Pearson Physics Level 20
Unit III Circular Motion, Work, and Energy: Unit III Review Solutions

Student Book pages 336–339

Vocabulary

1. **artificial satellite:** a human-made object in orbit around a celestial body
axis of rotation: an imaginary line that passes through the centre of rotation perpendicular to the circular motion
axle: a shaft on which a wheel rotates
centripetal acceleration: acceleration acting toward the centre of a circle for an object moving in a circular path
centripetal force: force acting toward the centre of a circle causing an object to move in a circular path
conservation of energy: within an isolated system energy may be transferred from one object to another or transformed from one form to another, but it cannot be increased or decreased.
conservative force: a force that acts within a system but does not change its mechanical energy; includes gravity and elastic forces
cycle: one complete oscillation
eccentricity: a number between 0 and 1 that represents the degree of elongation of a circle
efficiency: ratio of the energy output to the energy input of any system
elastic potential energy: energy resulting from an object being altered from its standard shape, without permanent deformation
ellipse: an elongated circle that has two foci
energy: ability to do work
frequency: the number of oscillations (cycles) per second
gravitational potential energy: energy of an object due to its position above the surface of Earth
isolated system: group of objects assumed to be isolated from all other objects in the universe
Kepler's constant: a constant that relates a planet's orbital period and radius $K = 1\,\text{y}^2/\text{AU}^3$ or $3.0 \times 10^{-19}\,\text{s}^2/\text{m}^3$ for objects orbiting the Sun
Kepler's laws: the three laws that Kepler deduced from Tycho Brahe's observations (see text for full definition)
kinetic energy: energy due to the motion of an object; symbol is E_k.
mean orbital radius: the average orbital radius of a satellite, measured from the centre of the body being orbited to the centre of the satellite; for objects with elliptical orbits, it is equal to the semi-major axis
mechanical energy: sum of potential and kinetic energies; symbol is E_m.
non-isolated system: system in which there is an energy change with the surroundings
orbital period: the time it takes for a satellite to make one complete orbit

orbital perturbations: an irregularity or disturbance in the predicted orbit of a planet
period: the time required for an object to make one complete oscillation (cycle)
potential energy: energy that is stored and held in readiness; includes gravitational and elastic potential energies; symbol is E_p
power: rate of doing work
reference point: arbitrarily chosen point from which distances are measured
revolution: one complete cycle for an object moving in a circular path
rpm: revolutions per minute; a unit of frequency for an object moving in a circular path
satellite: a celestial body that is in orbit around another celestial body
uniform circular motion: motion in a circular path at a constant speed
work: measure of the amount of energy transferred when a force acts over a given displacement; calculated as the product of the magnitude of applied force and the displacement of the object in the direction of the force
work-energy theorem: work done on a system is equal to the sum of the changes in the potential and kinetic energies of the system

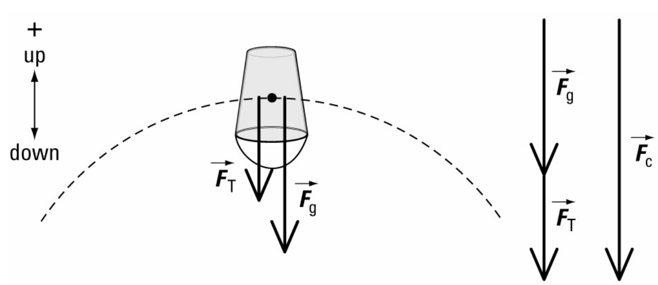
Knowledge

Chapter 5

2. If the centripetal acceleration remains constant, the speed will decrease as the radius decreases.

3. The car will be accelerating inward due to the friction between the wheels and the road making the car turn. The car's velocity is tangent to the circle, so if the friction does not provide the centripetal force necessary to keep the car turning, it will skid off tangentially.

4. (a) When the bucket is in the highest position, two forces work in concert to create the centripetal force: the force of gravity and the tension.

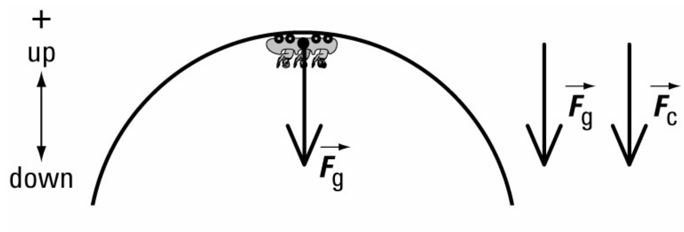


(b) The rope is likely to break when the bucket is in the bottom of the swing. In this position the tension on the rope is greatest because it must provide all the centripetal force necessary to keep the bucket moving around the circle.

5. The force you feel on your hand when spinning an object around in a circle is the reaction force the object is exerting on you. Your hand is providing a centripetal force on the object.

6. For planets orbiting the Sun, the centripetal force is the force of gravity. The force of gravity increases as the distance between the two objects decreases. Since a planet's orbit is elliptical, its orbital radius varies; therefore so does its centripetal force.

7. (a) The force of gravity is the centripetal force at the top of the loop (position C). In the top half of the loop, a component of the force of gravity acts as the centripetal force along with the track.



(b) The roller coaster exerts a force on the track in all positions except at the very top (position C).

(c)

$$F_c = F_g$$

$$F_c = \frac{mv^2}{r}$$

$$\frac{\cancel{m}v^2}{r} = \cancel{m}g \qquad \text{masses cancel}$$

$$\frac{v^2}{r} = g$$

8. There is no relationship between the frequency and radius of a rotating disc. The frequency is the same for all radii.

9. A table saw might use different blades of different diameters. The speed at the outer edge of different blades will vary, but the rotational frequency will not.

10. The period of the Moon (T_{Moon}) and radius of the Moon's orbit (r_{Moon}) must be known in order to determine Earth's mass.

11. (a) The planet with the least elliptical orbit is Venus with an eccentricity of 0.007.

(b) Venus's semi-major axis is closest in length to its semi-minor axis because it has the least eccentricity.

12. The three coins are identical except that they are placed at different radii. As the frequency of the platter increases, so does the centripetal force as shown by this equation:

$F_c = m4\pi^2 rf^2$

Since $m4\pi^2$ is constant the only factors affecting the centripetal force are:

$F_c \propto rf^2$

The coin at the outer radius will experience a greater centripetal force. If the force of friction (which is the same for all coins) is not great enough to provide the centripetal force, the outermost coin (coin C) will slide off first, followed by coin B then coin A.

13. Neptune was discovered when astronomers noticed perturbations in the orbit of Uranus. They noticed that the orbital velocity of Uranus was slowed. They reasoned that the force of gravity exerted on Uranus by another planet farther out must be the cause of the orbital perturbation. Based on the degree that Uranus' orbital velocity changed, they mathematically determined where the unknown planet should be. When they looked for it in the position predicted, they found it and called it Neptune.

14. Your friend is probably right. If Neptune was not a huge planet, the force of gravity that it exerted on Uranus would probably not be enough to make a noticeable perturbation in the orbit of Uranus. Neptune would likely have gone undiscovered for quite some time.

15. An extrasolar planet does not produce light of its own, and the light from the star it orbits is so bright that we can't see the planet. Visually sighting one with current telescopes is very unlikely. A planet may be detected by the perturbations in the star's movement if the planet is massive. To date, planets detected in this way are very large and close to their sun. This makes the likelihood of life on these planets remote.

Chapter 6

16. $1 \text{ J} = 1 \text{ kg}\cdot\text{m}^2/\text{s}^2$

17. The work done by a force depends on the angle between the directions of the force and the displacement over which it acts.

18. An object in free fall is constantly losing gravitational potential energy. If there were no air resistance, then the lost gravitational potential energy would all be transformed into kinetic energy. However, since there is air resistance, the air resistance transforms kinetic energy into heat in the surrounding air. As the speed of the object increases the magnitude of the force of air resistance increases until it reaches the magnitude of the force of gravity so that the net force on the object gradually becomes zero. At that time, the acceleration of the object becomes zero and the object continues its fall at a constant speed known as its terminal velocity. While falling at its terminal velocity, the object's gravitational potential energy is transformed into kinetic energy at the same rate as its kinetic energy is transformed into heat.

19. Since an object's kinetic energy varies as the square of its speed, doubling the speed will quadruple (2^2) the kinetic energy.

20. For a given force, the stopping distance varies directly as the initial kinetic energy. Since the two masses have equal kinetic energies, a force of equal magnitude acting on the each of the masses will stop them in equal distances.

21. As the rollercoaster rolls down the first hill gravitational potential energy is transformed into kinetic energy. In turn, the forces of friction (both rolling and air) act on the trolley to transform kinetic energy into thermal energy (heat). As the trolley rolls up the following hill, gravity transforms kinetic energy into gravitational potential energy. At the same time, friction is transforming kinetic energy into heat. Thus, the height of the second hill must be lower than the height of the first hill because the gravitational potential energy at the top of the second hill must be less than the gravitational potential energy at the top of the first hill by a magnitude at least equal to the amount of energy that was converted into heat by friction.

22. In terms of mechanical energy this is not an isolated system. The cart's gravitational potential energy is increasing while the kinetic energy is constant. Thus the force pulling the cart up the hill must be adding mechanical energy to the system. Therefore, the force pulling the cart up the hill cannot be considered a conservative force.

23. If no forces other than the force of gravity and the force of the inclined plane act on the cart, then a cart rolling down an inclined plane could be considered to be an isolated system with respect to mechanical energy. If, however, friction or some other force is applied to the cart, then those forces may reduce or add to the quantity of mechanical energy of the system. Regardless of whether or not other forces act on the cart, the force of gravity is still considered a conservative force.

24. Assuming no friction, as the slingshot is pulled back, chemical energy from your muscles is transformed into elastic potential energy in the sling. When the sling is released, its elastic potential energy is transformed into kinetic energy of the stone. As the stone rises, its kinetic energy is transformed into gravitational potential energy until the stone reaches its maximum height and no longer has any kinetic energy. If friction exists, energy is continually being transformed into heat at all stages except for the instants at which the stone is at rest, just before it is released and at the top of the rise.

25. According to the work-energy theorem, the work resulting from a net force causes changes in the kinetic energy. Thus the force causing a change in kinetic energy must be the sum of all the forces acting on the object.

26. On a force-displacement graph, the work is equivalent to the area under the curve.

27. The work-energy theorem states that there can be no energy transferred into or out of an isolated system.

28. Power describes the rate at which you are able to do work, not how much work you are able to do. A motor may be rated at 750 W; however, the actual work it does will depend on how long and how fast it runs.

Applications

29. ***Analysis and Solution***

$$\begin{aligned} T &= \frac{1}{f} \\ &= \frac{1}{60\,\text{Hz}} \\ &= 0.017\,\text{s} \end{aligned}$$

The period of the electrons is 0.017 s.

30. ***Analysis and Solution***

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{1}{0.0160\,\text{s}} \\ &= 62.5\,\text{Hz} \end{aligned}$$

The cell phone has a frequency of 62.5 Hz.

31. ***Analysis and Solution***

$$\begin{aligned} f &= \frac{300.0\,\text{rev}}{\text{min}} \times \frac{1\,\text{min}}{60\ \text{s}} = 5.00\ \text{Hz} \\ &= 5.00\,\text{Hz} \end{aligned}$$

$$\begin{aligned} T &= \frac{1}{f} \\ &= \frac{1}{5.00\,\text{Hz}} \\ &= 0.200\,\text{s} \end{aligned}$$

The frequency of the toy top is 5.00 Hz, and its period is 0.200 Hz.

32. ***Analysis and Solution***

$$\begin{aligned} T &= 27.3\,\cancel{\text{d}} \times \frac{86400\,\text{s}}{1\,\cancel{\text{d}}} \\ &= 2.359 \times 10^{6}\,\text{s} \end{aligned}$$

$$\begin{aligned} r &= 3.844 \times 10^{5}\,\cancel{\text{km}} \times \frac{1000\,\text{m}}{1\,\cancel{\text{km}}} \\ &= 3.844 \times 10^{8}\,\text{m} \end{aligned}$$

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{2\pi\left(3.844 \times 10^{8}\,\text{m}\right)}{2.359 \times 10^{6}\,\text{s}} \\ &= 1.02 \times 10^{3}\,\text{m/s} \end{aligned}$$

The speed of the Moon as it orbits Earth is 1.02×10^3 m/s.

33. ***Analysis and Solution***

$$v = \frac{2\pi r}{T}$$

$$\begin{aligned} T &= \frac{2\pi r}{v} \\ &= \frac{2\pi(4.0\,\text{m})}{1.57\,\frac{\text{m}}{\text{s}}} \\ &= 1.6 \times 10^{1}\,\text{s} \end{aligned}$$

The period of the ride is 1.6×10^1 s.

34. ***Analysis and Solution***

$$\begin{aligned} a_{\text{c}} &= \frac{v^2}{r} \\ &= \frac{\left(5.56\,\frac{\text{m}}{\text{s}}\right)^2}{10.0\,\text{m}} \\ &= 3.09\,\text{m/s}^2 \end{aligned}$$

She experiences an acceleration of 3.09 m/s^2.

35. ***Given***

$a_{\text{c}} = 4g$

r = 500.0 m

Required

speed of the airplane (v)

Analysis and Solution

Convert the acceleration of 4.00*g* to appropriate SI units. Use the equation for centripetal acceleration to determine the speed of the plane.

4 times the acceleration of gravity is:

$4.00 \times 9.81\,\text{m/s}^2 = 39.24\,\text{m/s}^2$

Now determine the speed of the plane.

$$a_c = \frac{v^2}{r}$$

$$\begin{aligned} v &= \sqrt{a_c r} \\ &= \sqrt{\left(39.24\frac{\text{m}}{\text{s}^2}\right)(500.0\,\text{m})} \\ &= 1.40 \times 10^2\ \text{m/s} \end{aligned}$$

Paraphrase

The airplane is travelling at a speed of 1.40×10^2 m/s .

36. ***Analysis and Solution***

$$\begin{aligned} F_c &= \frac{mv^2}{r} \\ &= \frac{(0.00288\,\text{kg})\left(0.314\,\frac{\text{m}}{\text{s}}\right)^2}{0.0400\,\text{m}} \\ &= 7.10 \times 10^{-3}\ \text{N} \end{aligned}$$

The centripetal force on the cork is 7.10×10^{-3} N.

37. ***Given***

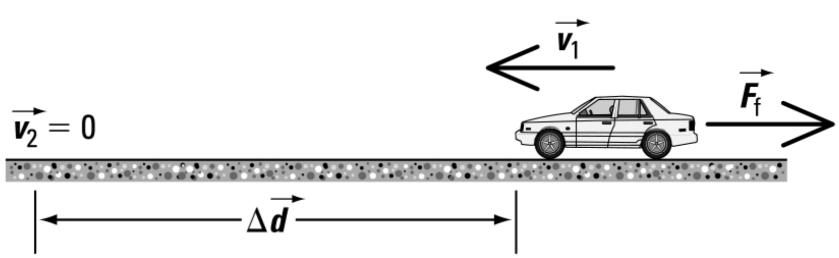
$\boldsymbol{m} = 1.25 \times 10^3$ kg

$\vec{F}_f = -3.20 \times 10^3$ N

(a) $\vec{v}_1 = 12.0\ \frac{\text{m}}{\text{s}}$

$E_{k_2} = 0$

(b) $\vec{v} = 2\vec{v}_1$



Required

(a) stopping distance for the given speed

(b) stopping distance for a speed that is twice as great.

Analysis and Solution

(a) When the car applies its brakes and comes to rest the force of friction acts to transform its kinetic energy into heat. Calculate the distance required for the force of friction to do work equal in magnitude to the car's initial kinetic energy. The direction of the force of friction is opposite to that of the initial velocity.

$$\begin{aligned} E_{k_1} &= \frac{1}{2}mv_1^2 \\ &= \frac{1}{2}\left(1.25\times10^3 \text{ kg}\right)\left(12.0 \ \frac{\text{m}}{\text{s}}\right)^2 \\ &= 9.00\times10^4 \text{ J} \end{aligned}$$

$$\begin{aligned} W &= \Delta E_k \\ \left(F_f \cos\theta\right)\Delta d &= E_{k_2} - E_{k_1} \\ \Delta d &= \frac{E_{k_2} - E_{k_1}}{F_f \cos\theta} \\ &= \frac{0 - 9.00\times10^4 \text{ J}}{3.20\times10^3 \text{ N}\left(\cos 180°\right)} \\ &= \frac{-9.00\times10^4 \text{ J}}{3.20\times10^3 \text{ N}\left(-1\right)} \\ &= 28.13 \text{ m} \\ &= 28.1 \text{ m} \end{aligned}$$

(b) Since the kinetic energy varies as the speed squared, doubling the speed quadruples the kinetic energy. When the force is constant, the displacement varies directly as the work it does. Hence, if the energy is four times larger, then the stopping distance must be four times larger.

$$\begin{aligned} 4\Delta d &= 4\left(28.13 \text{ m}\right) \\ &= 112.5 \text{ m} \\ &= 113 \text{ m} \end{aligned}$$

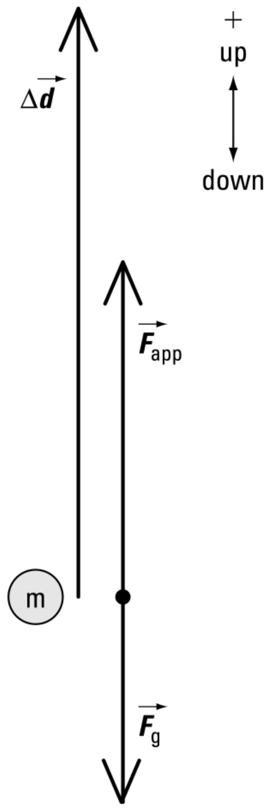
Paraphrase

(a) The force of friction brings the car to a stop in 28.1 m in the direction of the initial velocity.

(b) If the speed were doubled, the stopping distance would be four times as great: that is, 113 m.

38. ***Given***

$$\begin{aligned} \vec{F}_{\text{app}} &= 250 \text{ N[up]} \\ m &= 15.0 \text{ kg} \\ \Delta\vec{d} &= \Delta\vec{h} \\ &= 9.60 \text{ m[up]} \\ \vec{g} &= 9.81 \ \frac{\text{m}}{\text{s}^2}\text{[down]} \end{aligned}$$



Required
(a) work done by the applied force.
(b) change in gravitational potential energy for the mass.
(c) explanation of difference in answers (a) and (b), assuming isolated system
Analysis and Solution
(a) The work done by the applied force is found using the equation for work.

$$\begin{aligned} W &= (F\cos\theta)\Delta d \\ &= (250\text{ N}\cos 0°)9.60\text{ m} \\ &= 2400\text{ J} \end{aligned}$$

(b) The change in gravitational potential energy is found using that equation. The change in height is equal to the displacement.

$$\begin{aligned} \Delta E_{\mathrm{p}} &= mg\Delta h \\ &= (15.0\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(9.60\text{ m}) \\ &= 1413\text{ J} \\ &= 1410\text{ J} \end{aligned}$$

(c) If the applied force is greater in magnitude than the force of gravity the object will gain kinetic energy as well as gravitational potential energy as it moves up the incline. As the object moves upward it the applied force does 2400 J of work on it. Of that work 1410 J is transformed into gravitational potential energy and the remainder (about 990 J) is transformed into kinetic energy. If the system is not frictionless, some of the kinetic energy will have been transformed into heat.

Paraphrase

As the result of the 2400 J done by the applied force the object gained 1410 J. gravitational potential energy and 990 J of kinetic energy.

39. ***Given***

$m = 2.00 \times 10^3 \text{ kg}$

$\vec{v}_1 = 15.0 \ \dfrac{\text{m}}{\text{s}}[0°]$

$\vec{F}_{\text{app}} = 3.30 \times 10^3 \text{ N}[0°]$

$\Delta\vec{d} = 55.0 \text{ m}[0°]$

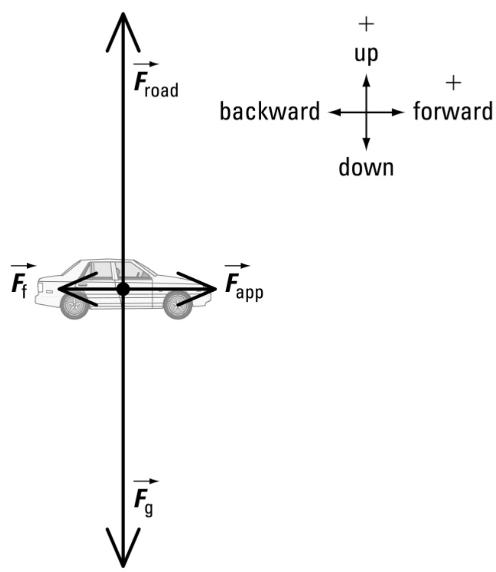
$\vec{F}_{\text{f}} = 5.00 \times 10^3 \text{ N}[180°]$

Required

(a) free body diagram and net force on the car
(b) work done by the net force
(c) final kinetic energy
(d) final speed of the car

Analysis and Solution

(a) Draw a diagram showing all the forces that act on the object.



(b) Find the net force by adding all the forces together then use that force to calculate the work it does. Since the upward force exerted by road surface and the force of gravity are equal in magnitude but opposite in direction the sum of those forces is zero.

$$\begin{aligned}
\vec{F}_{\text{net}} &= \vec{F}_{\text{app}} + \vec{F}_{\text{f}} + \left(\vec{F}_{\text{g}} + \vec{F}_{\text{road}}\right) \\
&= 3.30 \times 10^3 \text{ N}[0°] + 5.00 \times 10^2 \text{ N}[180°] + \left(\vec{F}_{\text{g}} + -\vec{F}_{\text{g}}\right) \\
&= 2.80 \times 10^3 \text{ N}[0°] + 0 \\
&= 2.80 \times 10^3 \text{ N}[0°]
\end{aligned}$$

$$W = (F_{\text{net}} \cos\theta)\Delta d$$
$$= (2.80\times10^3 \text{ N} \cos 0°)(55.0 \text{ m})$$
$$= 1.54\times10^5 \text{ J}$$

(c) The work-energy theorem states that the work done by the net force causes a change in kinetic energy. Calculate the final kinetic energy by adding the answer from part (b) to the initial kinetic energy.

$$E_{k_2} = \Delta E_k + E_{k_1}$$
$$= W + \frac{1}{2}mv_1^2$$
$$= 1.54\times10^5 \text{ J} + \frac{1}{2}(2.00\times10^3 \text{ kg})\left(15.0 \ \frac{\text{m}}{\text{s}}\right)^2$$
$$= 1.54\times10^5 \text{ J} + 2.25\times10^5 \text{ J}$$
$$= 3.79\times10^5 \text{ J}$$

(d) Use the final kinetic energy from part (c) to calculate the final speed of the object.

$$E_{k_2} = \frac{1}{2}mv_2^2$$
$$v_2 = \sqrt{\frac{2E_{k_2}}{m}}$$
$$= \sqrt{\frac{2(3.79\times10^5 \text{ J})}{2.00\times10^3 \text{ kg}}}$$
$$= 19.46 \ \frac{\text{m}}{\text{s}}$$
$$= 19.5 \ \frac{\text{m}}{\text{s}}$$

Paraphrase

(a) The free body diagram is as shown. The net force on the car is 2800 N [0º].
(b) The work done by the net force is equal to 1.54×10^5 J.
(c) The final kinetic energy of the car is 3.79×10^5 J.
(d) The final speed of the car is 19.5 m/s.

40. ***Given***

$m = 0.800\,kg$

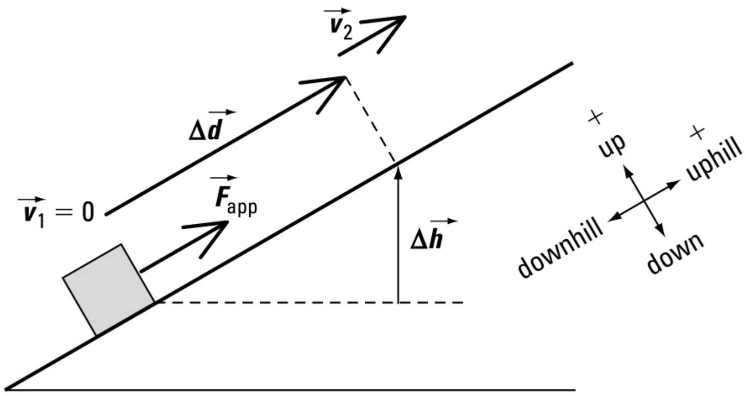
$v_1 = 0$

$F_{\text{app}} = 5.00\,\text{N}$

$\Delta d = 4.50\,\text{m}$

$v_2 = 6.00\frac{\text{m}}{\text{s}}$

$g = 9.81\frac{\text{m}}{\text{s}^2}$



Required
change in height through which block moved
Analysis and Solution
Since the plane is frictionless, all of the work done by the applied force will be transformed into kinetic and gravitational potential energy. The difference between the work done by the force and the gain in kinetic energy will give the gravitational potential energy. Use the gravitational potential energy to calculate the gain in height.

$$\begin{aligned}
W &= \Delta E_k + \Delta E_p \\
\Delta E_p &= W - \Delta E_k \\
&= F\Delta d - \left(E_{k_2} - E_{k_1}\right) \\
&= F\Delta d - \left(\tfrac{1}{2}mv_2{}^2 - \tfrac{1}{2}mv_1{}^2\right) \\
&= (5.00\,\text{N})(4.50\,\text{m}) - \left(\tfrac{1}{2}(0.800\,\text{kg})\left(6.00\,\frac{\text{m}}{\text{s}}\right)^2 - 0\right) \\
&= 22.5\,\text{J} - 14.4\,\text{J} \\
&= 8.10\ \text{J} \\
\Delta E_p &= mg\Delta h \\
\Delta h &= \frac{\Delta E_p}{mg} \\
&= \frac{8.10\,\text{J}}{(0.800\,\text{kg})\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)} \\
&= 1.032\,m \\
&= 1.03\,m
\end{aligned}$$

Paraphrase
As the force moves the block a distance of 4.50 m, the block gains 1.03 m in height.

41. ***Given***

$m = 25.0$ kg

$v_1 = 12.0\ \dfrac{\text{m}}{\text{s}}$

$F_1 = 15.0\,\text{N}$

$\Delta d_1 = 10.0\,\text{m}$

$F_2 = 45.0\,\text{N}$

$\Delta d_2 = 10.0\,\text{m}$

$F_3 = 0$

$\Delta d_3 = 20.0\,\text{m}$

Required

(a) work done by the forces

(b) final speed of the mass

Analysis and Solution

(a) The work is equal to the area under the curve. The area consists of three sections (i) a trapezoid, (ii) a rectangle and (iii) a triangle. Use the equations for the area for each of these figures to calculate the work in each region then add to get the total work done.

$$\begin{aligned}
W &= A_1 + A_2 + A_3 \\
&= \frac{1}{2}(F_1 + F_2)\Delta d_1 + F_2\Delta d_2 + \frac{1}{2}F_2\Delta d_3 \\
&= \frac{1}{2}(15.0\,\text{N} + 45.0\,\text{N})(10.0\ \text{m}) + (45.0\,\text{N})(10.0\ \text{m}) \\
&\quad + \frac{1}{2}(45.0\,\text{N})(20.0\ \text{m}) \\
&= 300\,\text{J} + 450\,\text{J} + 450\,\text{J} \\
&= 1200\,\text{J}
\end{aligned}$$

(b) Since the forces are net forces, the work results in a change in the object's kinetic energy. Calculate the initial kinetic energy and add the change in kinetic energy to give the final kinetic energy. Use the final kinetic energy to calculate the final speed.

$$\begin{aligned}
E_{\text{k}_2} &= \Delta E_\text{k} + E_{\text{k}_1} \\
&= W + \frac{1}{2}mv_1^{\,2} \\
&= 1200\ \text{J} + \frac{1}{2}(25.0\ \text{kg})\left(12.0\ \frac{\text{m}}{\text{s}}\right)^2 \\
&= 1200\ \text{J} + 1800\ \text{J} \\
&= 3000\ \text{J}
\end{aligned}$$

$$E_{k_2} = \frac{1}{2}mv_2^{\ 2}$$

$$v_2 = \sqrt{\frac{2E_{k_2}}{m}}$$

$$= \sqrt{\frac{2(3000\ \text{J})}{25.0\ \text{kg}}}$$

$$= 15.49\ \frac{\text{m}}{\text{s}}$$

$$= 15.5\ \frac{\text{m}}{\text{s}}$$

Paraphrase

(a) The total work done by the force is 1200 J.

(b) The initial kinetic energy is 1800 J. After the work is done on the object its final kinetic energy is 3000 J, so that its final speed is 15.5 m/s.

42. ***Given***

$m_A = 4.50\,\text{kg}$

$m_B = 5.50\,\text{kg}$

$g = 9.81\dfrac{\text{m}}{\text{s}^2}$

$\Delta h_B = 1.50\,m$

$\Delta d_A = 1.50\,m$

$v_1 = 0$

$v_2 = 3.00\ \dfrac{m}{s}$

Required

(a) change in gravitational potential energy for block A

(b) change in height through which block A moved

Analysis and Solution

(a) When the system is set in motion, block B will fall losing gravitational potential energy. Since there is no friction, by conservation of mechanical energy, the net loss in gravitational potential energy of the system must equal its net gain in kinetic energy. As block B falls, it loses gravitational potential energy. That gravitational potential energy is transformed into a gain in kinetic energy for blocks A and B as well as a gain in gravitational potential energy for A. The gain in block A's gravitational potential energy can be determined by subtracting the gain in kinetic energies for blocks A and B from the loss in gravitational potential energy for block B.

$$
\begin{aligned}
\Delta E_{\mathrm{p}} + \Delta E_{\mathrm{k}} &= 0 \\
\Delta E_{\mathrm{p}} &= -\Delta E_{\mathrm{k}} \\
\Delta E_{\mathrm{p_A}} + \Delta E_{\mathrm{p_B}} &= -\left(\Delta E_{\mathrm{k_A}} + \Delta E_{\mathrm{k_B}}\right) \\
\Delta E_{\mathrm{p_A}} &= -\Delta E_{\mathrm{p_B}} - \left(\Delta E_{\mathrm{k_A}} + \Delta E_{\mathrm{k_B}}\right) \\
&= -m_{\mathrm{B}} g \Delta h_{\mathrm{B}} - \left(\left(\tfrac{1}{2} m_{\mathrm{A}} v_2{}^2 - \tfrac{1}{2} m_{\mathrm{A}} v_1{}^2\right) + \left(\tfrac{1}{2} m_{\mathrm{B}} v_2{}^2 - \tfrac{1}{2} m_{\mathrm{B}} v_1{}^2\right)\right) \\
&= -(5.50\,kg)\left(9.81\frac{m}{s^2}\right)(-1.50\,m) \\
&\quad -\left(\left(\tfrac{1}{2}(4.50\,\mathrm{kg})\left(3.00\frac{m}{s}\right)^2 - \tfrac{1}{2}(4.50\,\mathrm{kg})(0)^2\right) + \left(\tfrac{1}{2}(5.50\,\mathrm{kg})\left(3.00\frac{m}{s}\right)^2 - \tfrac{1}{2}(4.50\,\mathrm{kg})(0)^2\right)\right) \\
&= +80.93\,\mathrm{J} - (20.25\,\mathrm{J} + 24.75\,\mathrm{J}) \\
&= 35.93\,\mathrm{J} \\
&= 35.9\,\mathrm{J}
\end{aligned}
$$

(b) Use the change in block A's gravitational potential energy to calculate its gain in height.

$$
\begin{aligned}
m_{\mathrm{A}} g \Delta h_{\mathrm{A}} &= \Delta E_{\mathrm{p_A}} \\
\Delta h_{\mathrm{A}} &= \frac{\Delta E_{\mathrm{p_A}}}{m_{\mathrm{A}} g} \\
&= \frac{35.93\,\mathrm{J}}{(4.50\,\mathrm{kg})\left(9.81\frac{\mathrm{m}}{\mathrm{s}^2}\right)} \\
&= 0.8139\,\mathrm{m} \\
&= 0.814\,\mathrm{m}
\end{aligned}
$$

Paraphrase

(a) As block B falls, it loses 80.9 J of gravitational potential energy, which is transformed into 45.0 J of kinetic energy for blocks A and B and 35.9 J of gravitational potential energy for block A.

(b) The gain in gravitational potential energy for block A is the result of a gain in height of 0.814 m.

43. ***Given***

As for Question 42

$m_{\mathrm{A}} = 4.50\,\mathrm{kg}$

$m_{\mathrm{B}} = 5.50\,\mathrm{kg}$

$g = 9.81\frac{\mathrm{m}}{\mathrm{s}^2}$

$\Delta h_{\mathrm{B}} = 1.50\,m$

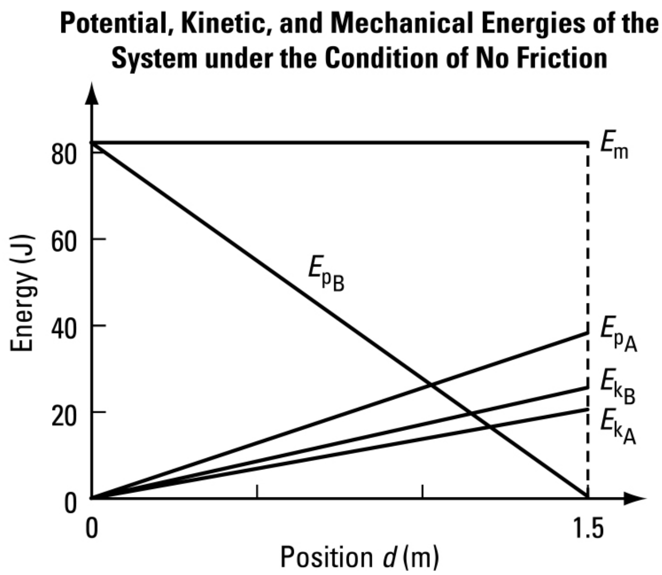
$\Delta d_{\mathrm{A}} = 1.50\,m$

$v_1 = 0$

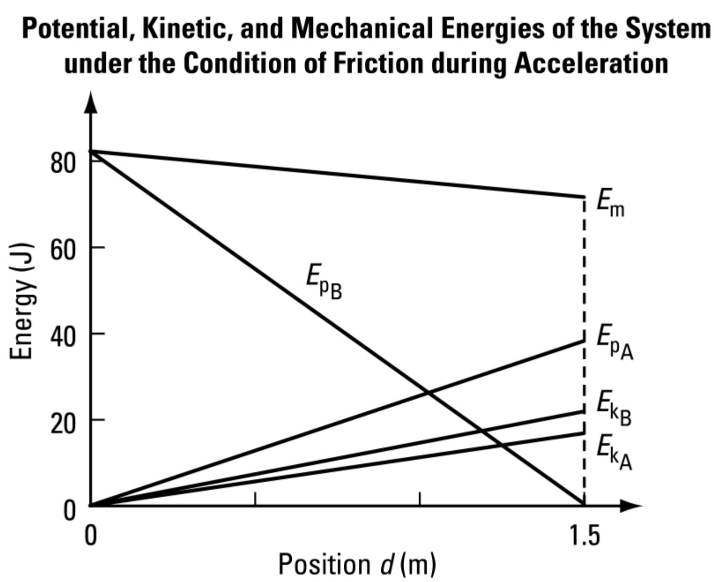
$v_2 = 3.00\,\frac{m}{s}$

Required
Graphs of potential, kinetic and mechanical energies of the system under conditions of (a) no friction, (b) friction but still acceleration

(a)



(b)



[**NOTE:** Since A and B are tied together, they undergo the same changes in gravitational potential energy on both graphs. On graph (b), however, the loss in energy to friction must come at the expense of the kinetic energies. Students may decrease the kinetic energies by any arbitrarily chosen amount but the ratio of the kinetic energies of A to B should remain at 4.5/5.5 (9/11). At the end of the motion, the sum of the kinetic energies of A and B plus the gravitational potential energy of A should be equal to the total mechanical energy.]

44. *Given*

(a) $m = 0.750\,\text{kg}$

$g = 9.81\,\dfrac{\text{m}}{\text{s}^2}$

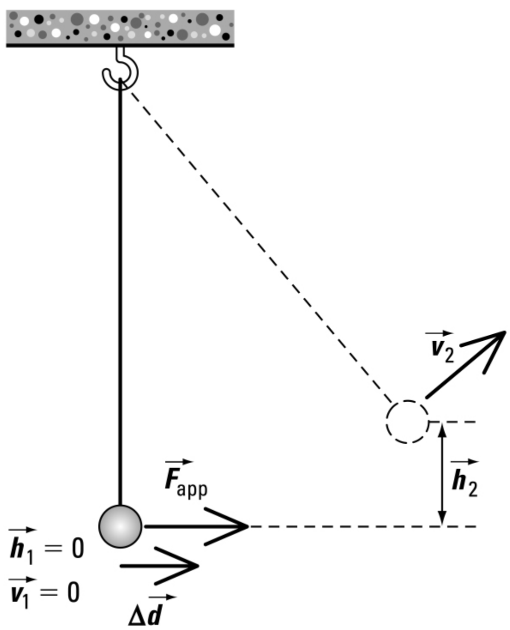
$v_1 = 0$

$h_1 = 0$

$h_2 = 0.150\,\text{m}$

$v_2 = 2.00\,\dfrac{\text{m}}{\text{s}}$

(b) $F_{\text{app}} = 40.0\,\text{N}$



Required

(a) work done on the pendulum by your push

(b) displacement over which you exerted force

Analysis and Solution

(a) The work done on the pendulum results in its gain in gravitational and potential energies. The sum of these energies must be equal to the work done to produce them. By assuming that the initial height is zero, the gain in energy is equal to the sum of the final kinetic and gravitational penitential energies.

$$\begin{aligned}
E_{\text{m}_2} &= E_{\text{p}_2} + E_{\text{k}_2} \\
&= mgh_2 + \tfrac{1}{2}mv_2{}^2 \\
&= (0.750\,\text{kg})\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)(0.150\,\text{m}) + \tfrac{1}{2}(0.750\,\text{kg})\left(2.00\,\frac{\text{m}}{\text{s}}\right)^2 \\
&= 1.104\,\text{J} + 1.50\,\text{J} \\
&= 2.604\,\text{J} \\
&= 2.60\,\text{J}
\end{aligned}$$

$$
\begin{aligned}
W &= E_{m_2} - E_{m_1} \\
&= 2.604\ \text{J} - 0 \\
&= 2.60\ \text{J}
\end{aligned}
$$

(b) Use the equation for work to calculate the magnitude of the displacement given the force.

$$
W = F\Delta d
$$

$$
\begin{aligned}
\Delta d &= \frac{W}{F} \\
&= \frac{2.604\ \text{J}}{40.0\ \text{N}} \\
&= 0.0651\ \text{m} \\
&= 0.065\ \text{m}
\end{aligned}
$$

Paraphrase

(a) To give the pendulum enough energy to produce the specified conditions would require 2.60 J of work.

(b) A force of 40.0 N would need to act over a distance of 0.065 m to do that amount of work.

[**NOTE:** In questions 44 and 45, the law of conservation of momentum is obeyed. In Q44, the collision is inelastic so that energy is lost but momentum is conserved. In Q45, the collision is elastic so that both kinetic energy and momentum are conserved. In an elastic collision of this nature the two balls move away at right angles to each other. When students have completed the unit on conservation of momentum in Physics 30, they can return to these questions and verify that momentum may be conserved in both elastic and inelastic collisions.]

45. ***Given***

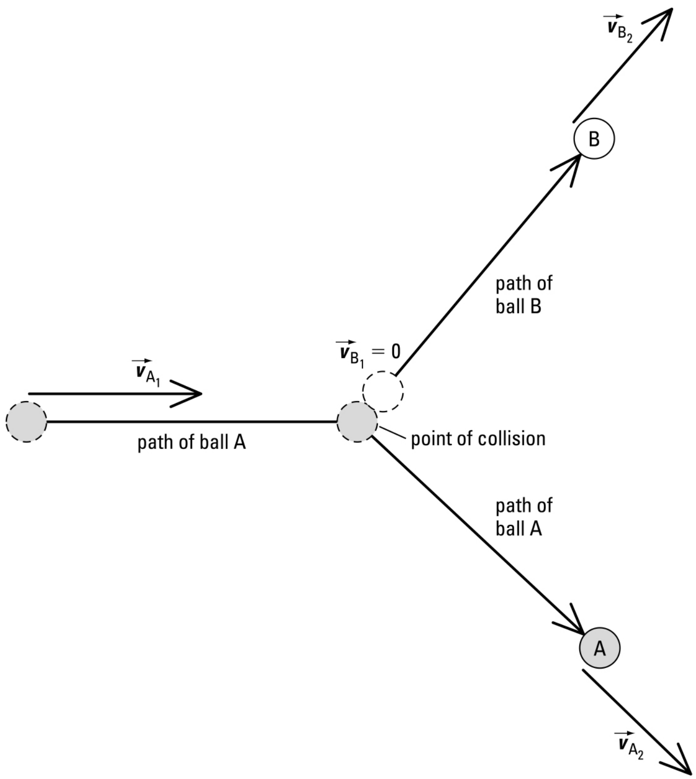
$m_A = 0.200\ \text{kg}$

$m_B = 0.200\ \text{kg}$

$v_{A_1} = 2.00\ \frac{\text{m}}{\text{s}}$

$v_{B_1} = 0$

$v_{A_2} = 1.50\ \frac{\text{m}}{\text{s}}$



Required
speed of ball B after the collision
Analysis and Solution
If mechanical energy is conserved and there is no change in potential energy, then the final kinetic energy must equal the initial kinetic energy. Find the total initial kinetic energy of the two balls and the final kinetic energy of ball A. The final kinetic energy of ball B is the difference between these two values. Use the final kinetic energy of ball B to find its final speed.

$$\begin{aligned}
E_{k_1} &= E_{k_{A_1}} + E_{k_{B_1}} \\
&= \tfrac{1}{2} m_A {v_{A_1}}^2 + \tfrac{1}{2} m_B {v_{B_1}}^2 \\
&= \tfrac{1}{2}(0.200 \text{ kg})\left(2.00 \ \frac{\text{m}}{\text{s}}\right)^2 + \tfrac{1}{2}(0.200 \text{ kg})(0)^2 \\
&= 0.400 \text{ J} + 0 \\
&= 0.400 \text{ J}
\end{aligned}$$

$$\begin{aligned}E_{k_1} &= E_{k_2} \\ &= E_{k_{A_2}} + E_{k_{B_2}} \\ &= \tfrac{1}{2}m_A {v_{A_2}}^2 + E_{k_{B_2}}\end{aligned}$$

$$\begin{aligned}E_{k_{B_2}} &= E_{k_1} - \tfrac{1}{2}m_A {v_{A_2}}^2 \\ &= 0.400\ \text{J} - \tfrac{1}{2}(0.200\ \text{kg})\left(1.50\ \frac{\text{m}}{\text{s}}\right)^2 \\ &= 0.400\ \text{J} - 0.225\ \text{J} \\ &= 0.175\ \text{J}\end{aligned}$$

$$E_{k_{B_2}} = \tfrac{1}{2}m_B {v_{B_2}}^2$$

$$\begin{aligned}v_{B_2} &= \sqrt{\frac{2\left(E_{k_{B_2}}\right)}{m_B}} \\ &= \sqrt{\frac{2(0.175\ \text{J})}{0.200\ \text{kg}}} \\ &= 1.322\ \frac{\text{m}}{\text{s}} \\ &= 1.32\ \frac{\text{m}}{\text{s}}\end{aligned}$$

Paraphrase

The final kinetic energy of ball B is equal to 0.175 J, which gives it a final speed of 1.32 m/s.

46. ***Given***

$m = 3.00\ \text{kg}$

$\vec{g} = 9.81\ \frac{\text{m}}{\text{s}^2}[\text{down}]$

$\vec{v}_1 = 280\ \frac{\text{m}}{\text{s}}[0°\ \text{up}\ 20°]$

$\vec{h}_1 = 450\ \text{m[up]}$

(b) $v_2 = v_1 \cos 20°$

(c) $h_2 = 0$

Required

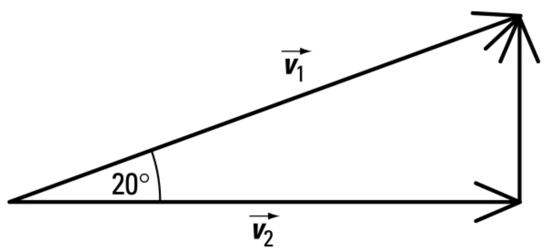
(a) initial mechanical energy for the cannonball
(b) highest point the cannonball attains
(c) speed of the cannonball when it lands

Analysis and Solution

(a) Using the ocean surface as height = 0, calculate the total mechanical energy by calculating the sum of the kinetic and gravitational potential energies.

$$
\begin{aligned}
E_{m_1} &= E_{p_1} + E_{k_1} \\
&= mgh_1 + \tfrac{1}{2}mv_1^{\,2} \\
&= (3.00 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(450 \text{ m}) + \tfrac{1}{2}(3.00 \text{ kg})\left(280 \ \frac{\text{m}}{\text{s}}\right)^2 \\
&= 1.324 \times 10^4 \text{ J} + 1.176 \times 10^5 \text{ J} \\
&= 1.308 \times 10^5 \text{ J} \\
&= 1.31 \times 10^5 \text{ J}
\end{aligned}
$$

(b) Assume that mechanical energy is conserved. To calculate gravitational potential energy, first find the kinetic energy at the highest point and then subtract that quantity from the mechanical energy. Use the gravitational potential energy to calculate the height. The speed at the highest point is the horizontal component of its initial velocity. Calculate the horizontal component of the speed using vector analysis.



$$
\begin{aligned}
v_2 &= v_1 \cos 20° \\
&= \left(280 \ \frac{\text{m}}{\text{s}}\right)(\cos 20°) \\
&= 263.1 \ \frac{\text{m}}{\text{s}} \\
&= 263 \ \frac{\text{m}}{\text{s}}
\end{aligned}
$$

$$
\begin{aligned}
E_{m_2} &= E_{k_2} + E_{p_2} \\
E_{p_2} &= E_{m_2} - E_{p_2} \\
&= E_{m_2} - E_{k_2} \\
&= E_{m_2} - \tfrac{1}{2}mv_2^{\,2} \\
&= 1.308 \times 10^5 \text{ J} - \tfrac{1}{2}(3.00 \text{ kg})\left(263.1 \ \frac{\text{m}}{\text{s}}\right)^2 \\
&= 1.308 \times 10^5 \text{ J} - 1.038 \times 10^5 \text{ J} \\
&= 2.70 \times 10^4 \text{ J}
\end{aligned}
$$

$$mgh_2 = E_{p_2}$$

$$h_2 = \frac{E_{p_2}}{mg}$$

$$= \frac{2.70 \times 10^4 \text{ J}}{(3.00 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)}$$

$$= 917.4 \text{ m}$$

$$= 917 \text{ m}$$

(c) At the surface of the ocean the gravitational potential energy of the ball is zero. Thus, assuming that mechanical energy is conserved, the kinetic energy is equal to its mechanical energy. Use the kinetic energy to calculate the speed of the ball when it hits the ocean.

$$E_{m_2} = E_{k_2} + E_{p_2}$$

$$E_{k_2} = E_{m_2} + E_{p_2}$$

$$= 1.308 \times 10^5 \text{ J} - 0$$

$$= 1.308 \times 10^5 \text{ J}$$

$$\tfrac{1}{2} m{v_2}^2 = E_{k_2}$$

$$v_2 = \sqrt{\frac{2E_{k_2}}{m}}$$

$$= \sqrt{\frac{2(1.308 \times 10^5 \text{ J})}{3.00 \text{ kg}}}$$

$$= 295.3 \ \frac{\text{m}}{\text{s}}$$

$$= 295 \ \frac{\text{m}}{\text{s}}$$

Paraphrase

(a) Initially, the mechanical energy of the cannonball is 1.31×10^5 J.

(b) At its highest point above the ocean, the cannon ball has a speed of 263 m/s and is 917 m above the ocean (or 467 m above its starting point).

(c) When the cannonball hits the surface of the ocean its speed is 295 m/s.

47. ***Given***

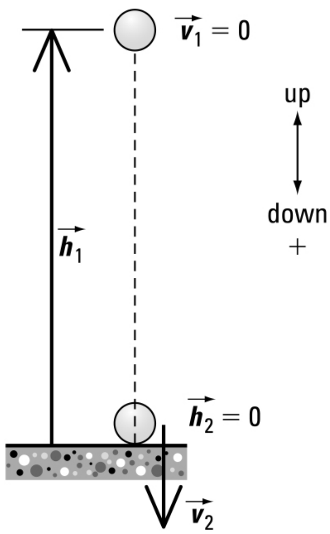
$$h_1 = 5.00 \text{ m}$$

$$m = 0.200 \text{ kg}$$

$$g = 9.81 \ \frac{\text{m}}{\text{s}^2}$$

$$v_1 = 0$$

$$v_2 = 3.00 \ \frac{\text{m}}{\text{s}}$$



Required

(a) change in the ball's mechanical energy as it falls.

(b) average force that air friction exerts on the ball while it falls.

Analysis and Solution

(a) As the ball falls it loses gravitational potential energy and gains kinetic energy. Since there is a force of friction, air friction will transform some of the kinetic energy into heat. When the ball lands its final mechanical energy will be less than its initial mechanical energy by the amount of energy transformed into heat.

$$\begin{aligned}
\Delta E_{\mathrm{m}} &= E_{\mathrm{m}_2} - E_{\mathrm{m}_1} \\
&= (E_{\mathrm{k}_2} + E_{\mathrm{p}_2}) - (E_{\mathrm{k}_1} - E_{\mathrm{p}_1}) \\
&= (\tfrac{1}{2}mv_2^{\,2} + mgh_2) - (\tfrac{1}{2}mv_1^{\,2} + mgh_1) \\
&= \left(\tfrac{1}{2}(0.200\ \mathrm{kg})\left(3.00\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2 + (0.200\ \mathrm{kg})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)(0) \right) \\
&\quad - \left(\tfrac{1}{2}(0.200\ \mathrm{kg})(0)^2 + (0.200\ \mathrm{kg})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)(5.00\ \mathrm{m}) \right) \\
&= (0.900\ \mathrm{J}) - (9.81\ \mathrm{J}) \\
&= -8.91\ \mathrm{J}
\end{aligned}$$

(b) To calculate the force of air friction, find the force required to cause the change in mechanical energy over the distance the ball falls. That force will be equal to the average force of friction.

$$W = \Delta E_{\mathrm{m}}$$
$$F\Delta h = \Delta E_{\mathrm{m}}$$
$$F = \frac{\Delta E_{\mathrm{m}}}{\Delta h}$$
$$= \frac{-8.91 \text{ J}}{5.00 \text{ m}}$$
$$= -1.782 \text{ N}$$
$$= -1.78 \text{ N}$$

Paraphrase

(a) During its fall the ball loses 8.91 J of mechanical energy.

(b) Friction exerts an average force of 1.78 N in the opposite direction (up) to the direction of the motion (down).

48. ***Given***

$m = 250 \text{ kg}$

$g = 9.81 \ \frac{\text{m}}{\text{s}^2}[\text{down}]$

$\Delta d = 30.0 \text{ m[up]}$

$\Delta t = 20.0 \text{ s}$

Required

average power output of the motor while it does the work.

Analysis and Solution

The work is equal to the increase in gravitational potential energy. Use the equation for power to calculate the output of the engine.

$$P = \frac{W}{\Delta t}$$
$$= \frac{\Delta Ep}{\Delta t}$$
$$= \frac{mg\Delta h}{\Delta t}$$
$$= \frac{(250 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(30.0 \text{ m})}{20.0 \text{ s}}$$
$$= \frac{7.358 \times 10^4 \text{ J}}{20.0 \text{ s}}$$
$$= 3.679 \times 10^3 \text{ W}$$
$$= 3.68 \times 10^3 \text{ W}$$

Paraphrase

The average power output of the engine is 3.68 kW.

49. ***Given***

$$v = 108\ \frac{\text{km}}{\text{h}}$$
$$= 108 \times 10^3\ \frac{\text{m}}{\text{h}} \times \frac{1\ \text{h}}{3.6 \times 10^3\ \text{s}}$$
$$= 30.0\ \frac{\text{m}}{\text{s}}$$
$$F = 540\ \text{N}$$

Required

power output (P)

Analysis and Solution

Use the equation that relates power to force and speed.

$$P = Fv$$
$$= 540\ \text{N}\left(30.0\ \frac{\text{m}}{\text{s}}\right)$$
$$= 1.62 \times 10^4\ \text{W}$$

Paraphrase

For a force of 540 N to maintain a speed of 108 km/h requires a power output of 16.2 kW.

50. ***Given***

$$P_{\text{motor}} = 150\ \text{kW}$$
$$F_{\text{friction}} = 2.50 \times 10^3\ \text{N}$$

Required

speed that results for power output of 150 kW.

Analysis and Solution

To maintain a constant speed, the force the motor exerts to move the plane forward (thrust) must be equal in magnitude to the force of friction (drag). The speed generated by a given power output can be found using the equation for power.

$$P_{\text{motor}} = F_{\text{motor}} v$$
$$v = \frac{P_{\text{motor}}}{F_{\text{motor}}}$$
$$= \frac{P_{\text{motor}}}{F_{\text{friction}}}$$
$$= \frac{1.50 \times 10^5\ \text{W}}{2.50 \times 10^3\ \text{N}}$$
$$= 60.0\ \frac{\text{m}}{\text{s}}$$

Paraphrase

The airplane's speed will be 60.0 m/s (216 km/h).

Extensions

51. ***Given***

$m = 90.9$ kg

$\vec{F}_{\mathrm{T}} = 108.18\,\mathrm{N}$

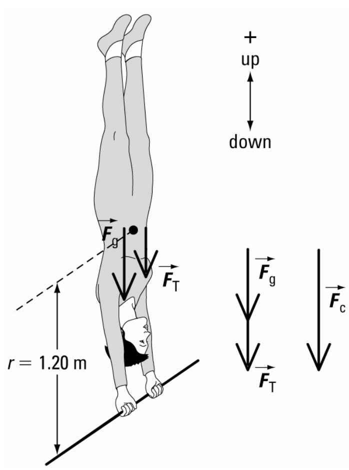
$r = 1.20$ m

Required

speed of the gymnast when he is above the bar (v)

Analysis and Solution

The gymnast's arms experience a tug of 108.18 N when he is above the bar. This is the tension in his arms. The force of gravity and tension are both down. Take down to be negative, and up to be positive. Determine the speed of the gymnast's body from the centripetal force that he experiences. First determine the centripetal force; then the speed.



$$\vec{F}_{\mathrm{c}} = \vec{F}_{\mathrm{g}} + \vec{F}_{\mathrm{T}}$$

$$\begin{aligned}\vec{F}_{\mathrm{c}} &= (mg) + (-108.18\,\mathrm{N}) \\ &= \left[(90.9\,\mathrm{kg})\left(-9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)\right] + \left(-108.18\,\mathrm{N}\right) \\ &= -1.00 \times 10^3\,\mathrm{N}\end{aligned}$$

The centripetal force is downward, but for the purposes of determining speed, it is a scalar quantity, so the negative sign is not needed.

$$F_{\mathrm{c}} = \frac{mv^2}{r}$$

$$\begin{aligned}v &= \sqrt{\frac{F_{\mathrm{c}}r}{m}} \\ &= \sqrt{\frac{(999.909\,\mathrm{N})(1.20\,\mathrm{m})}{90.9\ \mathrm{kg}}} \\ &= 3.63\ \mathrm{m/s}\end{aligned}$$

Paraphrase

The speed of the gymnast above the bar is 3.63 m/s.

52. (a) ***Given***

$T = 6.31 \times 10^{7}$ s

$r = 3.00 \times 10^{11}$ m

Required

mass of Tatooine's star (m_{Star})

Analysis and Solution

Determine the mass of Tatooine's star from Newton's version of Kepler's third law. Tatooine is a satellite of the Sun, so use its orbital radius and period.

$$F_c = F_g$$

$$\frac{4\pi^2 \cancel{m}_{\text{Tatooine}} r}{T^2} = \frac{G\cancel{m}_{\text{Tatooine}} m_{\text{Star}}}{r^2}$$

$$m_{\text{Star}} = \frac{4\pi^2 r^3}{T^2 G}$$

$$= \frac{4\pi^2 \left(3.00 \times 10^{11} \text{ m}\right)^3}{\left(6.31 \times 10^{7} \text{ s}\right)^2 \left(6.67 \times 10^{-11} \dfrac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)}$$

$$= 4.01 \times 10^{30} \text{ kg}$$

Paraphrase

The mass of Tatooine's star is 4.01×10^{30} kg .

(b) ***Given***

The orbital period and radius of Tatooine's moon:

$T = 1.73 \times 10^{6}$ s

$r = 6.00 \times 10^{8}$ m

Required

mass of Tatooine (m_{T})

Analysis and Solution

Determine the mass of Tatooine in the same manner as its star. The orbital period and radius of Tatooine's moon will be used to determine its mass.

$$F_c = F_g$$

$$\frac{4\pi^2 \cancel{m}_{\text{Moon}} r}{T^2} = \frac{G\cancel{m}_{\text{Moon}} m_{\text{T}}}{r^2}$$

$$m_{\text{T}} = \frac{4\pi^2 r^3}{T^2 G}$$

$$= \frac{4\pi^2 \left(6.00 \times 10^{8} \text{ m}\right)^3}{\left(1.73 \times 10^{6} \text{ s}\right)^2 \left(6.67 \times 10^{-11} \dfrac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)}$$

$$= 4.27 \times 10^{25} \text{ kg}$$

Paraphrase

The mass of Tatooine is 4.27×10^{25} kg .

(c) ***Given***

The orbital period and radius of Tatooine's moon:

$T = 1.73 \times 10^6 \text{ s}$

$r = 6.00 \times 10^8 \text{ m}$

Required

speed of Tatooine's moon (v)

Analysis and Solution

Determine the speed of the moon directly from its period and orbital radius.

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{2\pi\left(6.00 \times 10^8 \text{ m}\right)}{1.73 \times 10^6 \text{ s}} \\ &= 2.18 \times 10^3 \text{ m/s} \end{aligned}$$

Paraphrase

The speed of Tatooine's moon is 2.18×10^3 m/s.

53. ***Given***

$r = 3.05$ m

$f = 0.270$ Hz

Required

determine if the dirt will fall out at the top of the wheel

Analysis and Solution

This problem is similar to the roller coaster example. The centripetal force is a net force that is the sum of the force exerted by the bucket wall and the force of gravity. The dirt will not fall out if the centripetal force is greater than the force of gravity, because the wall will be providing a portion of the required force. If the force of gravity is greater than the centripetal force, the dirt will fall out. You don't know the mass of the dirt so you cannot determine the force, but mass is not needed:

$$\vec{F}_c = \vec{F}_s + \vec{F}_g$$

$$\cancel{m}\vec{a}_c = \cancel{m}\vec{a}_s + \cancel{m}\vec{g} \qquad \text{masses cancel}$$

$$\vec{a}_c = \vec{a}_s + \vec{g}$$

The same logic applies to the accelerations. Determine the centripetal acceleration and compare it with the acceleration of gravity. If $\vec{a}_c < \vec{g}$, the dirt will fall out; if $\vec{a}_c > \vec{g}$, the dirt will stay in the bucket.

$$\begin{aligned} \vec{a}_c &= 4\pi^2 r f^2 \\ &= 4\pi^2 (3.05 \text{ m})(0.270 \text{ Hz})^2 \\ &= 8.78 \text{ m/s}^2 \text{ [down]} \end{aligned}$$

$$\vec{g} = 9.81 \text{ m/s}^2 \text{ [down]}$$

Paraphrase

The centripetal acceleration is less than the acceleration of gravity, so the dirt will fall out.

54. ***Given***

$m_{\text{A}} = 6.00\text{ kg}$

$m_{\text{B}} = 4.00\text{ kg}$

$m_{\text{C}} = 2.00\text{ kg}$

$\vec{F}_{\text{app}} = 48.0\text{ N}[90°]$

$E_{\text{k}_1} = 0$

(a) $\Delta\vec{d} = 7.50\text{ m}[90°]$

(b) $\Delta\vec{d} = 2.00\text{ m}[90°]$

Required

(a) speed of the blocks after the force has acted over 7.50 m.

(b) speed of the block A after the force has acted over 2.00 m.

Analysis and Solution

(a) Since there is no friction and the blocks are moving on a horizontal surface, the applied force is a net force. The work-energy theorem says that the work done by a net force produces a change in the kinetic energy. Calculate the final kinetic energy from the change in kinetic energy and use that to calculate the final speed of the blocks as if they were a single block.

The mechanical energy of this system is not constant, so it is not considered an isolated system.

$$\begin{aligned} m &= m_{\text{A}} + m_{\text{B}} + m_{\text{C}} \\ &= 6.00\text{ kg} + 4.00\text{ kg} + 2.00\text{ kg} \\ &= 12.0\text{ kg} \end{aligned}$$

$$\begin{aligned} \Delta E_{\text{k}} &= W_{\text{net}} \\ E_{\text{k}_2} &= F_{\text{net}}\cos\theta\Delta d \\ \tfrac{1}{2}mv_2^2 &= F_{\text{net}}\cos\theta\Delta d \\ v_2 &= \sqrt{\frac{2F_{\text{net}}\cos\theta\Delta d}{m}} \\ &= \sqrt{\frac{2(48.0\text{ N})(\cos 0°)(7.50\text{ m})}{12.0\text{ kg}}} \\ &= \sqrt{60.0\ \frac{\text{m}^2}{\text{s}^2}} \\ &= 7.746\ \frac{\text{m}}{\text{s}} \\ &= 7.75\ \frac{\text{m}}{\text{s}} \end{aligned}$$

(b) When the force has acted over a displacement of 2.00 m, the string between blocks A and B has become tight so that block B is moving. However, the string between blocks B and C is not yet taut so that block C is still at rest. Calculate the work done by the force and solve for the kinetic energy in the same manner as in part (a) except that the mass in which the kinetic energy is stored is that of blocks A and B.

$$\begin{aligned}
m &= m_A + m_B \\
&= 6.00\ \text{kg} + 4.00\ \text{kg} \\
&= 10.0\ \text{kg} \\
\Delta E_k &= W_{net} \\
E_{k_2} &= F_{net} \cos\theta \Delta d \\
\tfrac{1}{2} m {v_2}^2 &= F_{net} \cos\theta \Delta d \\
v_2 &= \sqrt{\frac{2F_{net}\cos\theta\Delta d}{m}} \\
&= \sqrt{\frac{2(48.0\ \text{N})(\cos 0°)(2.00\ \text{m})}{10.0\ \text{kg}}} \\
&= \sqrt{19.2\ \frac{\text{m}^2}{\text{s}^2}} \\
&= 4.381\ \frac{\text{m}}{\text{s}} \\
&= 4.38\ \frac{\text{m}}{\text{s}}
\end{aligned}$$

Paraphrase

(a) After the force has acted over the displacement of 7.50 m, the three blocks are moving with a speed of 7.75 m/s.

(b) After the force has acted over a displacement of 2.00 m, only blocks A and B are moving, with a speed of 4.38 m/s.

Skills Practice

55. Satellites don't fall to Earth because they have a velocity that is tangent to their circular path. If gravity were to "disappear," they would fly off tangentially in a straight path. Gravity is pulling satellites toward the centre of Earth, so satellites are moving both toward the centre of Earth and off tangentially. The result is that a satellite "falls" around the Earth.

56. Students will find many websites dealing with meteor impact craters. Sites where data can be found are:

- http://wwwdsa.uqac.uquebec.ca/~mhiggins/MIAC/crater.htm
- http://exobio.ucsd.edu/Space_Sciences/earth_impact_craters.htm
- http://ottawa.rasc.ca/astronomy/earth_craters/map.jpg
- http://wwwdsa.uqac.uquebec.ca/~mhiggins/MIAC/impact.htm
- http://www.spacegrant.hawaii.edu/class_acts/CratersTe.html
- http://www.unb.ca/passc/ImpactDatabase/index.html

The last site is an Earth Impact Database maintained by the University of New Brunswick. If you click on the link (at left) for the Crater Inventory, by Name, you get a data table of the more important craters listed alphabetically. By location, gives you a world map showing the location of the craters in the database. Click on any area of the map and you will get a map of that region in more detail. If you click on Canada, you can easily locate the names and locations of the craters in Canada and near Alberta. Click on the number on the map and the details of that crater are given.

While this database lists the most important impact sites, there are hundreds of other smaller impact sites in Alberta. A bit of web surfing will reveal many sites on this topic.

57. The work done by external forces on a cart rolling down an inclined plane is equivalent to the loss in mechanical energy of the cart as it descends. Calculate the cart's mechanical energies at the top and bottom of the ramp. At the top of the ramp, the cart's mechanical energy is equal to its gravitational potential energy relative to the base of the ramp. At the bottom of the ramp the cart's mechanical energy is equal to its kinetic energy. Measure the speed of the cart when it reaches the base of the ramp using a photo gate timer or other appropriate method. Use that speed to calculate the final kinetic energy. Other variables that need to be determined are the initial height of the cart, the mass of the cart and the acceleration due to gravity at the location of the experiment.

Self-assessment

Students' answers in this section will vary greatly depending on their experiences, depth of understanding, and the depth of the research that they do into each of the topics.

