Pearson Physics Level 20
Unit IV Oscillatory Motion and Mechanical Waves: Chapter 8 Solutions

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8.1 Check and Reflect

Knowledge

1. The rays would be a set of vector arrows that diverge from the location of the source.

2. The angle between the incident wave fronts and the barrier is equal in magnitude to the angle between the reflected wave fronts and the barrier. The two angles lie on opposite sides of the point at which the wave is in contact with the barrier.

Applications

3. The wave front for this set of rays would be a circle that was growing smaller and converging on its centre. [**NOTE:** In nature this is the pattern of a set of waves that are being focussed on a point.]

4. The diagram will resemble Figure 8.12 except there should be several wave fronts parallel to the both the incident and reflected wave front. (Also see the diagram of the answer for chapter 8 LM-1.) The value of θ will be 40º. The speed, wavelength, and amplitude of the reflected wave train should be the same as those properties for the incident wave train. In a model of mechanical waves, the amplitude decreases slightly as the wave travels.

Extensions

5. The image in a plane mirror will be simulated by the reflection from a straight barrier. The experiment in 8-2 Inquiry Lab indicated that the reflected waves seem to come from a point behind the barrier that is symmetrical to the location of the origin point in front of the barrier. This would indicate that the image we see of ourselves in a plane mirror is located behind the mirror in a position symmetrical to our position in front of the mirror. [Students often think that the image in a mirror must be at the surface of the mirror.]

6. When wave lengths in a ripple tank slowed down, the wavelength of the wave train became shorter. When a sound wave enters a medium where its speed is faster, the wavelength of the sound must increase.

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Concept Check

The energy in a pulse is stored in the amplitude of the pulse, thus to increase the energy of the pulse you must increase its amplitude.

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Example 8.1 Practice Problems

1. ***Given***

$v = 5.30\ \frac{\text{m}}{\text{s}}$ (a) $\Delta t = 0.640$ s

Required

(a) length of the pulse (l)

(b) relationship between the amplitude and speed of the pulse

Analysis and Solution

(a) Use the equation for pulse length as given in the text on page 406.

$$\begin{aligned} l &= v\Delta t \\ &= \left(5.30\ \frac{\text{m}}{\cancel{\text{s}}}\right)(0.640\ \cancel{\text{s}}) \\ &= 3.39\ \text{m} \end{aligned}$$

(b) The speed of the pulse and the amplitude vary independently.

Paraphrase

(a) The length of the pulse is 3.39 m.

(b) The speed of the pulse and its amplitude vary independently.

2. ***Given***

$v = 3.60\ \frac{\text{m}}{\text{s}}$ $\quad l = 2.50$ m

Required

period required to generate the pulse (Δt)

Analysis and Solution

Use the equation relating pulse length to period and velocity and solve for period.

$$\begin{aligned} l &= v\Delta t \\ \Delta t &= \frac{l}{v} \\ &= \frac{2.50\ \cancel{\text{m}}}{3.60\ \frac{\cancel{\text{m}}}{\text{s}}} \\ &= 0.694\ \text{s} \end{aligned}$$

Paraphrase

The period required to generate the pulse is 0.694 s.

3. ***Given***

$l = 1.80$ m $\quad a = 0.50$ m

$\Delta t = 0.50$ s $\quad L = 5.0$ m

Required

time for the pulse to travel twice the length of the spring (Δt)

Analysis and Solution

First find the speed of the pulse using the equation for pulse length solved for speed. Calculate the time required for the pulse to travel a distance given by *2L* at that speed.

$$v = l\Delta t$$
$$= \frac{1.80 \text{ m}}{0.50 \text{ s}}$$
$$= 3.6 \ \frac{\text{m}}{\text{s}}$$

Find the time to travel 2L at that speed.

$$v = \frac{\Delta d}{\Delta t}$$
$$= \frac{2L}{\Delta t}$$
$$\Delta t = \frac{2L}{v}$$
$$= \frac{2(5.0 \ \cancel{\text{m}})}{3.6 \ \frac{\cancel{\text{m}}}{\text{s}}}$$
$$= 2.8 \text{ s}$$

Paraphrase

The pulse requires 2.8 s to travel the length of the spring and back.

4. ***Given***

$L = 6.0 \text{ m} \quad l = 1.50 \text{ m} \quad \Delta t_L = \ 3.6 \text{ s}$

Required

time required to generate the pulse (Δt)

Analysis and Solution

Use the length of the time for the pulse to travel along the known length of the spring to calculate the speed of the pulse. Use the speed and length of the pulse to calculate its period.

$$v = \frac{\Delta d}{\Delta t}$$
$$= \frac{2L}{\Delta t}$$
$$= \frac{2(6.0 \text{ m})}{3.6 \text{ s}}$$
$$= 3.3 \ \frac{\text{m}}{\text{s}}$$
$$l = v\Delta t$$
$$\Delta t = \frac{l}{v}$$
$$= \frac{1.50 \ \cancel{\text{m}}}{3.3 \ \frac{\cancel{\text{m}}}{\text{s}}}$$
$$= 0.45 \text{ s}$$

Paraphrase

The period required to generate the pulse is 0.45 s.

Example 8.2 Practice Problems

1. ***Given***

$f = 440\text{ Hz} \quad v = 350\ \dfrac{\text{m}}{\text{s}}$

Required

Wavelength (λ)

Analysis and Solution

Use the universal wave equation solved for wavelength.

$$\begin{aligned} v &= f\lambda \\ \lambda &= \frac{v}{f} \\ &= \frac{\left(350\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{440\ \frac{1}{\cancel{\text{s}}}} \\ &= 0.795\text{ m} \end{aligned}$$

Paraphrase

In this auditorium, the wavelength of the tuning note is 0.795 m.

2. ***Given***

$f = 325\text{ Hz} \quad \Delta t = 8.50\text{ s} \quad \lambda = 4.71\text{ m}$

Required

distance to the reflecting surface $\left(\dfrac{\Delta d}{2}\right)$

Analysis and Solution

Use the wavelength and frequency of the sound to find its speed.

$$\begin{aligned} v &= f\lambda \\ &= \left(325\ \frac{1}{\text{s}}\right)(4.71\text{ m}) \\ &= 1530\ \frac{\text{m}}{\text{s}} \end{aligned}$$

Use the speed of the sound and the given time interval to find out the distance travelled by the pulse (Δd).

$$\begin{aligned} \Delta d &= v\Delta t \\ &= \left(1530\ \frac{\text{m}}{\cancel{\text{s}}}\right)(8.50\ \cancel{\text{s}}) \\ &= 1.30\times10^{4}\text{ m} \end{aligned}$$

The distance to the reflecting surface is half the distance travelled by the pulse $\left(\dfrac{\Delta d}{2}\right)$.

$$\begin{aligned} \frac{\Delta d}{2} &= \frac{1.30\times10^{4}\text{ m}}{2} \\ &= 6.50\times10^{3}\text{ m} \qquad (6.50\text{ km}) \end{aligned}$$

Paraphrase
The sound reflected off a surface that was 6.50 km away.

3. ***Given***

$f = 8.0\ \frac{1}{\text{min}} \quad \Delta t = 3.00\text{ min} \quad \Delta d = 250\text{ m}$

Required
wavelength for the waves passing the boat (λ)

Analysis and Solution
Use the distance from shore and the time to travel that distance to calculate the speed of the waves. Use the speed of the waves and the frequency to calculate the wavelength.

$$v = \frac{\Delta d}{\Delta t}$$
$$= \frac{250\text{ m}}{3.00\text{ min}}$$
$$= 83.3\ \frac{\text{m}}{\text{min}}$$
$$v = f\lambda$$
$$\lambda = \frac{v}{f}$$
$$= \frac{83.3\ \frac{\text{m}}{\text{min}}}{8.0\ \frac{1}{\text{min}}}$$
$$= 10\text{ m}$$

Paraphrase
Waves passing the fisherman's boat are 10 m long.

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8.2 Check and Reflect

Knowledge

1. As a transverse wave moves through a medium, each segment of the medium moves in SHM perpendicular to the direction of motion of the wave. The wave is formed by a series of these motions with the phase of each segment of the medium lagging a bit behind the motion of the segment that precedes it.

2. In a longitudinal wave, the each segment of the medium moves in SHM along the line of the direction of the motion of the wave. As the wave moves through the medium, each segment oscillates so that its phase lags a bit behind the segment ahead of it.

3. Transverse waves oscillate at right angles to the direction in which the wave travels while longitudinal waves oscillate parallel to the direction in which the wave travels.

4. The amount of energy stored in a wave depends on the amplitude of the wave.

Applications

5. ***Given***

$v = 1.500 \times 10^3 \ \frac{\text{m}}{\text{s}} \qquad \lambda = 1.25 \text{ m}$

Required

frequency of the wave (f)

Analysis and Solution

Solve the universal wave equation for frequency.

$$v = f\lambda$$

$$\begin{aligned} f &= \frac{v}{\lambda} \\ &= \frac{1.500 \times 10^3 \ \frac{\cancel{\text{m}}}{\text{s}}}{1.25 \ \cancel{\text{m}}} \\ &= 1.20 \times 10^3 \ \frac{1}{\text{s}} \\ &= 1.20 \times 10^3 \text{ Hz} \end{aligned}$$

Paraphrase

If a wave 1.25 m long is travelling at 1500 m/s, it has a frequency of 1.20×10^3 Hz.

6. ***Given***

$\lambda_1 = 2.00 \text{ m} \qquad v_1 = 1.500 \times 10^3 \ \frac{\text{m}}{\text{s}}$

$v_2 = 1.550 \times 10^3 \ \frac{\text{m}}{\text{s}}$

Required

wavelength when the speed increases (λ_2)

Analysis and Solution

For a constant frequency, λ is directly proportional to v. Set up a ratio for this expression and solve for λ_2. [Students may solve for frequency then use that to solve for the wavelength but the method shown is easier and faster.]

$$\frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1}$$

$$\begin{aligned} \lambda_2 &= \lambda_1 \frac{v_2}{v_1} \\ &= (2.00 \text{ m})\left(\frac{1.55 \times 10^3 \ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}{1.50 \times 10^3 \ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}\right) \\ &= 2.07 \text{ m} \end{aligned}$$

Paraphrase

When the speed increases from 1500 m/s to 1550 m/s the wavelength increases from 2.00 m to 2.07 m.

7. ***Given***

$f = 2.400 \times 10^3 \text{ Hz} \qquad v = 325 \ \frac{\text{m}}{\text{s}}$

Required
wavelength of the wave (λ)
Analysis and Solution
Use the universal wave equation and solve for wavelength.

$$v = f\lambda$$

$$\lambda = \frac{v}{f}$$

$$= \frac{325 \ \frac{\text{m}}{\cancel{\text{s}}}}{2.40 \times 10^{3} \ \frac{1}{\cancel{\text{s}}}}$$

$$= 0.135 \text{ m}$$

Paraphrase
The wavelength of a sound with a frequency of 2400 Hz, which travels at 325 m/s, is 0.135 m.

8. For a wave generated in a spring, the wavelength varies inversely as the frequency while the amplitude is independent of either. This is because the wavelength is set by the displacement as you move your hand from the spring's equilibrium position, not how fast you move your hand.

9. ***Given***

$$f_{\text{A}} = f_{\text{B}} = 384 \frac{\text{m}}{\text{s}}$$

$$v_{\text{A}} = 350 \ \frac{\text{m}}{\text{s}} \qquad v_{\text{B}} = 1.50 \times 10^{3} \ \frac{\text{m}}{\text{s}}$$

Required
(a) wavelength for the tuning fork in air (λ_{A})
(b) wavelength for the tuning fork in water (λ_{B})
(c) description of the notes you would hear in air and water.
Analysis and Solution
Use the universal wave equation to solve for wavelength from speed and frequency.
(a)

$$v = f_{\text{A}}\lambda_{\text{A}}$$

$$\lambda_{\text{A}} = \frac{v}{f_{\text{A}}}$$

$$= \left(\frac{350 \ \frac{\text{m}}{\cancel{\text{s}}}}{384 \ \frac{1}{\cancel{\text{s}}}} \right)$$

$$= 0.911 \text{ m}$$

(b) Similarly

$$\lambda_B = \frac{v}{f_B}$$

$$= \left(\frac{1500\ \frac{\text{m}}{\cancel{\text{s}}}}{384\ \frac{1}{\cancel{\text{s}}}} \right)$$

$$= 3.91 \text{ m}$$

(c) Since the note is independent of the frequency, we would hear the same note both in the air and underwater.

Paraphrase and Verify

When a tuning fork creates a note in air or underwater, the note that we hear is the same. If a tuning fork with a frequency of 384 Hz creates a note that travels 350 m/s its wavelength is 0.911 m while at a speed of 1500 m/s its wavelength is 3.91 m.

Extension

10. The speaker creates sound waves by oscillating back and forth in the direction in which it broadcasts the sound. This means that the waves must be longitudinal. Similarly, when the sound waves travel down the auditory canal to the eardrum, the eardrum vibrates back and forth in the direction from which the waves travelled. This is further evidence that the sound waves are longitudinal. Air cannot exert the shear forces required to sustain a transverse wave. Its only elastic energy comes in the form of pressure. Thus sound waves are longitudinal pressure waves.

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Concept Check

In an ideal (frictionless) system with perfectly elastic reflecting surfaces, the incident wave should travel back and forth forever creating a permanent standing wave in the spring. In an ideal system, the only energy required is the energy to generate the incident wave. In a real system, the initial energy is required just as it is in an ideal system. However, once the standing wave is established, the energy required to sustain the standing wave is just enough to replace the energy lost to friction as the wave travels through the medium.

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Concept Check

The volume of a sound depends on the amplitude of the wave. If you want to make the note produced by a tuning fork louder, you strike it harder so that its arms vibrate with greater amplitude. Similarly, by striking a piano key harder you make the sound louder for the same reason. The damping pedal makes the notes softer (quieter) by reducing the amplitude of the vibration. If you hold a metre-stick on the top of a desk so that about 50 – 75 cm protrude beyond the edge of the desk, the stick can be made to vibrate. When it is set in motion, you will hear that the sound gets quieter as the amplitude of the vibration decreases.

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Example 8.3 Practice Problems

1. ***Given***

$f = 512\,\text{Hz} \quad L = 0.850\text{ m}$

Required

possible wavelengths that could produce resonance for this pipe (λ)

Analysis and Solution

Calculate the longest wavelength possible for this pipe and then calculate the speed of sound. If this speed is a reasonable speed for the speed of sound in air then you have your answer. If not, calculate the next longest wavelength and repeat the analysis.

Assuming that $L = \dfrac{\lambda}{4}$,

therefore

$$\begin{aligned}\lambda &= 4L\\ &= 4(0.850\text{ m})\\ &= 3.40\text{ m}\end{aligned}$$

$$\begin{aligned}v &= f\lambda\\ &= \left(512\ \frac{1}{\text{s}}\right)(3.40\text{ m})\\ &= 1.74\times10^{3}\ \frac{\text{m}}{\text{s}}\end{aligned}$$

This value is too large for the speed of sound in air.

Assume that $L = \dfrac{3\lambda}{4}$,

therefore

$$\lambda = \frac{4L}{3}$$
$$= \frac{4(0.850 \text{ m})}{3}$$
$$= 1.13 \text{ m}$$
$$v = f\lambda$$
$$= \left(512 \ \frac{1}{\text{s}}\right)(1.13 \text{m})$$
$$= 580 \ \frac{\text{m}}{\text{s}}$$

This value is too large for the speed of sound in air.

Assume that $L = \frac{5\lambda}{4}$,

therefore

$$\lambda = \frac{4L}{5}$$
$$= \frac{4(0.850 \text{ m})}{5}$$
$$= 0.680 \text{ m}$$
$$v = f\lambda$$
$$= \left(512 \ \frac{1}{\text{s}}\right)(0.680 \text{m})$$
$$= 348 \ \frac{\text{m}}{\text{s}}$$

This value is a reasonable value for the speed of sound in air.

Assume that $L = \frac{7\lambda}{4}$,

therefore

$$\lambda = \frac{4L}{7}$$
$$= \frac{4(0.850 \text{ m})}{7}$$
$$= 0.486 \text{ m}$$
$$v = f\lambda$$
$$= \left(512 \ \frac{1}{\text{s}}\right)(0.486 \text{m})$$
$$= 249 \ \frac{\text{m}}{\text{s}}$$

This value is less than the speed of sound in air.

Paraphrase and Verify

The only assumption that produces a reasonable value for the speed of sound is that the resonance resulted when the length of the pipe was $5\lambda/4$. This means that the wavelength of the sound in the close-pipe system was 0.680 m when the speed of sound in air is 348 m/s.

2. ***Given***

$f = 256 \text{ Hz} \quad \text{v} = 330 \ \frac{\text{m}}{\text{s}}$

Required

The four shortest lengths for which a closed pipe would produce resonance

Analysis and Solution

The four shortest lengths of a closed pipe for which resonance occurs are $L = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4}$. Calculate the wavelength of the sound, and then calculate the possible lengths for the tube.

$$v = f\lambda$$

$$\begin{aligned}\lambda &= \frac{v}{f} \\ &= \frac{330 \ \frac{\text{m}}{\cancel{\text{s}}}}{256 \ \frac{1}{\cancel{\text{s}}}} \\ &= 1.29 \text{ m}\end{aligned}$$

$$\begin{aligned}L_1 &= \frac{\lambda}{4} \\ &= \frac{1.29 \text{ m}}{4} \\ &= 0.323 \text{ m}\end{aligned}$$

$$\begin{aligned}L_2 &= \frac{3\lambda}{4} \\ &= \frac{3(1.29 \text{ m})}{4} \\ &= 0.968 \text{ m}\end{aligned}$$

$$\begin{aligned}L_3 &= \frac{5\lambda}{4} \\ &= \frac{5(1.29 \text{ m})}{4} \\ &= 1.61 \text{ m}\end{aligned}$$

$$\begin{aligned}L_4 &= \frac{7\lambda}{4} \\ &= \frac{7(1.29 \text{ m})}{4} \\ &= 2.26 \text{ m}\end{aligned}$$

Paraphrase

The lengths of the tube for which the first four resonance points occur are 0.323 m, 0.968 m, 1.61 m, and 2.26 m.

3. ***Given***

$L = 6.00 \text{ m} \quad f = 2.50 \text{ Hz}$

$L = \ 1.50\lambda$

Required

speed of the wave in the spring (v)

Analysis and Solution

Since there are 2 antinodes per wavelength, the three antinodes in the spring make up 1.50λ. Calculate the wavelength from the length of the spring; then use the universal wave equation to calculate the speed of the wave.

$$L = 1.50\lambda$$

$$\begin{aligned}\lambda &= \frac{L}{1.50} \\ &= \frac{6.00\text{ m}}{1.50} \\ &= 4.00\text{ m}\end{aligned}$$

$$\begin{aligned}v &= f\lambda \\ &= \left(2.50\ \frac{1}{\text{s}}\right)(4.00\text{ m}) \\ &= 10.0\ \frac{\text{m}}{\text{s}}\end{aligned}$$

Paraphrase

The speed of the wave in this spring is 10.0 m/s.

4. ***Given***

$L = 8.00\text{ m} \quad \text{v} = 5.00\ \frac{\text{m}}{\text{s}}$

$L = \frac{1}{2}\lambda$

Required

(a) frequency of the standing wave (f_1)

(b) next higher frequency that can produce a standing wave in this spring (f_2)

Analysis and Solution

(a) The wavelength for this frequency is twice the length of the spring.

$$L = \tfrac{1}{2}\lambda_1$$

$$\begin{aligned}\lambda_1 &= 2L \\ &= 2(8.00\text{ m}) \\ &= 16.0\text{ m}\end{aligned}$$

$$v = f_1\lambda_1$$

$$\begin{aligned}f_1 &= \frac{5.00\ \frac{\cancel{\text{m}}}{\text{s}}}{16.0\ \cancel{\text{m}}} \\ &= 0.3125\ \frac{1}{\text{s}} \\ &= 0.313\text{ hz}\end{aligned}$$

(b) The wavelength of the next standing wave pattern is one-half the wavelength of this standing wave, meaning that the frequency must double.

$$\begin{aligned}f_2 &= 2f_1 \\ &= 2\left(0.313\frac{1}{\text{s}}\right) \\ &= 0.626\ \frac{1}{\text{s}}\end{aligned}$$

Paraphrase
The lowest frequency producing a standing wave is 0.313 Hz. The next higher frequency that will produce a standing wave in the same spring is 0.626 Hz.

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8.3 Check and Reflect

Knowledge

1. Interference occurs when two or more waves pass simultaneously through the same point in a medium. At each point of overlap, the waves interfere with each other to form a single wave whose displacement is the algebraic sum of the displacements of the individual waves.

2. The amplitude of an antinode in a standing wave is the sum of the amplitudes of the waves that form it. To produce a standing wave, the two waves must have equal amplitudes or the nodes could not exist; thus the antinode has an amplitude twice that of either wave.

3. For a closed-pipe air column, the longest standing wavelength that can be sustained is four times the length of the column. For an open-pipe column, the longest standing wavelength that can be sustained is twice the length of the column.

4. The wavelengths of the three longest standing waves in a closed-pipe column of length L are $4L, \frac{4L}{3}$, and $\frac{4L}{5}$.

5. Two wave generators are said to be in phase if they have the same frequency and are simultaneously generating crests. Two waves are said to be in phase if, when they pass through a point in the medium, identical parts of the waves (e.g. crests) from each wave arrive at the same time.

Applications

6. (a) If the pulses are upright with respect to each other, then when two pulses with the same length totally overlap, the length of the net pulse is the same length as the individual pulses and the amplitude is the sum of the amplitudes, or in this case 7 units.

(b) If the pulses are the same length but inverted with respect to each other, then the net pulse is the same length as the individual pulses and the amplitude is the difference in their amplitudes, or in this case 1 unit.

7. ***Given***

$f = 768 \text{ Hz} \quad v = 325 \ \frac{\text{m}}{\text{s}}$

$L = \frac{\lambda}{4}$

Required
length of the shortest closed pipe for which resonance exists (L) under these conditions

Analysis and Solution
The shortest closed pipe that has an antinode at the open end has a length of $\lambda/4$. Use the universal wave equation to calculate the wavelength then calculate the length of the pipe.

$$v = f\lambda$$

$$\lambda = \frac{v}{f}$$

$$= \frac{325\ \frac{\text{m}}{\cancel{\text{s}}}}{768\ \frac{1}{\cancel{\text{s}}}}$$

$$= 0.423\ \text{m}$$

$$L = \frac{\lambda}{4}$$

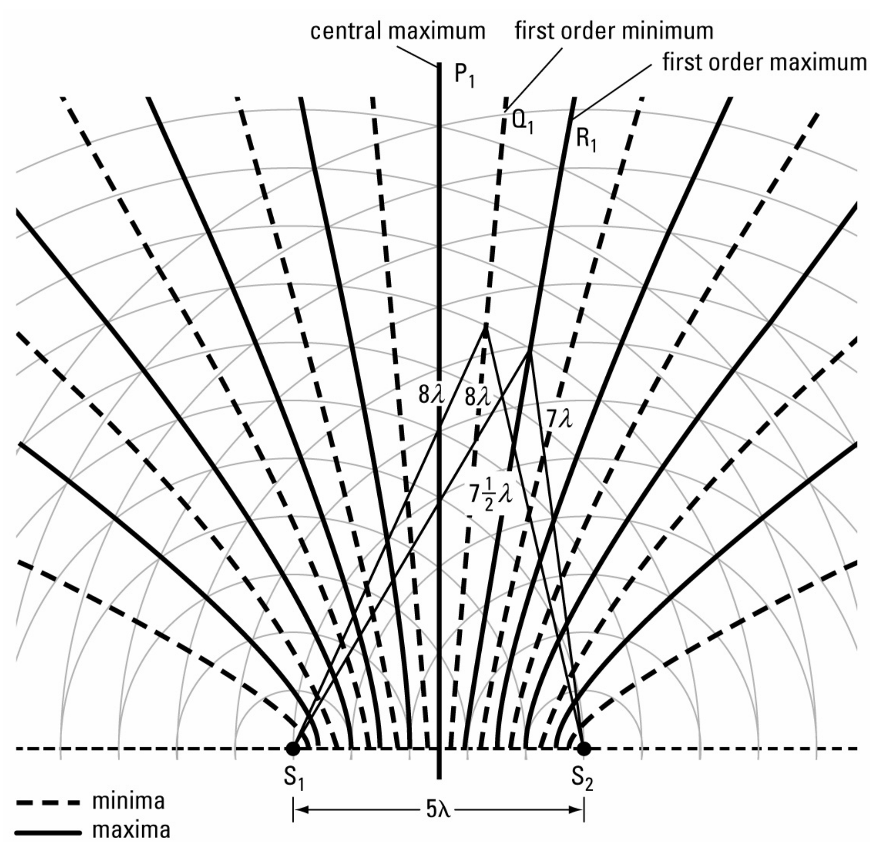
$$= \frac{0.423\ \text{m}}{4}$$

$$= 0.106\ \text{m}$$

Paraphrase and Verify

The shortest closed pipe that will have a resonance point at its open end under these conditions is 10.6 cm long.

8.



There are five minima on either side of the central maximum because the sources are five wavelengths apart. The greatest phase-shift that produces destructive interference is one-half wavelength less than the separation of the sources, so minima occur at phase-shifts of ½ λ, 3/2 λ, 5/2 λ, 7/2 λ and 9/2 λ.

9. ***Given***

$d_1 = 8.0 \text{ cm} \quad d_2 = 6.8 \text{ cm}$

Required

wavelength of the interference pattern (λ)

Analysis and Solution

On the second order maximum, the point must be two wavelengths farther from one source than the other. Thus the difference in the given distances must equal 2λ.

$$\begin{aligned} 2\lambda &= d_1 - d_2 \\ \lambda &= \frac{d_1 - d_2}{2} \\ &= \frac{8.0 \text{ cm} - 6.8 \text{ cm}}{2} \\ &= \frac{1.2 \text{ cm}}{2} \\ &= 0.60 \text{ cm} \end{aligned}$$

Paraphrase

The wavelength used to create this interference pattern is 0.60 cm.

Extension

10. Pipe organs actually use both open and closed pipes. Pipes (and strings) do not produce true harmonics. The physical dimensions of a pipe cause slight imperfections in the overtones. These result because the different frequencies require diameter-to-length measurements. Thus, a pipe that produces a true harmonic for one frequency will not produce overtones that are exact multiples of that frequency. As a result, each pipe produces a unique sound. The combinations of pipes that are used to produce the musical notes from a pipe organ are very complex. This complexity makes it virtually impossible to reproduce the sound using electronic reproduction.

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Concept Check

The Doppler effect is the result of the difference in your speed and the speed of the source of a sound. If you are travelling beside a train, at the same speed as the train, then there can be no Doppler effect. Thus, you will hear the true frequency of the train whistle, not a higher or a lower frequency.

Example 8.4 Practice Problems

1. ***Given***

$f_s = 264 \text{ Hz} \quad v_s = 60.0 \ \frac{\text{km}}{\text{h}} = 16.7 \ \frac{\text{m}}{\text{s}}$

$v_w = 340 \ \frac{\text{m}}{\text{s}}$

Required

apparent (Doppler) frequency of the sound (f_d)

Analysis and Solution

Use the equation for the Doppler effect for a wave source that is moving toward the observer.

$$f_d = \left(\frac{v_w}{v_w - v_s}\right) f_s$$

$$= \left(\frac{340 \ \frac{m}{s}}{340 \ \frac{m}{s} - 16.7 \ \frac{m}{s}}\right)(264 \text{ Hz})$$

$$= 278 \text{ Hz}$$

Paraphrase

As the car approaches you, the pitch of its horn would be increased to 278 Hz.

2. ***Given***

$$v_s = 360 \ \frac{km}{h} = 100 \ \frac{m}{s}$$

$$f_d = 512 \text{ Hz} \quad v_w = 345 \ \frac{m}{s}$$

Required

frequency of the source (f_s)

Analysis and Solution

Use the Doppler effect equation for a source moving toward the observer, and solve for the frequency of the source.

$$f_d = \left(\frac{v_w}{v_w - v_s}\right) f_s$$

$$f_s = \left(\frac{v_w - v_s}{v_w}\right) f_d$$

$$= \left(\frac{345 \ \frac{m}{s} - 100 \ \frac{m}{s}}{345 \ \frac{m}{s}}\right)(512 \text{ Hz})$$

$$= 364 \text{ Hz}$$

Paraphrase

Even though you hear it as a note with a frequency of 512 Hz, the frequency emitted by the airplane is actually 364 Hz.

3. ***Given***

$$v_s = 25.0 \ \frac{m}{s} \quad f_d = 260 \text{ Hz}$$

$$f_s = 240 \text{ Hz}$$

Required

speed of sound in air (v_w)

Analysis and Solution

Use the equation for the Doppler effect when the source is moving toward the observer, and solve for the speed of the wave.

$$f_d = \left(\frac{v_w}{v_w - v_s}\right) f_s$$

$$v_w = \left(\frac{f_d}{f_d - f_s}\right) v_s$$

$$= \left(\frac{260 \text{ Hz}}{260 \text{ Hz} - 240 \text{ Hz}}\right)\left(25.0 \ \frac{\text{m}}{\text{s}}\right)$$

$$= 325 \ \frac{\text{m}}{\text{s}}$$

Paraphrase

The speed of sound in air must be 325 m/s.

4. ***Given***

$f_d = 475 \text{ Hz}$ $f_s = 500 \text{ Hz}$

$v_w = 350 \ \frac{\text{m}}{\text{s}}$

Required

speed of the source (v_s)

Analysis and Solution

Use the equation for the Doppler effect for an wave source moving away from the observer and solve for the speed of the source.

$$f_d = \left(\frac{v_w}{v_w + v_s}\right) f_s$$

$$v_s = \left(\frac{f_s - f_d}{f_d}\right) v_w$$

$$= \left(\frac{500 \text{ Hz} - 475 \text{ Hz}}{475 \text{ Hz}}\right)\left(350 \ \frac{\text{m}}{\text{s}}\right)$$

$$= 18.4 \ \frac{\text{m}}{\text{s}}$$

Paraphrase

The train must have been moving at a speed of 18.4 m/s (66.3 km/h) away from the observer.

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8.4 Check and Reflect

Knowledge

1. When the source of waves is moving, the motion of the source results in changes to the wavelengths of waves which move away from the source. The wavelengths of the waves that move away from the source in the direction of its motion are decreased while the wavelengths of the waves that move away from the source in the direction opposite to its motion are increased in length. Since the speed of the waves is not dependent on the speed of the source, if the wavelength is decreased the frequency must be increased, and vice versa. Thus the frequency of the waves that move in the direction of the motion of the source must increase and those waves that move in the direction opposite to the motion of the source must decrease. The change in frequency is known as the Doppler effect.

2. The faster a sound source moves away from you the lower the frequency appears to be. Thus, if two sources with equal frequencies move away from you, the one that is moving faster will be the one for which you hear a lower frequency.

3. If the source of a sound wave is travelling at the same speed as sound then the wavelength of the sound that is moving in the direction of the source is zero. That means when the all sound waves pass you at the same time, rather than hearing a wave train with its energy spread out along the train, you hear all the energy in one single boom.

Applications

4. ***Given***

$f_s = 660\,\text{Hz} \quad v_\text{s} = 40.0\ \frac{\text{m}}{\text{s}}$

$v_\text{w} = 340\ \frac{\text{m}}{\text{s}}$

Required

frequency of the sound that you hear (f_d)

Analysis and Solution

Use the Doppler effect equation for a source moving towards you.

$$\begin{aligned} f_\text{d} &= \left(\frac{v_\text{w}}{v_\text{w} - v_\text{s}}\right) f_\text{s} \\ &= \left(\frac{340\ \frac{\text{m}}{\text{s}}}{340\ \frac{\text{m}}{\text{s}} - 40.0\ \frac{\text{m}}{\text{s}}}\right)(660\ \text{Hz}) \\ &= 748\ \text{Hz} \end{aligned}$$

Paraphrase

The siren of the police car appears to have a frequency of 748 Hz.

5. ***Given***

$f_\text{s} = 850\ \text{Hz} \quad f_\text{d} = 890\ \text{Hz}$

$v_\text{w} = 350\ \frac{\text{m}}{\text{s}}$

Required

speed of the police car (v_s)

Analysis and Solution

Since the frequency is increased the car must be moving toward you. Use the Doppler equation solved for the speed of the source.

$$f_d = \left(\frac{v_w}{v_w - v_s}\right) f_s$$

$$v_s = \left(\frac{f_d - f_s}{f_d}\right) v_w$$

$$= \left(\frac{890\text{ Hz} - 850\text{ Hz}}{890\text{ Hz}}\right)\left(350\ \frac{\text{m}}{\text{s}}\right)$$

$$= 15.7\ \frac{\text{m}}{s}$$

Paraphrase

The car is travelling at a speed of 15.7 m/s (56.5 km/h).

6. ***Given***

$v_s = v_w \quad f_s = 1.000 \times 10^3$ Hz

Required

(a) frequency that you would hear for the sound of the plane as it approaches you

(b) frequency that you would hear for the sound of the plane as it moves away from you

(c) description of what you would hear as the plane moves towards then away from you

Analysis and Solution

Use the equation for the Doppler effect in calculations of:

(a) the apparent frequency as the plane moves toward you

$$f_d = \left(\frac{v_w}{v_w - v_s}\right) f_s$$

$$= \left(\frac{v_w}{v_w - v_w}\right) f_s$$

$$= \left(\frac{v_w}{0}\right) f_s$$

$$= \text{ undefined}$$

(b) the apparent frequency as the plane moves away from you.

$$f_d = \left(\frac{v_w}{v_w + v_s}\right) f_s$$

$$= \left(\frac{v_w}{v_w + v_w}\right) f_s$$

$$= \left(\frac{v_w}{2v_w}\right) f_s$$

$$= \left(\frac{1}{2}\right) f_s$$

(c) Between these two cases, you hear a sonic boom at the instant of transition from case (a) to case (b).

Paraphrase and Verify

When a plane approaches at the speed of sound there is no sound wave preceding the plane. When a plane moves away from you at the speed of sound the apparent frequency of the sound is one-half the source frequency. Between these two cases, you hear a sonic boom at the instant of transition from case (a) to case (b).

Extension

7. The frequency of red light is less than the frequency of blue light. Thus if the light from a star is shifted from the blue end of the spectrum toward the red end, the frequency of the light has decreased. That means that the star must be moving away from us. If the light from all distant stars shows red shift, then all those stars must be moving away from Earth, proving that the universe must be enlarging.

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Chapter 8 Review

Knowledge

1. **(a)** When a transverse crest of a water wave moves, the water at the top of the crest moves downward. The water below the crest then has to move downward and sideways to make room for the water in the crest and to evacuate a space for the trough.

(b) When an incident wave train is reflected from a barrier, the angle between the incident wave front and the barrier is equal to the angle between the reflected wave front and the barrier.

(c) The oscillations of the tuning fork would set up a compression when they moved in the direction of the wave motion and a rarefaction when they moved away from the direction of the wave motion. The compression and rarefactions would move through the medium in ever expanding circles in a sequence of SHM.

2. **(a)** The speed of a wave in water is determined by the depth of the water.

(b) When a longitudinal wave moves through a medium, the particles of the medium oscillate, in sequence, in SHM.

(c) The wavelength varies directly as the speed of the wave while the amplitude varies independently of the speed.

(d) As a wave moves through a medium each particle of the medium oscillates in SHM either across the direction of motion of the wave (transverse waves) or along the direction of motion (longitudinal waves).

(e) When the speed of the wave is constant, the wavelength varies inversely as the frequency of a wave.

3. **(a)** Interference results when two waves occupy the same position in a medium. If the waves displace the medium in the same direction from the equilibrium, then constructive interference results. If the waves displace the medium in opposite directions from the equilibrium, then destructive interference results.

(b) When two pulses of equal length and amplitude interfere to produce no apparent pulse, destructive interference is taking place. The principle of superposition states that the displacement of the net pulse is the algebraic sum of the displacement of the pulses at each point in the medium. The pulses must have had equal but opposite displacements so that the sum at each point is zero.

(c) A node is the point in an interference pattern where interference produces only total (or nearly total) destruction of the wave. An antinode is the point in an interference pattern where the waves interfere to produce maximum construction.

A standing wave results when two waves of equal frequency and amplitude move in opposite directions through a medium. The effect is to produce a series of fixed nodes with antinodes between them.

(d) In a standing wave in a spring, there is a node at the fixed end of the spring. As you move away from that end of the spring there are nodes at every one-half wavelength along the spring with the last node at the opposite end of the spring. Between each node there is a region which is one-half a wavelength long with an antinode at the middle. Thus the length of the spring is an integral multiple of half wavelengths.

(e) If the frequency applied to a spring generates a wave such that the length of the spring is not an integral number of half wavelengths, then the waves travelling in one direction along the spring arrive at the other end of the spring out of phase with the waves leaving that end. In this case the waves interact destructively so that there is no wave pattern established. If, on the other hand, the length of the spring is an integral number of half wavelengths, then the waves arriving at opposite ends of the spring are in-phase with the waves leaving that end. This results in constructive interference and a strengthening of the wave motion in the phenomenon known as resonance.

4. **(a)** The Doppler effect applies to all forms of waves. For light waves, the Doppler effect is used to explain the red-shift in the spectra of light from stars. The Doppler effect is used with radio waves to measure moving objects (cars, planes, etc.). [**NOTE:** Students may not realize that radio waves and light waves are different forms of the same phenomenon.]

(b) The waves that a source emits in the direction of its motion are decreased in length. Thus observers detect these waves to have a higher frequency than that at which they were generated.

Applications

5. ***Given***

$v = 15.0\ \frac{\text{m}}{\text{s}} \quad l = 2.00\ \text{m}$

Required

period required to generate the pulse (Δt)

Explanation of why frequency is not used to describe pulses.

Analysis and Solution

Use the equation for calculating pulse length and solve for period.

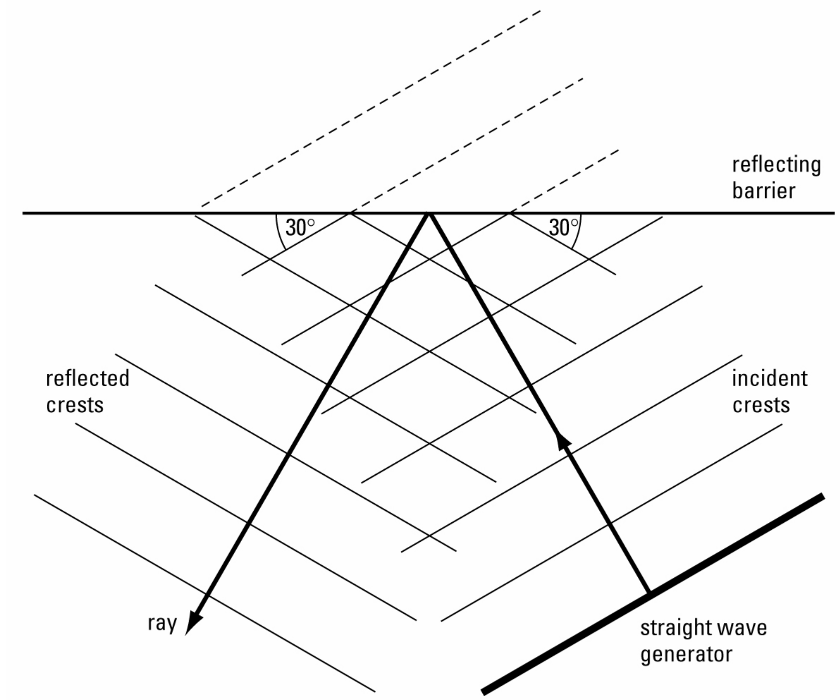
$$l = v\Delta t$$

$$\begin{aligned}\Delta t &= \frac{l}{v}\\ &= \frac{2.00\ \text{m}}{15.0\ \frac{\text{m}}{\text{s}}}\\ &= 0.133\ \text{s}\end{aligned}$$

Paraphrase and Verify

It required 0.133 s to generate the pulse. Since pulses are single occurrences and frequency implies a repeated occurrence, it makes no sense to convert the period required to generate a pulse into a frequency.

6. ***Analysis and Solution***



7. ***Given***

$v_1 = 15.0\ \frac{\text{cm}}{\text{s}}$ $v_2 = 10.0\ \frac{\text{cm}}{\text{s}}$

$f = 12.0\ \text{Hz}$

Required

changes in the waves as they move into shallow water

Analysis and Solution

Since the waves slow down but the frequency is unchanged the wavelengths will decrease.

$$v = f\lambda$$

$$\lambda_1 = \frac{v_1}{f}$$

$$= \frac{15.0\ \frac{\text{cm}}{\text{s}}}{12.0\ \frac{1}{\text{s}}}$$

$$= 1.25\ \text{cm}$$

Similarly

$$\lambda_2 = \frac{v_2}{f}$$

$$= \frac{10.0\ \frac{\text{cm}}{\text{s}}}{12.0\ \frac{1}{\text{s}}}$$

$$= 0.833\ \text{cm}$$

Paraphrase and Verify

When the waves pass from the deep water to the shallow water, their speed is reduced from 15.0 cm/s to 10.0 cm/s. Because the frequency is constant, the decrease in speed results in a corresponding decrease in wavelengths from 1.25 cm to 0.833 cm. Since the wave fronts are parallel to the line between the deep and shallow water they do not change direction.

8. ***Given***

$v_1 = 12.0\ \frac{\text{cm}}{\text{s}} \quad v_2 = 9.0\ \frac{\text{cm}}{\text{s}}$

$\lambda_1 = 11.5\ \text{cm}$

Required

wavelength in the shallow water (λ_2)

Analysis and Solution

Since the frequency is constant, the wavelength varies directly as the speed. Express this relationship as a ratio and solve for the final wavelength.

$$\lambda \propto v$$

$$\frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1}$$

$$\begin{aligned}\lambda_2 &= \lambda_1 \frac{v_2}{v_1} \\ &= 11.5\ \text{cm}\left(\frac{9.0\ \frac{\text{cm}}{\text{s}}}{12.0\ \frac{\text{cm}}{\text{s}}}\right) \\ &= 8.6\ \text{cm}\end{aligned}$$

Paraphrase

When the wave moves from the deep to the shallow water the wavelength decreases from 11.5 cm to 8.6 cm.

9. ***Given***

$f = 12.8\ \text{kHz} = 1.28 \times 10^4\ \text{Hz}$

$v = 350\ \frac{\text{m}}{\text{s}}$

Required

wavelength (λ)

Analysis and Solution

Use the universal wave equation to solve for wavelength.

$$v = f\lambda$$

$$\begin{aligned}\lambda &= \frac{v}{f} \\ &= \frac{350\ \frac{\text{m}}{\text{s}}}{1.28 \times 10^4\ \frac{1}{\text{s}}} \\ &= 2.73 \times 10^{-2}\ \text{m}\end{aligned}$$

Paraphrase

The wavelength of the ultrasound is 2.73×10^{-2} m.

10. ***Given***

$L = 7.0 \text{ m} \quad f = 2.0 \text{ Hz}$

$L = \dfrac{5\lambda}{2}$

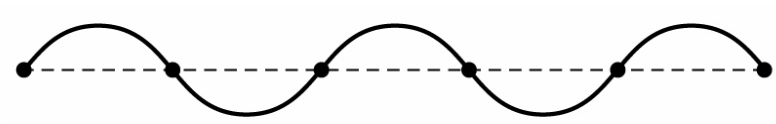
Required

(a) wave pattern for this system

(b) velocity of the wave (v)

Analysis and Solution

(a) The standing wave has 6 nodes and 5 antinodes, which means that it is five half-wavelengths, or 2½ λ, long.



(b) Use the length of the spring to calculate one wavelength; then use the frequency and wavelength to calculate wave velocity.

$$L = \frac{5\lambda}{2}$$

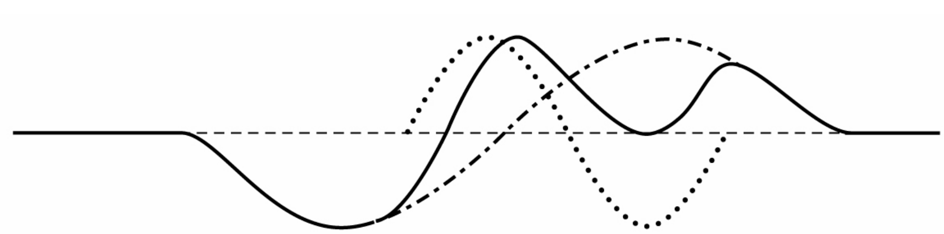
$$\begin{aligned}\lambda &= \frac{2}{5}L \\ &= \frac{2}{5}(7.0\,\text{m}) \\ &= 2.8 \text{ m}\end{aligned}$$

$$\begin{aligned}v &= f\lambda \\ &= \left(2.0\ \frac{1}{\text{s}}\right)(2.8 \text{ m}) \\ &= 5.6\ \frac{\text{m}}{\text{s}}\end{aligned}$$

Paraphrase

The wavelength as determined from the standing wave pattern is 2.8 m. This produces a wave velocity of 5.6 m/s.

11. ***Analysis and Solution***



Paraphrase

The solid line indicates the result of the interference of the two dotted waves.

12. ***Given***

$f_1 = 1.5\text{ Hz} \quad \lambda_1 = \frac{2L}{3}$

$\lambda_2 = \frac{2L}{5} \quad \lambda_3 = 2L$

Required

(a) frequency that produces the second standing wave pattern (f_2)

(b) fundamental frequency for the system (f_3)

Analysis and Solution

(a) Since the speed of the wave in the spring is constant, the frequency varies inversely as the wavelength. Use the ratio to calculate the frequency for the second standing wave pattern.

$$f \propto \frac{1}{\lambda}$$

$$\frac{f_2}{f_1} = \frac{\lambda_1}{\lambda_2}$$

$$\begin{aligned} f_2 &= f_1 \frac{\lambda_1}{\lambda_2} \\ &= \left(1.5 \frac{1}{\text{s}}\right)\left(\frac{\frac{2L}{3}}{\frac{2L}{5}}\right) \\ &= \left(1.5 \frac{1}{\text{s}}\right)\left(\frac{5}{3}\right) \\ &= 2.5 \ \frac{1}{\text{s}} \end{aligned}$$

(b) Similarly, calculate the frequency for the third standing wave pattern. This is the fundamental frequency where the length of the spring equals one-half a wavelength.

$$\frac{f_3}{f_1} = \frac{\lambda_1}{\lambda_3}$$

$$\begin{aligned} f_3 &= f_1 \frac{\lambda_1}{\lambda_3} \\ &= \left(1.5 \frac{1}{\text{s}}\right)\left(\frac{\frac{2L}{3}}{2L}\right) \\ &= \left(1.5 \frac{1}{\text{s}}\right)\left(\frac{1}{3}\right) \\ &= 0.50 \ \frac{1}{\text{s}} \end{aligned}$$

Paraphrase

(a) The frequency that produces a standing wave with 5 antinodes is 2.5 Hz.

(b) The fundamental frequency for the spring is 0.50 Hz.

13. *Given*

$L_1 = 33.0 \text{ cm} \quad f = 659 \text{ Hz}$

$\lambda_1 = 2L_1 \quad L_2 = 28.0 \text{ cm}$

$\lambda_2 = 2L_2$

Required

(a) speed of waves in the string (v)

(b) frequency when the length of the string is shortened (f_2)

Analysis and Solution

(a) Use the universal wave equation to calculate the velocity from frequency and wavelength.

$$\begin{aligned}\lambda_1 &= 2L_1 \\ &= 2(33.0 \text{ cm}) \\ &= 66.0 \text{ cm} \\ &= 0.660 \text{ m}\end{aligned}$$

$$\begin{aligned}v &= f_1\lambda_1 \\ &= \left(659 \ \frac{1}{\text{s}}\right)(0.660 \text{ m}) \\ &= 4.35\times10^2 \ \frac{\text{m}}{\text{s}}\end{aligned}$$

(b) Since the tension is unchanged, the velocity is the same as in part (a). Use the velocity and wavelength to calculate the frequency.

$$\begin{aligned}\lambda_2 &= 2L_2 \\ &= 2(28.0 \text{ cm}) \\ &= 56.0 \text{ cm} \\ &= 0.560 \text{ m}\end{aligned}$$

$$v = f_2\lambda_2$$

$$\begin{aligned}f_2 &= \frac{v}{\lambda_2} \\ &= \frac{4.35\times10^2 \ \frac{\text{m}}{\text{s}}}{0.560 \text{ m}} \\ &= 7.77\times10^2 \ \frac{1}{\text{s}}\end{aligned}$$

Paraphrase

(a) The speed of the wave in the string is 435 m/s.

(b) When the string is shortened to 28.0 cm the frequency increases to 777 Hz.

14. *Given*

$f = 426 \text{ Hz} \quad \text{v} = 335 \ \frac{\text{m}}{\text{s}}$

(a) $L_1 = \frac{\lambda}{4}$ (b) $L_2 = \frac{3\lambda}{4}$

Required

(a) length of the shortest closed pipe that can produce resonance (L_1)

(b) length of the next longest closed pipe that can produce resonance (L_2)

Analysis and Solution

(a) The shortest closed pipe for which resonance occurs is 0.25 wavelengths long. Calculate the wavelength using the universal wave equation; then calculate the length of the pipe.

$$v = f\lambda$$

$$\lambda = \frac{v}{f}$$

$$= \frac{335\ \frac{\text{m}}{\text{s}}}{426\ \frac{1}{\text{s}}}$$

$$= 0.786\ \text{m}$$

$$L_1 = \frac{\lambda}{4}$$

$$= \frac{0.786\ \text{m}}{4}$$

$$= 0.197\ \text{m}$$

(b) Use the same wavelength to calculate the length of the second-shortest pipe, which would be 0.75 wavelengths long.

$$L_2 = \frac{3\lambda}{4}$$

$$= \frac{3(0.786\ \text{m})}{4}$$

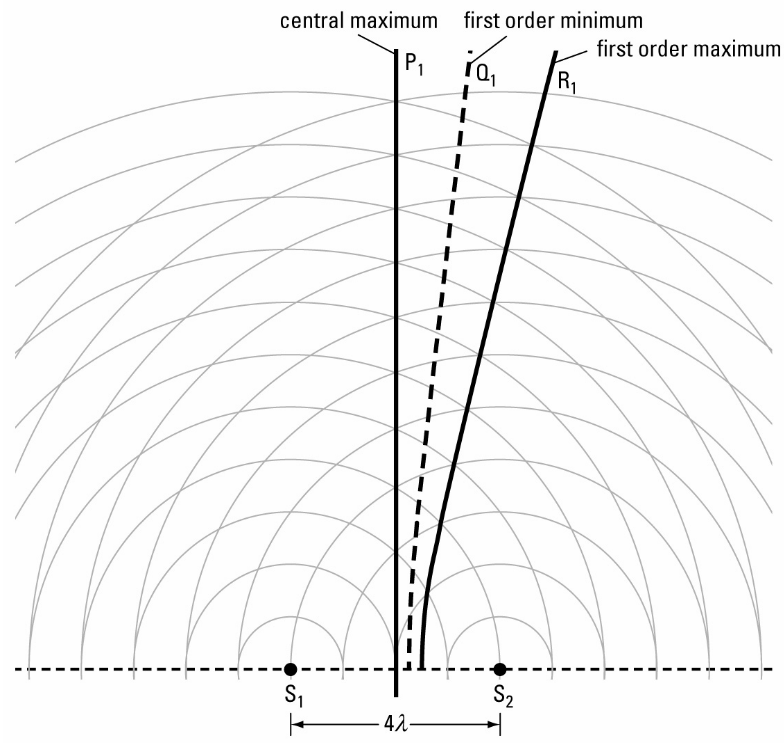
$$= 0.590\ \text{m}$$

Paraphrase

(a) The shortest closed pipe that will produce resonance for this frequency is 19.7 cm long.

(b) The second-shortest closed pipe that will produce resonance for this frequency is 59.0 cm long.

15. ***Analysis and Solution***



16. ***Given***

$2\lambda = 2.8\text{ cm}$

Required

wavelength (λ)

Analysis and Solution

On a second order maximum the points are 2λ farther from one source than the other. Thus the given distance equals 2λ. Use this relationship to calculate λ.

$$2\lambda = 2.8\text{ cm}$$

$$\lambda = \frac{2.8\text{ cm}}{2} = 1.4\text{ cm}$$

Paraphrase

The wavelength of the interference pattern is 1.4 cm.

17. ***Given***

$f_s = 290\text{ Hz} \quad v_w = 340\ \frac{\text{m}}{\text{s}}$

$v_s = 72.0\ \frac{\text{km}}{\text{h}} = 20.0\ \frac{\text{m}}{\text{s}}$

Required

apparent frequency of the sound (f_d)

Analysis and Solution

Use the Doppler effect equation for a source moving toward the observer.

$$f_d = \left(\frac{v_w}{v_w - v_s}\right) f_s$$

$$= \left(\frac{340\ \frac{\text{m}}{\text{s}}}{340\ \frac{\text{m}}{\text{s}} - 20.0\ \frac{\text{m}}{\text{s}}}\right)(290\ \text{Hz})$$

$$= 308\ \text{Hz}$$

Paraphrase

The apparent frequency of the approaching car horn has increased to 308 Hz.

18. ***Given***

$f_d = 580\ \text{Hz} \quad f_s = 540\ \text{Hz}$

$v_w = 350\ \frac{\text{m}}{\text{s}}$

Required

speed of the source (v_s)

Analysis and Solution

Use the Doppler equation for a sound moving away from the observer and solve for the speed of the source.

$$f_d = \left(\frac{v_w}{v_w - v_s}\right) f_s$$

$$v_s = \left(\frac{f_d - f_s}{f_d}\right) v_w$$

$$= \left(\frac{580\ \text{Hz} - 540\ \text{Hz}}{580\ \text{Hz}}\right)\left(350\ \frac{\text{m}}{\text{s}}\right)$$

$$= 24.1\ \frac{\text{m}}{\text{s}}$$

$$= 86.9\ \frac{\text{km}}{\text{h}}$$

Paraphrase

In order produce the increase in the apparent frequency, the sound must be moving toward you at a speed of 24.1 m/s (86.9 km/h).

19. ***Given***

$v_w = 350\ \frac{\text{m}}{\text{s}} \quad f_d = \frac{1}{2} f_s$

Required

speed of the source (v_s)

Analysis and Solution

Use the Doppler equation for a source moving away from the observer. Calculate the speed of the source.

$$f_{\text{d}} = \left(\frac{v_{\text{w}}}{v_{\text{w}} + v_{\text{s}}}\right) f_{\text{s}}$$

$$v_{\text{s}} = \left(\frac{f_{\text{s}} - f_{\text{d}}}{f_{\text{d}}}\right) v_{\text{w}}$$

$$= \left(\frac{f_{\text{s}} - \frac{1}{2} f_{\text{s}}}{\frac{1}{2} f_{\text{s}}}\right)\left(350\ \frac{\text{m}}{\text{s}}\right)$$

$$= \left(\frac{\frac{1}{2} f_{\text{s}}}{\frac{1}{2} f_{\text{s}}}\right)\left(350\ \frac{\text{m}}{\text{s}}\right)$$

$$= 350\ \frac{\text{m}}{\text{s}}$$

$$= 1260\ \frac{\text{km}}{\text{h}} \text{ or } 1.26 \times 10^3\ \frac{\text{km}}{\text{h}}$$

A source moving toward you at the speed of sound would not be heard until it produced a sonic boom.

Paraphrase and Verify

For the frequency that you hear to be one half of the true frequency, the speed of the source would need to be 350 m/s or 1.26×10^3 km/h. The source is moving away from you at the speed of sound. If the source had been moving toward you, you would not hear it until it passed you and you heard a sonic boom.

20. ***Given***

$$\lambda_{\text{s}} = 0.550\text{ m} \quad v_{\text{s}} = 120\ \frac{\text{km}}{\text{h}} = 33.3\ \frac{\text{m}}{\text{s}}$$

$$v_{\text{w}} = 345\ \frac{\text{m}}{\text{s}}$$

Required

apparent frequency of the siren (f_{d})

Analysis and Solution

Use the wavelength from the source and the speed of sound to calculate the frequency of the source; then use the Doppler effect equation for a source moving toward the observer to calculate the apparent frequency of the sound.

$$v_{\text{w}} = f_{\text{s}} \lambda_{\text{s}}$$

$$f_{\text{s}} = \frac{v_{\text{w}}}{\lambda_{\text{s}}}$$

$$= \frac{345\ \frac{\text{m}}{\text{s}}}{0.550\text{ m}}$$

$$= 627\ \frac{1}{\text{s}}$$

$$f_d = \left(\frac{v_w}{v_w - v_s}\right) f_s$$

$$= \left(\frac{345\ \frac{m}{s}}{345\ \frac{m}{s} - 33.3\ \frac{m}{s}}\right)(627\ Hz)$$

$$= 694\ Hz$$

Paraphrase

The movement of the car toward the observer causes its apparent frequency to increase from 627 Hz to 694 Hz.

21. ***Given***

$v_w = 350\ \frac{m}{s}$ $\quad f_d = 2 f_s$

Required

speed of the source (v_s)

Analysis and Solution

Use the Doppler effect equation for a sound moving toward you and calculate the speed of the source. Then use this calculated speed of the object to calculate the apparent frequency if the source had been moving away from you.

$$f_d = \left(\frac{v_w}{v_w - v_s}\right) f_s$$

$$v_s = \left(\frac{f_d - f_s}{f_d}\right) v_w$$

$$= \left(\frac{2f_s - f_s}{2f_s}\right)\left(350\ \frac{m}{s}\right)$$

$$= \left(\frac{f_s}{2f_s}\right)\left(350\ \frac{m}{s}\right)$$

$$= 175\ \frac{m}{s}$$

When the source moves away from you:

$$f_d = \left(\frac{v_w}{v_w + v_s}\right) f_s$$

$$= \left(\frac{350\ \frac{m}{s}}{350\ \frac{m}{s} + 175\ \frac{m}{s}}\right) f_s$$

$$= (0.667) f_s$$

A more elegant way to state the argument is:

$$\begin{aligned} f_d &= \left(\frac{v_w}{v_w + v_s}\right) f_s \\ &= \left(\frac{v_w}{v_w + \frac{1}{2}v_w}\right) f_s \\ &= \left(\frac{v_w}{\frac{3}{2}v_w}\right) f_s \\ &= \left(\frac{2}{3}\right) f_s \end{aligned}$$

Paraphrase

The object is moving towards you at half the speed of sound or 175 m/s. If the source is moving away from you, the apparent frequency would be 2/3 the source frequency.

Extensions

22. The interference pattern is for two in-phase point sources at a separation of 3 wavelengths. If the speed of sound is 350 m/s, then the wavelength for a source with a frequency of 512 Hz is about 0.683 m. To create this interference pattern in sound would require sources that are 2.05 m apart (3 × 0.683 m). The in-phase sources might be a pair of small speakers connected to an audio frequency generator set at the desired separation. To detect the maxima and minima using just your ears would be very difficult. You would need to use a microphone connected to an oscilloscope or computer. As the requirements to detect this interference pattern are so restricted, it is very unlikely that anyone would detect the pattern unless they had set up a system with the intention to do so.

23. The ratio of the wavelength to the distance between two in-phase point sources affects the interference pattern because the pattern depends on the phase shift produced when the waves from one source travel farther than the waves from the other source. The greatest phase shift occurs on the extension of the line between the sources. Thus if the sources are 4λ apart, then the greatest possible phase shift is 4λ , which occurs on the fourth order maximum. Thus this pattern will contain a central maximum with four maxima and four minima on either side of it.