Pearson Physics Level 20
Unit IV Oscillatory Motion and Mechanical Waves: Chapter 7 Solutions

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Example 7.1 Practice Problems

1. ***Analysis and Solution***

$$T = 5.00 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}}$$
$$= 300 \text{ s}$$
$$f = \frac{1}{T}$$
$$= \frac{1}{300 \text{ s}}$$
$$= 3.33 \times 10^{-3} \text{ Hz}$$

The frequency of the earthquake waves is 3.33×10^{-3} Hz.

2. ***Analysis and Solution***

$$T = \frac{1}{f}$$
$$= \frac{1}{78 \text{ Hz}}$$
$$= 0.013 \text{ s}$$

The period of the hummingbird's wing is 0.013 s.

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7.1 Check and Reflect

Knowledge

1. For motion to be oscillatory, it must repeat at regular intervals.
2. Equivalent units are revolutions/s and Hz.
3. Period: The time required for one complete oscillation (cycle)
Frequency: The number of oscillations (cycles) per second
4. $T = \frac{1}{f}$. They are the inverse of one another.
5. No, it is not possible. Frequency and period are inversely related to one another, so if the period increases, the frequency must decrease.
6. Students' answers will vary. Some possible answers could include: clock, wheel, bird/insect wings, planets, toys.

Applications

7. ***Analysis and Solution***

$$\frac{1\ \text{rev}}{\text{s}} = \frac{1\ \text{cycle}}{\text{s}}$$

$$\frac{20.0\,\text{rev}}{\text{s}} = \frac{20.0\,\text{cycles}}{\text{s}} = 20.0\,\text{Hz}$$

The frequency of the toy is 20.0 Hz.

8. The oars on a rowboat move with oscillatory motion if the rowing motion is at a constant rate.

9. ***Analysis and Solution***

$$f = \frac{1}{T} = \frac{1}{0.004\ 00\ \text{s}} = 2.50 \times 10^2\ \text{Hz}$$

The guitar string has a frequency of 2.50×10^2 Hz.

10. ***Analysis and Solution***

$$T = \frac{1}{f} = \frac{1}{38\,\text{Hz}} = 0.026\,\text{s}$$

The period of the dragonfly's wings is 0.026 s.

11. ***Analysis and Solution***

$$f = \frac{4800\ \text{beats}}{\cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{s}} = 80.00\ \text{Hz}$$

$$T = \frac{1}{f} = \frac{1}{80.00\ \text{Hz}} = 0.01250\ \text{s}$$

The period of the bird's wings is 0.01250 s.

12. (a) ***Analysis and Solution***

$$T = \frac{1}{f} = \frac{1}{2.50\ \text{Hz}} = 0.400\ \text{s}$$

The period of the dog's tail is 0.400 s.

(b) ***Analysis and Solution***

$$\#\text{wags} = 2.50 \text{ Hz} \times \frac{60 \text{ s}}{1 \text{ min}}$$

$$= 1.50 \times 10^2 \text{ wags}$$

The dog will wag its tail 1.50×10^2 times in 1 min.

Extensions

13. Students' answers will vary. Possible insects researched could include:

Insect	Frequency (Hz)
Mosquito	550
Housefly	330
Wasp	110
Ladybug (beetle)	46–90
Dragonfly	38
Butterfly	9–12

14. **(a)** This motion is periodic because the skater performs the same motion at the same speed.

(b) This motion can be considered periodic if the car's speed doesn't change. In practice, the motion is not periodic because the car's speed varies.

(c) This motion can be considered periodic when a person is at rest, but not when he or she changes the level of physical exertion.

15.

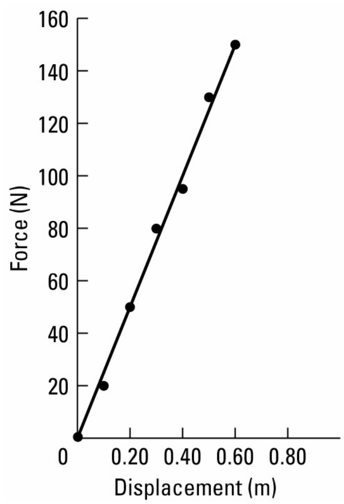
	Frequency (Hz)
(a) fluorescent light bulbs	120 Hz
(b) overhead power lines	60 Hz
(c) human voice range	Approx. 250 Hz–3000 Hz
(d) FM radio range	88 MHz–108 MHz
(e) lowest note on a bass guitar	41.2 Hz

Example 7.2 Practice Problems

1. ***Analysis and Solution***

Plotting the values gives the following graph:

Force vs. Displacement



The slope is:

$$k = \text{slope}$$
$$= \frac{(150 - 0.0)\,\text{N}}{(0.60 - 0.0)\,\text{m}}$$
$$= 2.5 \times 10^2\ \text{N/m}$$

The spring constant of this spring is 2.5×10^2 N/m.

2. ***Analysis and Solution***

Determine the slope (spring constant) by taking two values from the line on the graph. Below is an example of a calculation. Actual values chosen may vary.

$$k = \frac{(30.0 - 0.0)\,\text{N}}{(2.0 - 0.0)\,\text{m}}$$
$$= 15\ \text{N/m}$$

The spring constant of this spring is 15 N/m.

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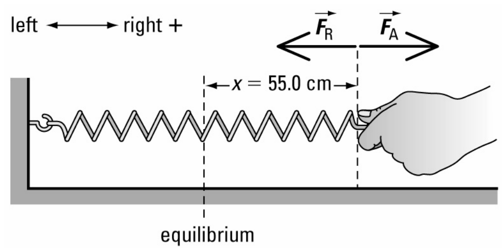
Example 7.3 Practice Problems

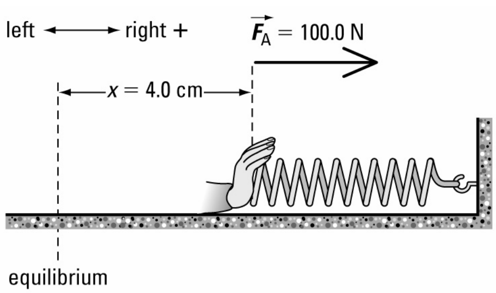
1. ***Analysis and Solution***



$$\vec{F} = -k\vec{x}$$
$$= -\left(48.0\,\frac{\text{N}}{\cancel{\text{m}}}\right)\left(0.550\,\cancel{\text{m}}\right)$$
$$= -26.4\,\text{N}$$

The restoring force of the spring is –26.4 N.

2. ***Analysis and Solution***



$$k = \frac{\vec{F}_A}{\vec{x}}$$
$$= \frac{100.0\,\text{N}}{0.040\,\text{m}}$$
$$= 2.5 \times 10^3\ \text{N/m}$$

The spring constant is 2.5×10^3 N/m.

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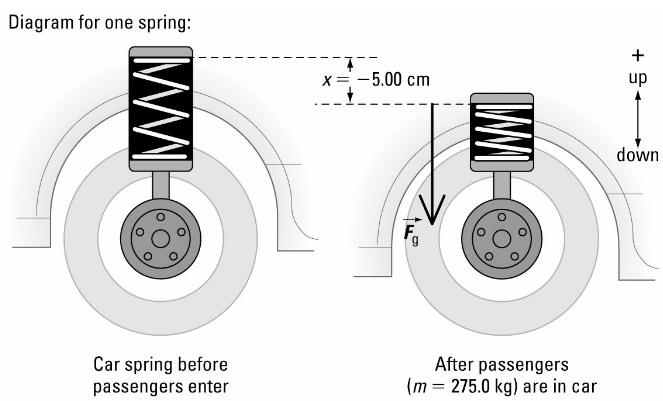
Example 7.4 Practice Problems

1. ***Given***

$m = 275.0\,\text{kg}$

$x = 0.0500\,\text{m}$

Required
spring constant (k)
Analysis and Solution
All four springs are compressed a distance of 5.00 cm by the force of gravity, so the weight on each spring is one-quarter of the combined weight. Determine the spring constant of each spring by dividing the total spring constant by 4.



$$F_g = \left(9.81\frac{\text{m}}{\text{s}^2}\right)(275.0\,\text{kg})$$
$$= 2697.75\,\text{N}$$

$$F = kx$$
$$k = \frac{2697.75\,\text{N}}{0.0500\,\text{m}}$$
$$= 53\,955\,\text{N/m}$$

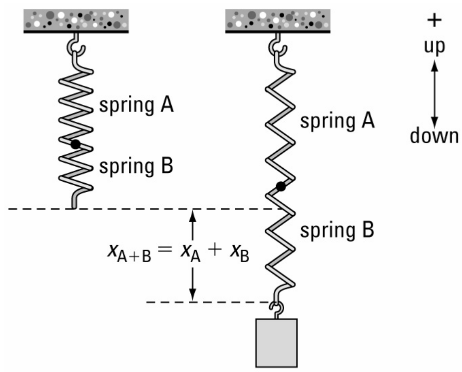
$$\frac{k}{4} \text{ for each spring}$$

$$k = \frac{53\,955\frac{\text{N}}{\text{m}}}{4}$$
$$= 1.35\times10^4\ \text{N/m}$$

Paraphrase
This spring constant of each spring in the car is 1.35×10^4 N/m .

2. ***Given***
$\vec{F}_g$ = 40.0 N [down]
k_A = 25 N/m
k_B = 60.0 N/m
Required
displacement (x)
Analysis and Solution
Both springs are affected by the weight of the mass attached to the end. Each spring will deform as if it has a 40.0 N mass attached to it. Assume that the springs are

massless. Determine the displacement for each, and then add the displacements together.



$$\vec{F}_g = k_A \vec{x}_A$$

$$\vec{x}_A = \frac{\vec{F}_g}{k_A} = \frac{-40.0\ \text{N}}{25\ \frac{\text{N}}{\text{m}}} = -1.6\ \text{m}$$

$$\vec{F}_g = k_B \vec{x}_B$$

$$\vec{x}_B = \frac{\vec{F}_g}{k_B} = \frac{-40.0\ \text{N}}{60.0\ \frac{\text{N}}{\text{m}}} = -0.667\ \text{m}$$

$$\vec{x}_A + \vec{x}_B = (-1.6\ \text{m}) + (-0.667\ \text{m}) = -2.3\ \text{m}$$

Paraphrase

The mass has a displacement of 2.3 m [down].

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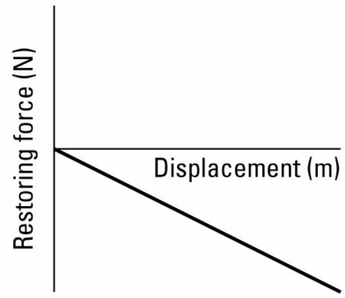
Concept Check

1. The force-displacement graph must always intercept the origin because there will be no displacement if no force is applied.

2. The line will become non-linear if the elastic limit of the spring is exceeded, and the spring permanently deforms or breaks.

3. The mass on the end of the spring will momentarily come to a stop at its maximum displacements—the top or bottom of its movement. The student should take a picture when the mass is in one of these positions.

4.



5. The negative sign indicates that the restoring force is always opposite to the spring's displacement.

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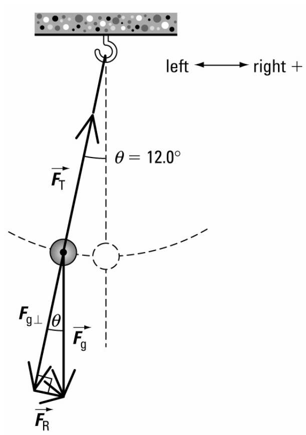
Example 7.5 Practice Problems

1. ***Analysis and Solution***



$$\begin{aligned} F_{g\perp} &= F_g \sin\theta \\ &= (0.3000\,\text{kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)(\sin 12.0°) \\ &= 0.612\,\text{N} \end{aligned}$$

The restoring force on the pendulum is 0.612 N [right].

2. ***Analysis and Solution***

$$F_{g\perp} = F_g \sin\theta$$

$$\begin{aligned} \theta &= \sin^{-1}\left(\frac{F_{g\perp}}{F_g}\right) \\ &= \sin^{-1}\left(\frac{4.00\,\text{N}}{(0.500\,\text{kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)}\right) \\ &= 54.6° \end{aligned}$$

An angle of 54.6° results in a restoring force of 4.00 N [left].

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7.2 Check and Reflect

Knowledge

1. Two factors affecting the restoring force of a pendulum are:
(i) the angle through which it is displaced
(ii) the mass of the bob

2.

Mass-spring System	**Displacement**	**Acceleration**	**Velocity**	**Restoring Force**
max $\vec{x}$		max	0	max
max $\vec{a}$	max		0	max
max $\vec{v}$	0	0		0
min $\vec{F}$	0	0	max	

Pendulum System	**Displacement**	**Acceleration**	**Velocity**	**Restoring Force**
max $\vec{x}$		max	0	max
max $\vec{a}$	max		0	max
max $\vec{v}$	0	0		0
min $\vec{F}$	0	0	max	

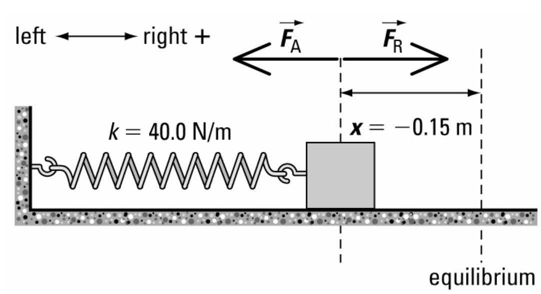
3. A pendulum is not a true simple harmonic oscillator because the restoring force does not vary linearly with displacement.

Applications

4. ***Given***
m = 2.0 kg
k = 40.0 N/m
$\vec{x}$ = −0.15 m

Required
restoring force ($\vec{F}_R$)

Analysis and Solution
Use Hooke's law to determine the restoring force. Mass is not required in the solution.



$$\vec{F}_R = -k\vec{x}$$
$$= -\left(40.0\frac{\text{N}}{\text{m}}\right)(-0.15\text{ m})$$
$$= 6.0\text{ N}$$

Paraphrase

The restoring force is 6.0 N in the opposite direction of the displacement.

5. ***Given***

$m = 4.0$ kg

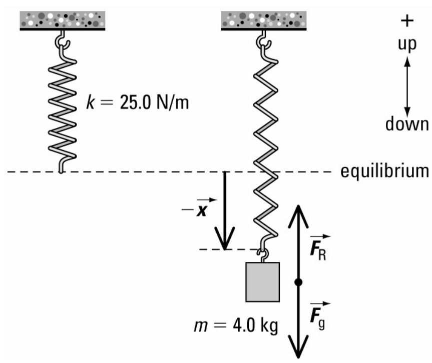
$k = 25.0$ N/m

Required

displacement ($\vec{x}$)

Analysis and Solution

Use the equation $\vec{F} = k\vec{x}$ to determine the displacement. You do not need to use a negative sign ($\vec{F} = -k\vec{x}$) because the force of gravity pulling the mass down is the applied force, not the restoring force.



$$\vec{F}_g = (4.0\text{ kg})\left(-9.81\frac{\text{m}}{\text{s}^2}\right)$$
$$= -39.24\text{ N}$$
$$\vec{F}_g = k\vec{x}$$
$$\vec{x} = \frac{\vec{F}_g}{k}$$
$$= \frac{-39.24\text{ N}}{25.0\frac{\text{N}}{\text{m}}}$$
$$= -1.6\text{ m}$$

Paraphrase

The displacement of the mass is 1.6 m [down].

6. ***Given***

$\vec{F} = -25.0$ N

$\vec{x} = -0.20$ m

Required

force required to pull the mass to a displacement of +0.15 m ($\vec{F}$)

Analysis and Solution
Use the initial force and displacement values to determine the spring constant. Then use the spring constant to determine the force necessary to displace the mass +0.15 m. Use Hooke's law without the negative sign because the force is applied.

$$\vec{F} = k\vec{x}$$

$$k = \frac{\vec{F}}{\vec{x}} = \frac{-25.0\text{ N}}{-0.20\text{ m}} = 125\text{ N/m}$$

$$\vec{F} = k\vec{x} = \left(125\ \frac{\text{N}}{\cancel{\text{m}}}\right)\left(0.15\ \cancel{\text{m}}\right) = 1.9 \times 10^{1}\text{ N}$$

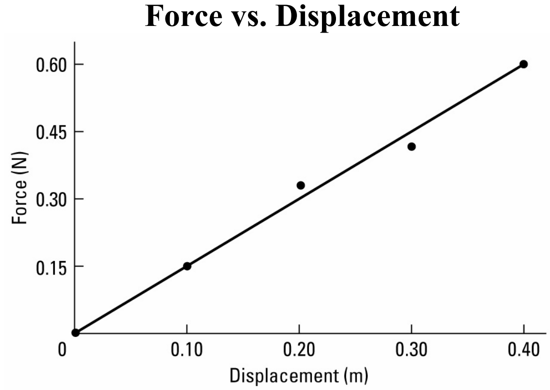
Paraphrase
The force required to move the mass through a displacement of 0.15 m is 19 N.

7. ***Given***

Displacement (m)	**Force (N)**
0.00	0.00
0.10	0.15
0.20	0.33
0.30	0.42
0.40	0.60

Required
spring constant (k)
Analysis and Solution
Using the data provided, plot a graph and find the slope.

Force vs. Displacement



You can choose any values from the line of best fit. For example, the following calculation uses these values:
$(x_1, y_1) = (0.0, 0.0)$ and $(x_2, y_2) = (0.34, 0.51)$

$$\begin{aligned}\text{slope} &= k \\ &= \frac{(0.51-0.0)\,\text{N}}{(0.34-0.0)\,\text{m}} \\ &= 1.5\,\text{N/m}\end{aligned}$$

Paraphrase

The spring constant is 1.5 N/m.

8. ***Analysis and Solution***

$$\begin{aligned}F_{g\perp} &= F_g \sin\theta \\ &= (0.400\,\text{kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)\sin 5.00° \\ &= 0.342\,\text{N}\end{aligned}$$

The restoring force on the pendulum is 0.342 N.

9. ***Given***

$\vec{x}$ = 20.0 cm [left]
m = 0.0100 kg
$\vec{a}$ = 0.55 m/s^2 [right]

Required

spring constant (k)

Analysis and Solution

Use Newton's second law ($\vec{F} = m\vec{a}$) to determine the restoring force. Determine the spring constant from the restoring force by using Hooke's law.

$$\begin{aligned}\vec{F} &= -k\vec{x} \\ m\vec{a} &= -k\vec{x} \\ k &= \frac{-m\vec{a}}{\vec{x}} \\ &= \frac{-(0.0100\,\text{kg})\left(0.55\frac{\text{m}}{\text{s}^2}\right)}{-0.20\,\text{m}} \\ &= 0.028\,\text{N/m}\end{aligned}$$

Paraphrase

The spring constant of the car's spring is 0.028 N/m.

Extension

10. ***Given***

Sample data:

Ruler	Deflection $\vec{x}$ (cm)	Mass (g)	$\vec{F}_g$ (N)
Plastic	−4.6	200.0	−1.96
Wood	−2.2	500.0	−4.91
Metal	−3.0	40.0	−0.392

Required
spring constant of each ruler (k)
Analysis and Solution
The force of gravity is different for each ruler, so you must determine it for each one. The values are recorded in the table. Use the equation $\vec{F} = k\vec{x}$ to find k for each ruler.

$$\vec{F}_{\text{g}} = k_{\text{plastic}}\vec{x}$$

$$\begin{aligned} k_{\text{plastic}} &= \frac{\vec{F}_{\text{g}}}{\vec{x}} \\ &= \frac{-1.96\ \text{N}}{-0.046\ \text{m}} \\ &= 43\ \text{N/m} \end{aligned}$$

$$\vec{F}_{\text{g}} = k_{\text{wood}}\vec{x}$$

$$\begin{aligned} k_{\text{wood}} &= \frac{\vec{F}_{\text{g}}}{\vec{x}} \\ &= \frac{-4.91\ \text{N}}{-0.022\ \text{m}} \\ &= 2.2 \times 10^{2}\ \text{N/m} \end{aligned}$$

$$\vec{F}_{\text{g}} = k_{\text{metal}}\vec{x}$$

$$\begin{aligned} k_{\text{metal}} &= \frac{\vec{F}_{\text{g}}}{\vec{x}} \\ &= \frac{-0.392\ \text{N}}{-0.030\ \text{m}} \\ &= 13\ \text{N/m} \end{aligned}$$

Paraphrase
The rank of the rulers from highest to lowest spring constants is:

$$k_{\text{wood}} = 2.2 \times 10^{2}\ \text{N/m}$$

$$k_{\text{plastic}} = 43\ \text{N/m}$$

$$k_{\text{metal}} = 13\ \text{N/m}$$

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Concept Check

1. According to Hooke's Law, the restoring force always acts to return the object to its equilibrium position. To do this, it must act opposite to the displacement. Since the restoring force is simply the product of mass and acceleration, the acceleration must also be opposite of the displacement.
2. The object is accelerating, and the acceleration is not uniform. This results in a curved velocity-time graph. From chapter 1, we know that the slope of the velocity-time graph represents acceleration. The acceleration of the object is greatest at the object's maximum displacement, which is why the velocity-time graph is steep at the extremes of the object's motion. When the object is in the equilibrium position, it does not

experience any acceleration, so the slope of the velocity-time graph in this position is zero.

3. For the other half of the oscillator's journey, the acceleration-displacement graph doesn't change. The object simply moves backward from (c) to (b) to (a). The velocity-displacement graph is different. For the second half of the object's motion, the velocity is in the opposite direction, so it is negative and follows the path from (c) to (b) to (a) as well.

4. The equation for maximum velocity is:

$$v_{\max} = A\sqrt{\frac{k}{m}}$$

If the amplitude (A) is doubled, the maximum velocity will also be doubled as shown below:

$$v_{\max} = 2A\sqrt{\frac{k}{m}} \quad \text{or}$$

$$v_{\max} = 2\left(A\sqrt{\frac{k}{m}}\right)$$

The magnitude of the velocity would be twice as large.

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Example 7.6 Practice Problems

1. ***Given***

$m = 0.724$ kg

$k = 8.21$ N/m

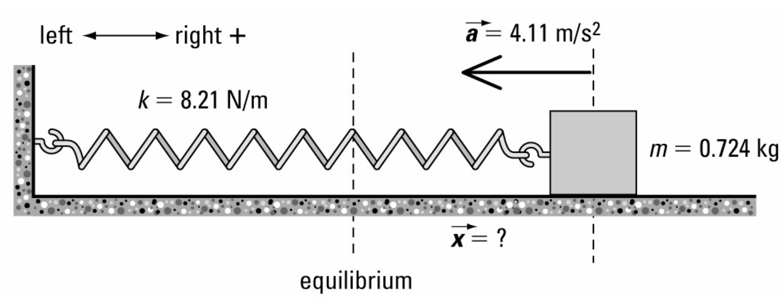
$\vec{a} = 4.11 \text{ m/s}^2$ [left]

Required

displacement ($\vec{x}$)

Analysis and Solution

Determine the displacement from the acceleration equation. Take right to be positive and left to be negative.



$$\vec{F} = -k\vec{x}$$

$$m\vec{a} = -k\vec{x}$$

$$\vec{x} = \frac{m\vec{a}}{-k}$$

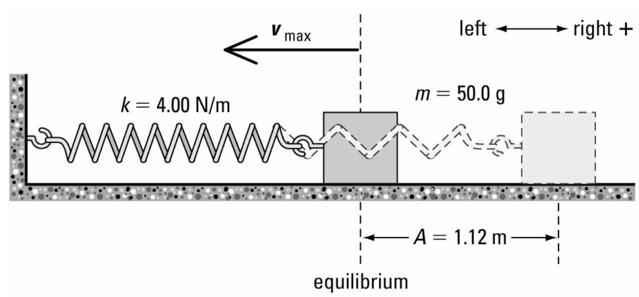
$$= \frac{(0.724\,\text{kg})\left(-4.11\,\frac{\text{m}}{\text{s}^2}\right)}{-8.21\,\frac{\text{N}}{\text{m}}}$$

$$= +0.362\,\text{m}$$

Paraphrase

The displacement of the mass when it experiences an acceleration of 4.11 m/s^2 [left] is 0.362 m [right].

2. ***Analysis and Solution***



$$v_{\text{max}} = A\sqrt{\frac{k}{m}}$$

$$= 1.12\text{ m}\sqrt{\frac{4.00\ \frac{\text{N}}{\text{m}}}{0.0500\text{ kg}}}$$

$$= 10.0\text{ m/s}$$

The maximum speed of the mass is 10.0 m/s.

3. ***Given***

k = 6.05 N/m

A = 81.7 cm

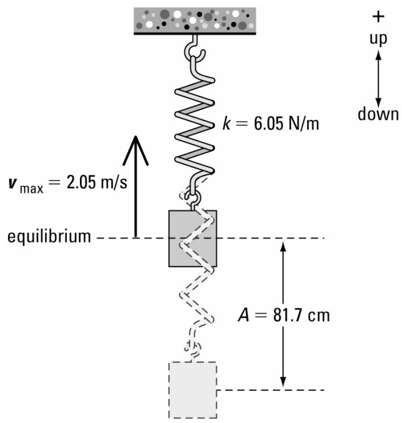
$v_{\text{max}} = 2.05$ m/s

Required

mass of the oscillator (m)

Analysis and Solution

Find the mass of the oscillator by manipulating the maximum velocity equation.



$$v_{max} = A\sqrt{\frac{k}{m}}$$

$$v_{max}^2 = A^2\frac{k}{m}$$

$$m = \frac{A^2 k}{v_{max}^2}$$

$$= \frac{(0.817\text{ m})^2\left(6.05\ \frac{\text{N}}{\text{m}}\right)}{\left(2.05\ \frac{\text{m}}{\text{s}}\right)^2}$$

$$= 0.961\text{ kg}$$

Paraphrase

The mass of the oscillator is 0.961 kg.

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Example 7.7 Practice Problems

1. (a) ***Given***

$m = 2.50$ kg

$k = 40.0$ N/m

$A = 0.800$ m

Required

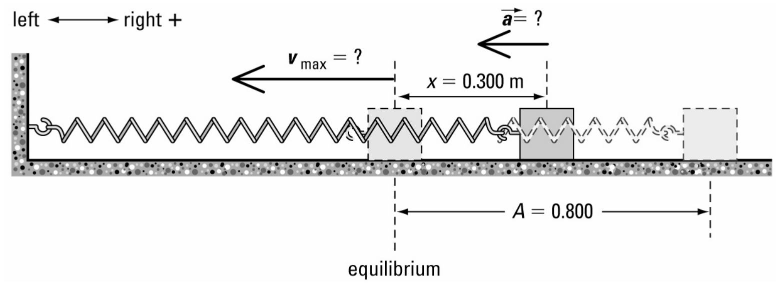
(a) acceleration at a displacement of 0.300 m to the right ($\vec{a}$)

(b) maximum speed (v_{max})

(c) period (T)

Analysis and Solution

Take left to be negative and right to be positive.



a)

$$\vec{F} = -k\vec{x}$$
$$m\vec{a} = -k\vec{x}$$
$$\vec{a} = \frac{-k\vec{x}}{m}$$
$$= \frac{-\left(40.0\,\frac{\text{N}}{\cancel{\text{m}}}\right)(0.300\,\cancel{\text{m}})}{2.50\text{ kg}}$$
$$= -4.80\,\text{m/s}^2$$

b)

$$v_{\text{max}} = A\sqrt{\frac{k}{m}}$$
$$= 0.800\,\text{m}\sqrt{\frac{40.0\,\frac{\text{N}}{\text{m}}}{2.50\text{ kg}}}$$
$$= 3.20\text{ m/s}$$

c)

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$T = 2\pi\sqrt{\frac{2.50\,\text{kg}}{40.0\,\frac{\text{N}}{\text{m}}}}$$
$$= 1.57\text{ s}$$

Paraphrase

(a) The acceleration of the mass when it is 0.300 m to the right is −4.80 m/s^2.

(b) The maximum speed of the mass is 3.20 m/s.

(c) The period of the mass is 1.57 s.

2. ***Given***

m = 2.60 g

$\vec{a}$ = 20.0 m/s^2

$\vec{x}$ = 0.700 m

Required

spring constant (k)

Analysis and Solution

Determine the spring constant from the acceleration equation. No directions are given, so take the quantities of acceleration and displacement as scalars. No negative sign is required in the equation for acceleration.

$$\vec{F} = k\vec{x}$$
$$m\vec{a} = k\vec{x}$$
$$\vec{a} = \frac{k\vec{x}}{m}$$
$$k = \frac{ma}{x}$$
$$= \frac{(0.00260\ \text{kg})\left(20.0\ \frac{\text{m}}{\text{s}^2}\right)}{0.700\ \text{m}}$$
$$= 0.0743\ \text{N/m}$$

Paraphrase

The spring constant of the spring is 0.0743 N/m.

3. ***Given***

$T = 2.0$ s

$k = 20.0$ N/m

Required

mass (m)

Analysis and Solution

Manipulate the equation for the period of a simple harmonic oscillator to solve for the mass.

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$T^2 = 4\pi^2\frac{m}{k}$$
$$m = \frac{T^2k}{4\pi^2}$$
$$= \frac{(2.0\ \text{s})^2\left(20.0\ \frac{\text{N}}{\text{m}}\right)}{4\pi^2}$$
$$= 2.0\ \text{kg}$$

Paraphrase

The mass of the oscillator is 2.0 kg.

4. ***Given***

$k = 12.07$ N/m

$A = 71.3$ cm

$v_{\text{max}} = 7.02$ m/s

Required

period (T)

Analysis and Solution
You cannot determine the period of the oscillator directly. Determine the mass first; then use the mass to solve for the period. All values are scalars so you do not need to specify directions. Convert the amplitude to appropriate SI units.

Solve for mass first:

$$v_{max} = A\sqrt{\frac{k}{m}}$$

$$v_{max}^2 = A^2\frac{k}{m}$$

$$\begin{aligned} m &= A^2\frac{k}{v_{max}^2} \\ &= \frac{(0.713\ \text{m})^2\left(12.07\,\frac{\text{N}}{\text{m}}\right)}{\left(7.02\,\frac{\text{m}}{\text{s}}\right)^2} \\ &= 0.1245\ \text{kg} \end{aligned}$$

Use the mass to solve for period:

$$\begin{aligned} T &= 2\pi\sqrt{\frac{m}{k}} \\ &= 2\pi\sqrt{\frac{0.1245\ \text{kg}}{12.07\,\frac{\text{N}}{\text{m}}}} \\ &= 0.638\ \text{s} \end{aligned}$$

Paraphrase
The period of the oscillator is 0.638 s.

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Concept Check

1. Doubling the displacement has no effect on the period of oscillation of a simple harmonic oscillator.

2. The two conditions that must be satisfied are:
(i) The period of the oscillator and the circular motion must be the same.
(ii) The amplitude of the oscillator must match the radius of the circular motion.

3. The period of a mass-spring oscillator is given by the equation $T = 2\pi\sqrt{\frac{m}{k}}$. If k and m are both doubled then we could write the equation as $T = 2\pi\sqrt{\frac{2m}{2k}}$. Since both are increased by a factor of two, there is no change. This can be shown mathematically by cancelling the 2 in the numerator and the 2 in the denominator in the fraction:

$T = 2\pi\sqrt{\frac{\cancel{2}m}{\cancel{2}k}}$. This leaves the equation that we started with.

4. The equation for period is:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Solving for k gives:

$$k = \frac{4\pi^2 m}{T^2}$$

If T is doubled, then the equation can be written:

$$k = \frac{4\pi^2 m}{(2T)^2} \text{ or } k = \frac{4\pi^2 m}{4T^2}$$

The difference between the last equation and the initial equation for k is a factor of one-quarter. The spring constant of the oscillator with twice the period is one-quarter of that of the other spring.

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Example 7.8 Practice Problems

1. ***Analysis and Solution***

The gravitational field strength is always directed toward the centre of the planet. Solve for the magnitude of the field strength only.

$$\begin{aligned} g &= \frac{4\pi^2 l}{T^2} \\ &= \frac{4\pi^2 (0.500 \text{ m})}{(2.30 \text{ s})^2} \\ &= 3.73 \text{ m/s}^2 \end{aligned}$$

The magnitude of the gravitational field strength on Mercury is 3.73 m/s^2 .

2. ***Analysis and Solution***

Only the scalar quantity of pendulum length is required.

$$\begin{aligned} l &= \frac{gT^2}{4\pi^2} \\ &= \frac{\left(1.63 \frac{\text{m}}{\cancel{\text{s}^2}}\right)(5.00 \cancel{\text{s}})^2}{4\pi^2} \\ &= 1.03 \text{ m} \end{aligned}$$

The length of the pendulum is 1.03 m.

3. ***Analysis and Solution***

Convert the pendulum length to SI units.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$= 2\pi\sqrt{\frac{0.300 \text{ m}}{\left(3.71\frac{\text{m}}{\text{s}^2}\right)}}$$

$$= 1.79 \text{ s}$$

The period of the pendulum on Mars is 1.79 s.

Concept Check

1. The velocity of the swinging apple will be the slowest near the extremes of its motion. It will come to an instantaneous stop at its maximum displacements. It would be easiest to hit in these positions.

2. The factors that affect the accuracy of pendulum clocks are:

- temperature: A change in temperature will cause the metal arm to expand or contract, changing its length and changing its period.
- altitude and latitude: A pendulum clock's period depends on the acceleration of gravity, which changes with these factors.

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7.3 Check and Reflect

Knowledge

1. (a) Changing the amplitude has no effect.

(b) Changing the spring constant has an inverse effect on the period as shown by the equation:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

If the spring constant gets larger, the period decreases. If the spring constant becomes smaller, the period increases.

(c) The period is directly related to the square root of the mass. If the mass increases, so does the period, as shown by this equation:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

2. (a) The effect of changing the amplitude of a pendulum is the same as the effect of changing it in a mass-spring system; it has no effect. This is true for any simple harmonic oscillator.

(b) The gravitational field strength is inversely related to the period. Therefore, they will have the opposite effect on one another, just as the spring constant does for the mass-spring system.

(c) Mass has no effect on the period of a pendulum.

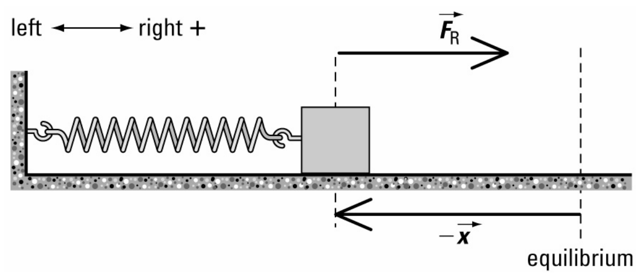
3. (a) At maximum acceleration, both systems are at their maximum displacement.

(b) The velocity of both systems is a maximum when the oscillator is in the equilibrium position.

(c) The restoring force varies as the acceleration does. Therefore, its maximum will be at the maximum displacements.

4. The acceleration of a simple harmonic oscillator is not uniform because the force varies with displacement according to Hooke's law. The closer the mass is to the equilibrium position, the less force acts on it. Therefore, it experiences less acceleration than if it were farther away. The acceleration varies linearly with displacement just as the force does.

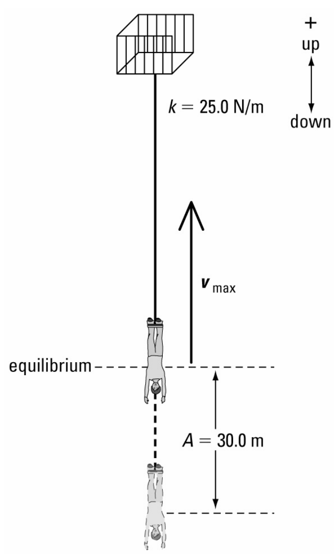
5. Restoring force and acceleration act in the same direction, so the acceleration is also positive.



Applications

6. ***Analysis and Solution***

The bungee jumper will experience the fastest speed when he is passing through the equilibrium position.



Determine his fastest speed using the following equation:

$$v_{max} = A\sqrt{\frac{k}{m}}$$

$$= 30.0\ \frac{\text{m}}{\text{s}}\sqrt{\frac{25.0\ \frac{\text{N}}{\text{m}}}{60.0\ \text{kg}}}$$

$$= 19.4\ \text{m/s}$$

The fastest the bungee jumper will move is 19.4 m/s.

7. ***Given***

$T = 4.0$ s

$g_{\text{Mars}} = 3.71\,\text{m/s}^2$

Required

length of the pendulum (l)

Analysis and Solution

The period of a pendulum varies with gravitational field strength. Modify the pendulum period equation to solve for length.

$$l = \frac{T^2 g}{4\pi^2}$$

$$= \frac{(4.0\ \text{s})^2 \left(3.71\ \frac{\text{m}}{\text{s}^2}\right)}{4\pi^2}$$

$$= 1.5\,\text{m}$$

Paraphrase

The length of the pendulum is 1.5 m

8. ***Given***

$m = 3.08$ kg

$T = 0.323$ s

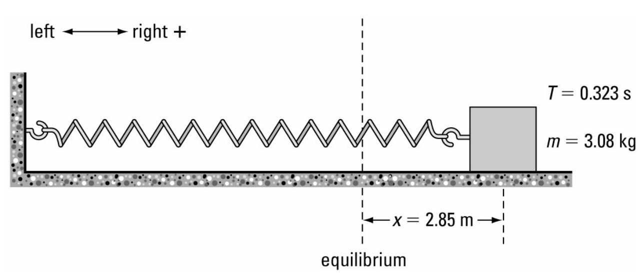
$\vec{x} = 2.85\,\text{m}$ [right]

Required

acceleration ($\vec{a}$)

Analysis and Solution

The oscillator's acceleration changes every half cycle, so we need to pay close attention to its vector nature. Acceleration to the right is positive, and to the left is negative. You cannot solve for the acceleration in one step. Determine the spring constant first; then use the spring constant to find the acceleration.



Solve for the spring constant:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T^2 = 4\pi^2 \frac{m}{k}$$

$$k = 4\pi^2 \frac{m}{T^2}$$

$$= 4\pi^2 \frac{3.08\,\text{kg}}{(0.323\,\text{s})^2}$$

$$= 1165.48\,\text{N/m}$$

Now solve for acceleration:

$$\vec{F} = -k\vec{x}$$

$$m\vec{a} = -k\vec{x}$$

$$\vec{a} = \frac{-k\vec{x}}{m}$$

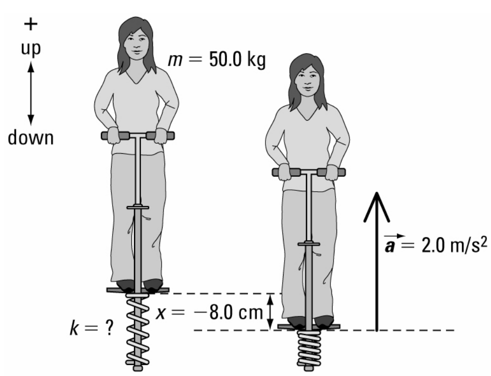
$$= \frac{-\left(1165.48\frac{\text{N}}{\text{m}}\right)(+2.85\,\text{m})}{3.08\,\text{kg}}$$

$$= -1.08 \times 10^3\ \text{m/s}^2$$

Paraphrase

The acceleration of the mass is $1.08 \times 10^3\ \text{m/s}^2$ [left].

9. ***Analysis and Solution***



$$\vec{F} = -k\vec{x}$$

$$m\vec{a} = -k\vec{x}$$

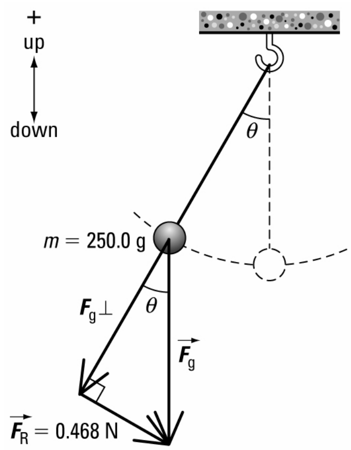
$$k = \frac{-m\vec{a}}{\vec{x}}$$

$$= \frac{-(50.0\,\text{kg})\left(2.0\frac{\text{m}}{\text{s}^2}\right)}{-0.080\,\text{m}}$$

$$= +1.3 \times 10^3\ \text{N/m}$$

The pogo stick has a spring constant of 1.3×10^3 N/m.

10. ***Analysis and Solution***



$$F_{g\perp} = F_g \sin\theta$$

$$\sin\theta = \frac{F_{g\perp}}{F_g}$$

$$\theta = \sin^{-1}\left(\frac{F_{g\perp}}{mg}\right)$$

$$= \sin^{-1}\left(\frac{0.468\,\text{N}}{(0.2500\,\text{kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)}\right)$$

$$= 11.0°$$

The pendulum bob is displaced through an angle of 11.0°.

11. ***Analysis and Solution***

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$= 2\pi\sqrt{\frac{0.500\,\text{m}}{1.64\frac{\text{m}}{\text{s}^2}}}$$

$$= 3.47\,\text{s}$$

The period of the pendulum is 3.47 s.

Extensions

12. (a) ***Given***

Horizontal mass-spring system:

A = 1.50 m

k = 10.00 N/m

Mass moving in circular path:

r = 1.50 m

v = 5.00 m/s

Required
period of the mass-spring system (T)
Analysis and Solution
Both the mass moving in the circular path, and the mass-spring system are synchronized, so they must have the same period. Determine the period of the mass moving in the circular path, which gives the period of the mass-spring system.

$$v = \frac{2\pi r}{T}$$

$$\begin{aligned} T &= \frac{2\pi r}{v} \\ &= \frac{2\pi (1.50\ \cancel{\text{m}})}{5.00\ \frac{\cancel{\text{m}}}{\text{s}}} \\ &= 1.88\ \text{s} \end{aligned}$$

Paraphrase
The period of the mass-spring system is the same as the circular system, so it is also 1.88 s.

(b) ***Given***
Horizontal mass-spring system:
A = 1.50 m
k = 10.00 N/m
T = 1.88 s
Mass moving in circular path:
r = 1.50 m
v = 5.00 m/s
T = 1.88 s
Required
mass of the mass-spring system (m)
Analysis and Solution
The maximum speed of the mass-spring system matches the speed of the mass moving in the circular path. Now that you know the period, determine the mass using the equation for speed.

$$\text{v}_{\text{max}} = A\sqrt{\frac{k}{m}}$$

$$\begin{aligned} m &= \frac{A^2 k}{v_{\text{max}}^2} \\ &= \frac{(1.50\,\text{m})^2 \left(10.00\,\frac{\text{N}}{\text{m}}\right)}{\left(5.00\,\frac{\text{m}}{\text{s}}\right)^2} \\ &= 0.900\,\text{kg} \end{aligned}$$

Paraphrase
The mass of the mass-spring system is 0.900 kg.

(c) ***Given***

Mass-spring system:

$A = 1.50$ m

$k = 10.00$ N/m

$T = 1.88$ s

$m = 0.900$ kg

Required

maximum acceleration of the mass-spring system ($\vec{a}$)

Analysis and Solution

The maximum acceleration for any simple harmonic oscillator occurs at the maximum displacement. You are solving only for the magnitude of the acceleration, as it could be in either direction. Therefore, omit the negative sign in the acceleration equation.

$$\begin{aligned} a &= \frac{kx}{m} \\ &= \frac{\left(10.00\,\frac{\text{N}}{\text{m}}\right)(1.50\,\text{m})}{0.900\,\text{kg}} \\ &= 16.7\,\text{m/s}^2 \end{aligned}$$

Paraphrase

The magnitude of the acceleration is 16.7 m/s^2.

13. ***Given***

$f = 10.0$ kHz

$A = 0.0500$ mm

Required

maximum speed of quartz oscillator (v_{max})

Analysis and Solution

The quartz crystal is a simple harmonic oscillator. Convert the frequency of its oscillations to period to solve for its mass. Once you know the mass, determine the maximum speed. The speed is a scalar quantity, so the maximum displacement (amplitude) does not require a direction.

$$\begin{aligned} T &= \frac{1}{f} \\ &= \frac{1}{1.00 \times 10^4\ \text{Hz}} \\ &= 1.00 \times 10^{-4}\ \text{s} \end{aligned}$$

Solve for the spring constant:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$k = \frac{4\pi^2 m}{T^2}$$
$$= \frac{4\pi^2 (2.00 \times 10^{-4}\ \text{kg})}{(1.00 \times 10^{-4}\ \text{s})^2}$$
$$= 7.896 \times 10^5\ \text{N/m}$$

Find the maximum speed of the crystal:

$$v_{\text{max}} = A\sqrt{\frac{k}{m}}$$
$$= (5.00 \times 10^{-5}\ \text{m})\sqrt{\frac{7.896 \times 10^5\ \frac{\text{N}}{\text{m}}}{2.00 \times 10^{-4}\ \text{kg}}}$$
$$= 3.14\ \text{m/s}$$

Paraphrase

The quartz crystal oscillates with a maximum speed of 3.14 m/s.

14. ***Given***

$m = 0.200$ kg

$A = 0.120$ m

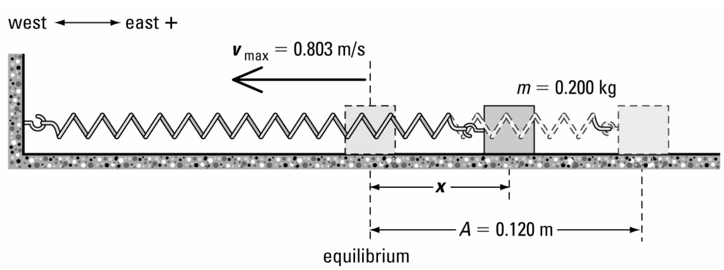
$v_{\text{max}} = 0.803\ \text{m/s}$ [west]

Required

displacement ($\vec{x}$)

Analysis and Solution

You can determine the displacement for a specific acceleration only if you know the spring constant. Determine the spring constant by using the equation for maximum speed, and solving for k. Then use k to determine the displacement. Take east to be positive, and west to be negative.



$$v_{\text{max}} = A\sqrt{\frac{k}{m}}$$
$$k = \frac{v_{\text{max}}^2 m}{A^2}$$
$$= \frac{\left(0.803\ \frac{\text{m}}{\text{s}}\right)^2 (0.200\ \text{kg})}{(0.120\ \text{m})^2}$$
$$= 8.956\ \text{N/m}$$

Now determine the displacement using the spring constant:

$$\vec{F} = -k\vec{x}$$
$$m\vec{a} = -k\vec{x}$$
$$\vec{a} = \frac{-k\vec{x}}{m}$$
$$\vec{x} = \frac{-m\vec{a}}{k}$$
$$= \frac{-(0.200\,\text{kg})\left(-3.58\,\frac{\text{m}}{\text{s}^2}\right)}{8.956\,\frac{\text{N}}{\text{m}}}$$
$$= +7.99\,\text{cm}$$

Paraphrase

The displacement of the mass when it experiences an acceleration of 3.58 m/s^2 [west] is 7.99 cm [east].

15. ***Given***

h = 12.31 km

l = 20.0 cm

Required

period of the pendulum (T)

Analysis and Solution

The pendulum's period depends on the gravitational field strength at the height of 12.31 km. Determine the gravitational field strength first, remembering to add Earth's radius to the altitude. Then determine the period of the pendulum.

$$g = \frac{Gm_{\text{Earth}}}{r^2}$$
$$= \frac{\left(6.67\times10^{-11}\,\frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)\left(5.98\times10^{24}\,\text{kg}\right)}{\left(6.37\times10^{6}\,\text{m}+1.231\times10^{4}\,\text{m}\right)^2}$$
$$= 9.79\ \text{m/s}^2$$

Now find the period of the pendulum:

$$T = 2\pi\sqrt{\frac{l}{g}}$$
$$= 2\pi\sqrt{\frac{0.200\,\text{m}}{9.79\,\frac{\text{m}}{\text{s}^2}}}$$
$$= 0.898\,\text{s}$$

Paraphrase

The period of the pendulum aboard the plane is 0.898 s.

7.4 Check and Reflect

Knowledge

1. The wind causes a phenomenon called vortex shedding. It can create a forced vibration that matches the natural frequency of a building or bridge.

2. Forced frequency is the frequency at which an external force is applied to a resonating object.

3. Engineers use tuned mass dampers in buildings to minimize the amplitude of resonant vibration caused by the wind. The tuned mass dampers are computer controlled so that a large mass (either a mass-spring system or a pendulum) is set oscillating with the same frequency as the building but in the opposite direction.

4. A force applied with the same frequency as the resonant frequency of the object has the effect of increasing the amplitude of its oscillations.

5. The limitations were:

(i) It only worked at specific elevations or altitudes due to the variations in gravity caused by Earth's shape.

(ii) The metal arm of the pendulum expanded or contracted with changes in temperature, changing its period. (Recall $T = 2\pi\sqrt{\frac{l}{g}}$).

(iii) It could not be used on a moving vessel (boat).

6. No, the radius of Earth is longer at the equator than at the poles. The temperature is very different as well. The pendulum clock's period of oscillation would change significantly. A pendulum clock built to operate at a specific geographic location must remain there to be accurate.

7. Damping is the process where the amplitude of oscillations of an object is reduced. This can be accomplished by increasing or decreasing the tension on the object, or by providing a periodic force out of phase with the oscillations. An example could include the damping of a bridge by using stiffer materials that require a large forced frequency to oscillate and reduce wind resistance.

Applications

8. The walker would change the frequency of his or her walking step across the bridge.

9. The musical note is a specific frequency that matches the resonant frequency of the champagne glass, which increases the amplitude of its vibration. If the energy (amplitude) of the forced vibration is large enough, the glass will break.

10. The tuning forks' vibration at a specific frequency creates a vibration in the piano strings. The piano tuner listens to see if the proper piano string begins to vibrate at the same resonance as the fork, and will adjust the tension on the string accordingly until it vibrates when the tuning fork is held near.

11. a) A forced frequency that is different from the resonant frequency will not change the resonant frequency but will likely dampen the vibrations.

b) The friction created by the water changes the acceleration of the pendulum as it swings back and forth and affects the period of the oscillations so its resonant frequency will change. The restoring force is the net force and will be the sum of the perpendicular component of gravity and the force of friction.

$$\vec{F}_{\text{R}} = \vec{F}_{\text{net}}$$

$$\vec{F}_{\text{net}} = \vec{F}_{\text{g}\perp} + \vec{F}_{\text{f}_{\text{kinetic}}}$$

Friction will also dampen the oscillations.

c) Increasing the mass of the pendulum bob will not change the resonant frequency or dampen the oscillations.

d) Moving the pendulum to a higher altitude will change the acceleration of gravity (it will be less). This will change the resonant frequency of the pendulum because the period will be greater. It will not dampen the oscillations

12. The size and shape of the crystal affect its resonant frequency.

13. a) Student answers will vary. Some valid answers are:

- The quartz clock is much more accurate than the pendulum clock.
- The quartz clock is not affected by temperature.
- The quartz clock is not affected by latitude or elevation.
- The quartz clock is not affected by motion so it can be used aboard moving vehicles.
- The quartz clock is subject to much less mechanical friction.
- The quartz clock does not require gravity to function.

b) Student answers will vary. Some valid answers are:

- The pendulum clock does not require a battery.
- The pendulum clock is more pleasing to look at.

Extensions

14. ***Given***

$g = 9.81 \text{ m/s}^2$

$f = 1.00 \text{ Hz}$

Required

length of the pendulum (l)

Analysis and Solution

The period of a pendulum is determined by the length of the arm and the acceleration of gravity. You can determine the length of a pendulum with equation 9. To use this equation, you must convert the frequency to period.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T^2 = 4\pi^2 \frac{l}{g}$$

$$\begin{aligned} l &= \frac{T^2 g}{4\pi^2} \\ &= \frac{(1.00 \text{ s})^2 \left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)}{4\pi^2} \\ &= 0.248 \text{ m} \end{aligned}$$

Paraphrase

A pendulum arm 0.248 m long has a frequency of 1.00 Hz in Alberta. This pendulum would be most accurate if the temperature remained constant, the pendulum was not moved, and friction was kept to a minimum.

15. Automobile manufacturers will do the following things:

- Place damping material in the doors.
- Use epoxies/glues to glue components together so they won't rattle (electronic circuitry in ignition systems is covered with an epoxy resin so the electrical components won't rattle and break over time).
- Use self-locking washers/nuts that aren't as susceptible to loosening from vibration.

16. They use light, stiff materials that will not vibrate easily. This is a form of damping.

17. They are placed in large ships to reduce the amount of rolling caused by the waves because the lateral back-and-forth motion can cause seasickness. The tuned mass dampers perform the same role on a ship as they do in a building.

18. Orbital resonance in Saturn's rings occurs when the period of some of the orbital debris (small rocks) in a ring has the same orbital period (or a multiple of the period) as one of Saturn's moons. For example a moon may go around once, and in that time some of the debris in the ring goes around exactly two times. Over time the gravitational pull of the moon works to increase the amplitude of the debris' orbital radius every time the moon and the debris are aligned, creating a gap in the rings.

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Chapter 7 Review

Knowledge

1. Oscillatory motion is uniform back-and-forth motion.

2. A ball will bounce with oscillatory motion if it has a uniform period. This means that it must bounce to the same height in each oscillation.

3. An elastic material will deform due to a stress. Once the stress is removed, the material will return to its original shape.

4. The restoring force is the only force that acts on an isolated, frictionless, simple harmonic oscillator.

5. The direction of the restoring force is always opposite to the displacement.

6. Slope is the spring constant.

7. It is not in its equilibrium position.

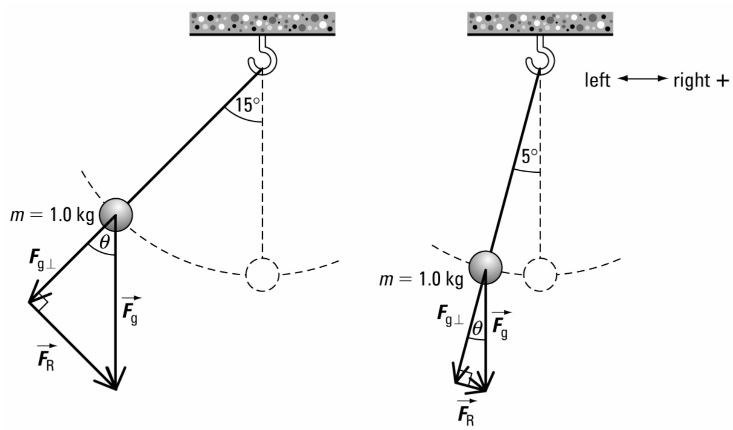
8. Acceleration depends on displacement, as given by $\vec{a} = \frac{-k\vec{x}}{m}$. Therefore, it is not uniform.

9. The restoring force varies with the sine of the angle of displacement. For small angles, this relationship is almost linear. As the displacement angle increases, the relationship is no longer linear. This difference begins to show at approximately 15°.

10. The forced frequency will increase the amplitude of the resonant frequency.

Applications

11. (a) ***Analysis and Solution***



Note that the angles in the diagram have been exaggerated for illustration purposes.

$$F_{g\perp} = F_g \sin\theta$$

$$F_{g\perp} = \left(9.81\frac{\text{m}}{\text{s}^2}\right)(1.0\,\text{kg})\sin 15^\circ$$
$$= 2.5\,\text{N}$$

The restoring force on the pendulum is 2.5 N [forward].

(b) ***Analysis and Solution***

$$F_{g\perp} = F_g \sin\theta$$

$$F_{g\perp} = \left(9.81\frac{\text{m}}{\text{s}^2}\right)(1.0\,\text{kg})\sin 5^\circ$$
$$= 0.9\,\text{N}$$

The restoring force on the pendulum is 0.9 N.

12. ***Analysis and Solution***

$$f = \frac{1}{T}$$
$$= \frac{1}{0.004\,00\,\text{s}}$$
$$= 250\,\text{Hz}$$

The frequency of the guitar string is 250 Hz.

13. ***Analysis and Solution***

$$T = \frac{1}{f}$$
$$= \frac{1}{0.67\ \text{Hz}}$$
$$= 1.5\ \text{s}$$

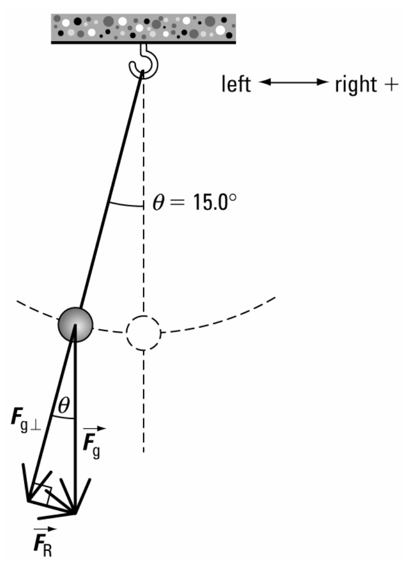
The period is 1.5 s.

14. ***Analysis and Solution***

$$f = \frac{1}{T}$$
$$= \frac{1}{0.100 \text{ s}}$$
$$= 10.0 \text{ Hz}$$

The frequency is 10.0 Hz.

15. ***Analysis and Solution***



$$F_{g\perp} = F_g \sin\theta$$
$$= (2.0 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(\sin 15°)$$
$$= 5.1 \text{ N}$$

The restoring force acting on the pendulum is 5.1 N [forward].

16. ***Analysis and Solution***

Any two points from the line of best fit can be used to determine the slope.
The following points were used for the solution below:
$(x_1, y_1) = (0.00, 0.00)$ and $(x_2, y_2) = (0.75, 150)$

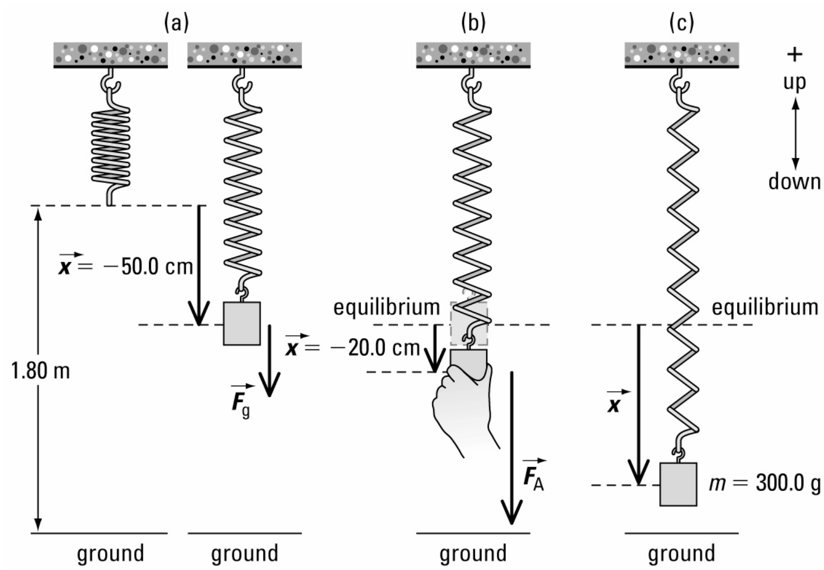
$$k = \text{slope}$$
$$= \frac{(150 - 0.00)\text{ N}}{(0.75 - 0.0)\text{ m}}$$
$$= 2.0 \times 10^2 \text{ N/m}$$

The spring constant of the spring is 2.0×10^2 N/m.

17. ***Given***

height of spring from floor = 1.80 m
height of spring from floor with 100.0-g mass attached =1.30 m

Required
(a) spring constant (k)
(b) force needed to pull the 100.0-g mass through a displacement of 20.0 cm ($\vec{F}$)
(c) displacement of the 300.0-g mass ($\vec{x}$)
Analysis and Solution
Draw diagram of the situation described in each part the problem.



You need to know the spring constant of the spring to determine the force and displacements in parts (b) and (c). To determine the spring constant, use the force of gravity for the 100.0-g mass. The displacement of the mass-spring is 50.0 cm down.

(a)

$$\vec{x} = 0.500\,\text{m [down]}$$

$$\vec{F}_g = k\vec{x}$$

$$k = \frac{\vec{F}_g}{\vec{x}}$$

$$= \frac{(0.100\,\text{kg})\left(-9.81\,\frac{\text{m}}{\text{s}^2}\right)}{-0.500\,\text{m}}$$

$$= 2.00\,\text{N/m}$$

(b) Take the displacement of the 100.0-g mass (1.30 m) from the floor as the new equilibrium position, and determine the force required to pull it down 20.0 cm.

$$\vec{x} = 0.200\,\text{m [down]}$$

$$\vec{F} = k\vec{x}$$

$$= (2.00\,\frac{\text{N}}{\text{m}})(-0.200\,\text{m})$$

$$= -0.400\,\text{N}$$

(c) Determine the displacement for the 300.0-g mass from the spring's original equilibrium position (1.80 m) from the floor. Use the spring constant found in part (a) to determine this displacement.

$$\vec{x} = \frac{\vec{F}_g}{k}$$

$$= \frac{(0.300\,\text{kg})\left(-9.81\,\dfrac{\text{m}}{\text{s}^2}\right)}{2.00\,\dfrac{\text{N}}{\text{m}}}$$

$$= -1.47\,\text{m}$$

The height from the floor is:

$1.80\text{ m} - 1.47\text{ m} = 0.33\text{ m}$

Paraphrase

(a) The spring constant of the spring is 2.00 N/m.

(b) A person must apply a force of 0.400 N to pull the hanging 100.0-g mass down through a displacement of 20.0 cm.

(c) If the 100.0-g mass is removed and a 300.0-g mass is attached, it will hang 0.33 m from the floor.

18. ***Given***

$k_A = 100.0\,\text{N/m}$

$k_B = 50.0\,\text{N/m}$

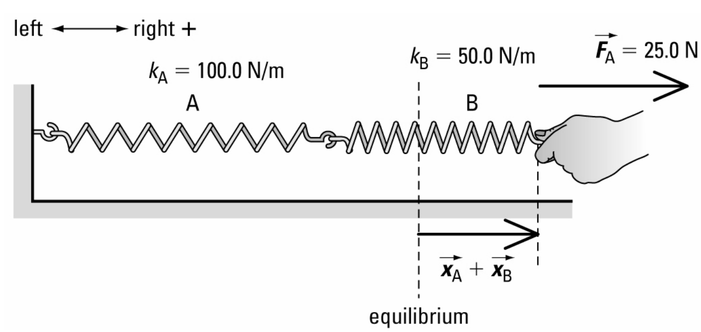
$\vec{F} = 25.0\,\text{N}$

Required

total displacement of both springs (x_T)

Analysis and Solution

Each spring will be displaced differently by the force. To determine the total displacement, calculate the displacement of each spring; then add the two displacements together.



Spring A:

$$k_A = 100.0 \text{ N/m}$$

$$\vec{F}_A = k\vec{x}_A$$

$$\vec{x}_A = \frac{\vec{F}_A}{k_A}$$

$$= \frac{25.0 \text{ N}}{100.0 \frac{\text{N}}{\text{m}}}$$

$$= 0.250 \text{ m}$$

Spring B:

$$k_B = 50.0 \text{ N/m}$$

$$\vec{F}_B = k\vec{x}_B$$

$$\vec{x}_B = \frac{\vec{F}_B}{k_B}$$

$$= \frac{25.0 \text{ N}}{50.0 \frac{\text{N}}{\text{m}}}$$

$$= 0.500 \text{ m}$$

$$\vec{x}_T = \vec{x}_A + \vec{x}_B$$

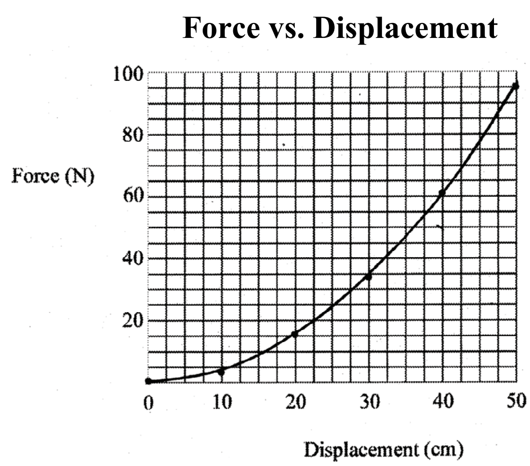
$$= 0.250 \text{ m} + 0.500 \text{ m}$$

$$= 0.750 \text{ m}$$

Paraphrase

The combined displacement of both springs when a force of 25.0 N is applied is 0.750 m.

19.



The graph shows that the elastic band does not obey Hooke's law because the graph is not linear.

20. ***Analysis and Solution***

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$l = \frac{T^2 g}{4\pi^2}$$

$$= \frac{(1.00\ \cancel{\text{s}})^2\left(9.81\ \frac{\text{m}}{\cancel{\text{s}^2}}\right)}{4\pi^2}$$

$$= 0.248\,\text{m or } 24.8\,\text{cm}$$

The length of the pendulum must be 24.8 cm to swing with a period of 1s under the influence of a gravitational field of 9.81 m/s^2.

21. ***Analysis and Solution***

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{10.0\,\text{kg}}{44.0\,\frac{\text{N}}{\text{m}}}}$$

$$= 3.00\,\text{s}$$

The period of the mass is 3.00 s.

22. ***Analysis and Solution***

$$v_{\text{max}} = A\sqrt{\frac{k}{m}}$$

$$= 0.120\ \text{m}\sqrt{\frac{200.0\,\frac{\text{N}}{\text{m}}}{2000\ \text{kg}}}$$

$$= 0.0379\ \text{m/s}$$

The maximum speed of the crate is 0.0379 m/s.

23. ***Given***

A = 0.040 m

v_{max} = 0.100 m/s

m = 0.480 g

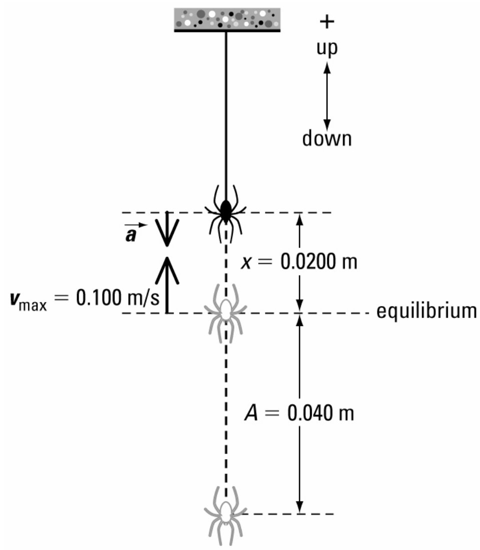
$\vec{x}$ = 0.0200 m [upward]

Required

acceleration at displacement of 0.0200 m [up] ($\vec{a}$)

Analysis and Solution

To determine the acceleration of the spider, find the spring constant of its thread. Do this first, and then use the spring constant in the acceleration equation. Make sure to use the appropriate SI units for the spider's mass. Take up as positive, and down as negative.



$$v_{max} = A\sqrt{\frac{k}{m}}$$

$$v_{max}^2 = A^2\frac{k}{m}$$

$$k = \frac{v_{max}^2 m}{A^2}$$

$$= \frac{\left(0.100\frac{\text{m}}{\text{s}}\right)^2 (4.80\times10^{-4}\text{ kg})}{(0.040\text{ m})^2}$$

$$= 3.0\times10^{-3}\text{ N/m}$$

Use this spring constant to determine the spider's acceleration at a displacement of 0.0200 m [up].

$$\vec{F} = -k\vec{x}$$

$$m\vec{a} = -k\vec{x}$$

$$\vec{a} = \frac{-k\vec{x}}{m}$$

$$= \frac{-\left(3.0\times10^{-3}\frac{\text{N}}{\text{m}}\right)(0.0200\text{ m})}{4.80\times10^{-4}\text{ kg}}$$

$$= -0.13\text{ m/s}^2$$

Paraphrase

The spider experiences an acceleration of 0.13 m/s^2 [down], when it is at a displacement of 0.0200 m [up].

24. ***Analysis and Solution***

$$\begin{aligned} T &= 2\pi\sqrt{\frac{l}{g}} \\ &= 2\pi\sqrt{\frac{0.2585\ \text{m}}{\left(9.81\frac{\text{m}}{\text{s}^2}\right)}} \\ &= 1.02\ \text{s} \end{aligned}$$

The period of a pendulum with a length of 25.85 cm is 1.02 s.

25. ***Analysis and Solution***

$$\begin{aligned} T &= \frac{1}{f} \\ &= \frac{1}{0.182\ \text{Hz}} \\ &= 5.49\ \text{s} \end{aligned}$$

$$\begin{aligned} g &= \frac{4\pi^2 l}{T^2} \\ &= \frac{4\pi^2 0.50\ \text{m}}{(5.49\ \text{s})^2} \\ &= 0.65\ \text{m/s}^2 \end{aligned}$$

Pluto's gravitational field strength is 0.65 m/s^2.

26. (a) X represents $\frac{l}{g}$.

(b) ***Analysis and Solution***

$$X = \frac{l}{g}$$

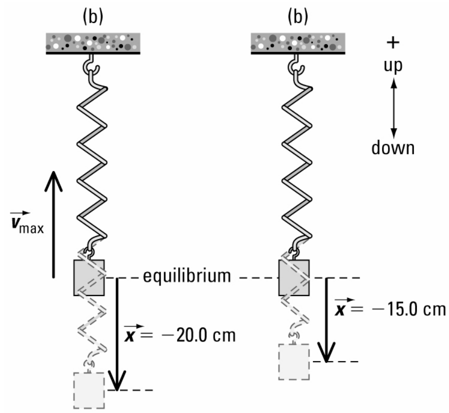
$$\begin{aligned} \frac{l}{g} &= \frac{T^2}{4\pi^2} \\ &= \frac{(1.79\ \text{s})^2}{4\pi^2} \\ &= 0.081\,16\ \text{s}^2 \end{aligned}$$

$$\begin{aligned} l &= 0.081\,16\ \cancel{\text{s}^2}\left(9.81\frac{\text{m}}{\cancel{\text{s}^2}}\right) \\ &= 0.796\ \text{m} \end{aligned}$$

The length of the pendulum is 0.796 m.

Extensions

27.



(a) ***Analysis and Solution***

$$v_{max} = A\sqrt{\frac{k}{m}}$$

$$= 0.200\ \text{m}\sqrt{\frac{10.0\ \frac{\text{N}}{\text{m}}}{0.250\ \text{kg}}}$$

$$= 1.26\ \text{m/s}$$

The maximum speed of the mass is 1.26 m/s.

(b) ***Analysis and Solution***

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{0.250\ \text{kg}}{10.0\ \frac{\text{N}}{\text{m}}}}$$

$$= 0.993\ \text{s}$$

The mass will have a period of vibration of 0.993 s. The period is independent of the amplitude.

28. (a) ***Given***

f =0.800 Hz

Required

frequency of the oscillator when the mass is doubled (f)

Analysis and Solution

To find the frequency when the mass is doubled, modify the equation for the period to solve for frequency. Substitute 2m in place of m in the equation to determine the factor by which the frequency increases or decreases. Then determine the actual change to the frequency by multiplying this factor by the original frequency.

$T = 2\pi\sqrt{\frac{m}{k}}$

$f = \frac{1}{T}$

$= \frac{1}{2\pi\sqrt{\frac{m}{k}}}$

$= \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Change m to $2m$.

$f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}$ $\qquad \sqrt{2} = 1.414$

$= \frac{1}{2\pi(1.414)}\sqrt{\frac{k}{m}}$

$= \left(\frac{1}{1.414}\right)\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$= (0.707)\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

The frequency has changed by a factor of 0.707. Now determine the new frequency.

$f = 0.707 \times 0.800 \text{ Hz}$

$= 0.566 \text{ Hz}$

Paraphrase

The frequency of the oscillator will change from 0.800 Hz to 0.566 Hz.

(b) For any simple harmonic oscillator, the amplitude of vibration does not affect the period, and therefore does not affect the frequency. The frequency will stay at 0.800 Hz.

29. (a) A bouncing ball is not a simple harmonic oscillator because the force does not vary with displacement. The force of gravity is constant regardless of the position of the ball.

(b) The movement of a puck on the ice cannot be considered SHM. The force applied by the stick is constant throughout its displacement.

(c) A plucked guitar string is an example of SHM. The guitar string's restoring force varies directly with its displacement.

