengths of the planets appears below:

Planet	***g*** **(m/s^2)**
Mercury	3.70
Venus	8.87
Earth	9.81

The gravitational field strength determined by the pendulum is:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$g = \frac{4\pi^2 l}{T^2}$$

$$= \frac{4\pi^2(1.0\ \mathrm{m})}{(3.27\ \mathrm{s})^2}$$

$$= 3.7\ \mathrm{m/s^2}$$

This gravitational field strength matches Mercury's.

Paraphrase

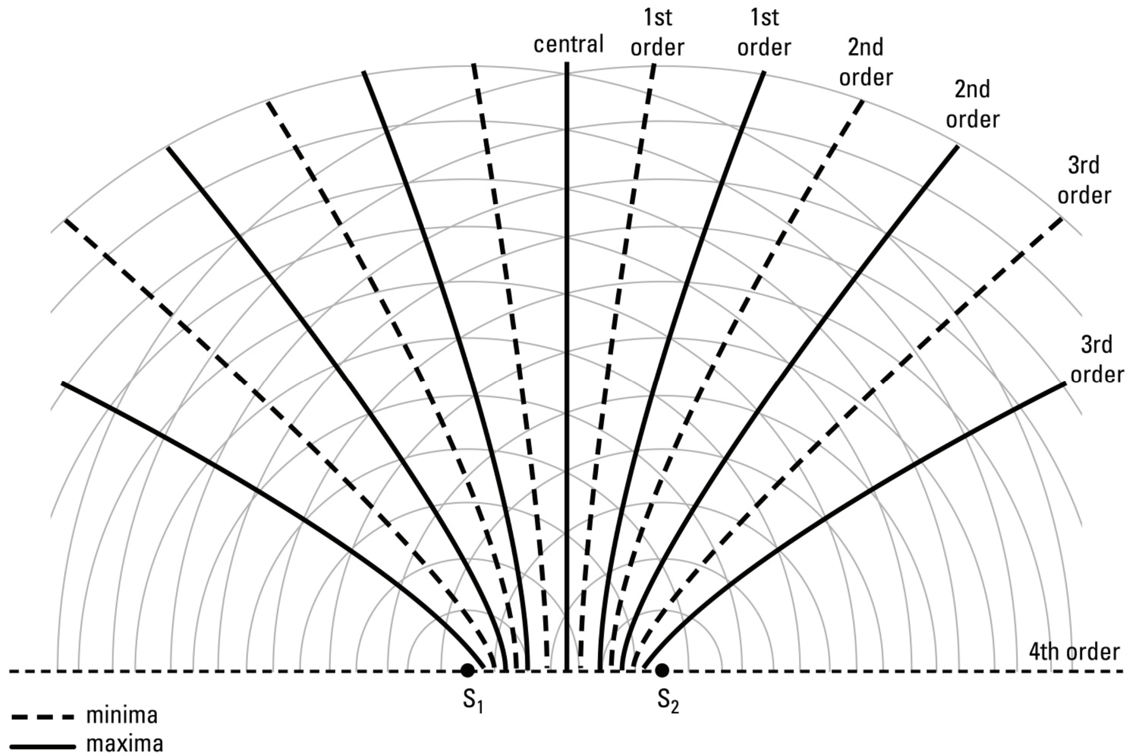
The gravitational field strength determined by the pendulum is 3.7 m/s^2, which matches Mercury. The alien landed on Mercury.

53. ***Given***

Two in-phase point sources separated by 3.5λ

Required

Diagram of interference pattern



In the diagram in the text, the sources are 3λ apart so that a 3rd order maximum lies on the line through the sources (S_1S_2) extended. In the diagram above, that location is occupied by a 4th order minimum. Because of the wider separation of the sources (3 ½ λ) the third order maxima have moved so that they lie roughly at an angle of 59º to either side of the central maximum. This diagram and the text diagram both have a central maximum. The text diagram has three maxima and three minima to the right and to the left of the central maximum. This diagram, because the sources are farther apart, has three maxima and four minima to each side of the central maximum. That means the pattern has behaved much like a hand fan that has been closed slightly, in that all the maxima and minima are a bit closer together.

54. The interference patterns, such as those on pages 425 or 426, are the result of in-phase sources. Since the sound from speakers is not of a single frequency and not in phase it should not be expected that a stationary interference pattern would be generated. In fact, as the frequencies vary, the interference pattern shifts. Hence, for a set of speakers, the interference pattern is not detectable because it is rapidly shifting more or less randomly.

55. ***Given***

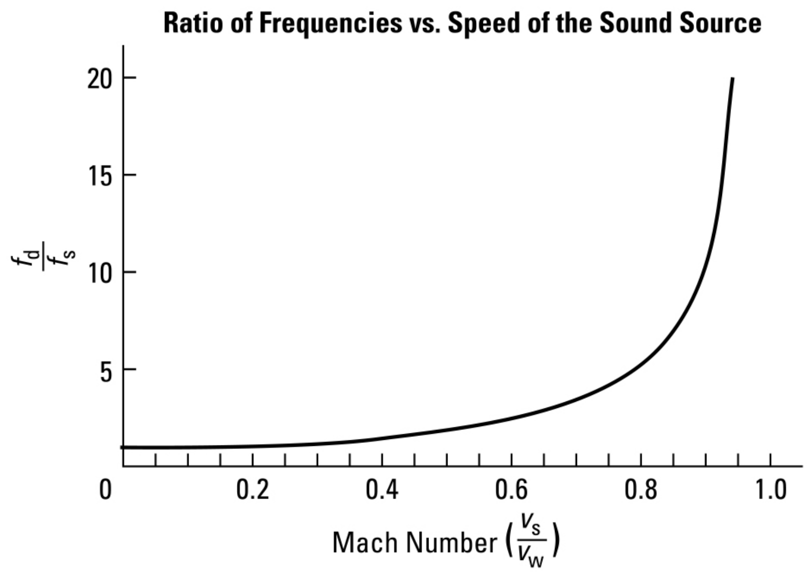
Approaching sound source with speed increasing from zero to Mach I

Required

Graph of ratio of frequencies vs. speed of sound source.

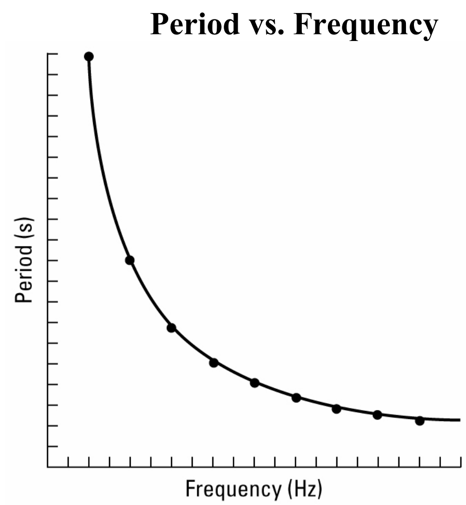
Analysis and Solution

If the speed of the source is Mach 1 (the speed of sound) the ratio of the frequencies is indeterminate as it requires division by zero.



Skills Practice

56.

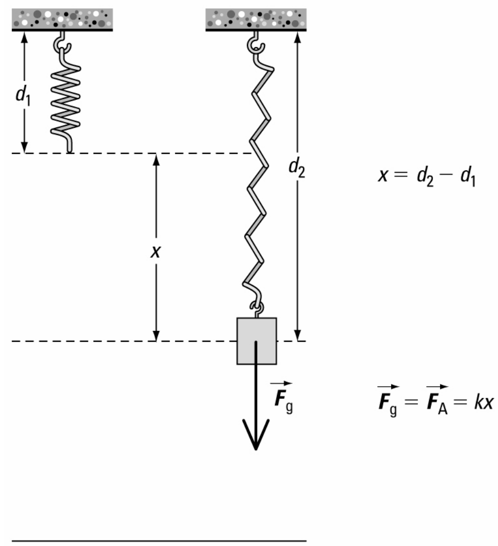


The graph of period versus frequency is an inverse relation.

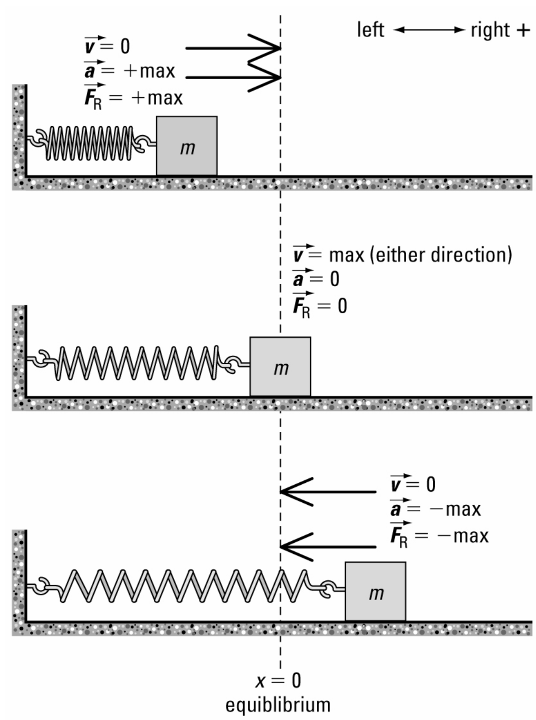
57. Students' answers will vary. A suitable procedure might be:

1. Hang a spring from a ceiling.
2. Measure the height of the free end from the ceiling.
3. Attach a mass of a predetermined amount to the free end of the spring, and allow it to come to rest in a stretched position.
4. Measure the height of the end of the spring to the ceiling.
5. Determine the displacement of the spring by subtracting the unstretched distance from the stretched distance.
6. Determine the applied force on the mass by using $\vec{F} = m\vec{g}$.
7. Use Hooke's law ($\vec{F} = k\vec{x}$) to solve for k: $k = \frac{\vec{F}}{\vec{x}}$.

   Students might include a diagram as follows:



58.



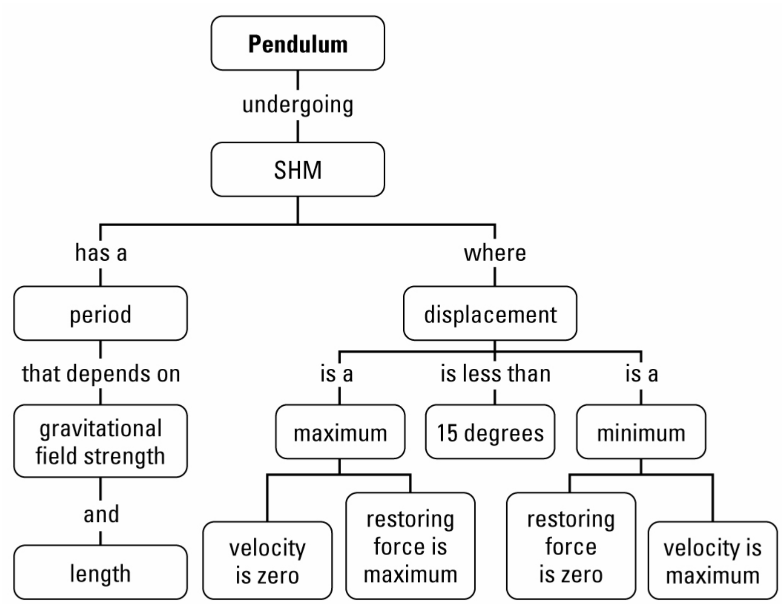
59. Students' answers will vary. A possible procedure is:

1. Determine the spring constant of the spring. (Use a force meter to pull the mass through a measured displacement; then apply $k = \frac{\vec{F}}{\vec{x}}$).
2. Pull the mass attached to the spring through some displacement and hold it.
3. Using a stopwatch, release the mass and let it vibrate on the spring. Time 20 complete oscillations.
4. Determine the period of an oscillation (T) by dividing the time for 20 oscillations by 20.

5. Use the equation $T = 2\pi\sqrt{\frac{m}{k}}$ to solve for m.

60. No, it will not because mass has no relevance to the period of oscillation of a pendulum. The period of a pendulum is only determined by it length and g.

61.



62. Huygens' pendulum clock was the first clock of this type. It relied on the mechanical resonance of a swinging pendulum. If cut to the right length, the pendulum arm would swing at a specific frequency that could be used to keep time reliably. This is its resonant frequency. To prevent the arm from succumbing to friction a small force was imparted on the arm on each swing. This small push given to the arm was the forced frequency.

The clock had limitations, however. It was only accurate for specific geographical regions as the period of the pendulum swing depended on g, which varies across Earth due to variations in the gravitational field strength at different latitudes and elevations. The metal arm would expand or contract in hot or cold weather, affecting its period. The pendulum could not be used on ships because of the rolling motion of the vessel.

63. Students should find that the term red shift in astronomy refers to the Doppler effect applied to light. If the light source is travelling away from the observer, then the apparent frequency of the light will be lower than the actual frequency of the light emitted by the source. The effect of decreasing frequency and increasing wavelength is to make the colour of the visible light received from the source appear to be shifted toward the red end of the spectrum. This is given as evidence that the universe is expanding as all other stars (galaxies) seem to be moving away from ours.

64. If two springs with different elastic constants are connected end-to-end, a wave will change speed when it passes from one spring to the other. In this manner, one can observe the changes in a pulse as it moves from a medium of high (low) speed into a medium of low (high) speed.

65. Following are several points that the students may include in their arguments.

Objects	**Mechanical Waves**
• The object carries the energy from place to place. • Energy is transferred from one object to another when the objects interact. • Masses may carry energy through space, including a vacuum. • Energy moves from place to place at the speed of the mass, which also is a factor in the quantity of the energy the mass carries. • The energy stays concentrated in the object until it interacts with other objects or its surroundings. The interaction of the object with its surroundings may cause it to lose energy to them.	• The medium transmits the wave which carries the energy. • The wave interacts with objects or another medium to transfer energy. • Mechanical waves can only move through a space in which a medium exists to enable their motion. • The nature of the medium determines the properties of the waves including the speed at which they propagate. This determines the speed at which energy can move through from place to place. The wave may spread out (diverge) so that its energy becomes more and more diluted along its wave front so that at any point along the wave front the energy available for transfer may be diminished.

Self-assessment

Students' answers in this section will vary greatly depending on their experiences, depth of understanding, and the depth of the research that they do into each of the topics.

