Pearson Physics Level 20
Unit III Circular Motion, Work, and Energy: Chapter 6
Solutions

Student Book page 294

Concept Check

The centripetal force acts at an angle of 90º to the motion. Since cos 90º = 0, the equation for work says that the force can do no work.

Example 6.1 Practice Problems

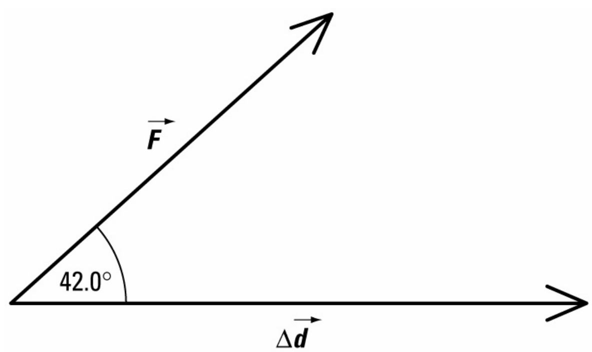
1. ***Given***

$F = 620 \text{ N} \quad \theta = 42.0° \quad \Delta d = 160 \text{ m}$

Required

work (ΔE)

Analysis and Solution



The equation for work (on page 293 and used in the example on page 294) applies.

$$\begin{aligned} W &= (F\cos\theta)\Delta d \\ &= (620 \text{ N})(\cos 42.0°)(160 \text{ m}) \\ &= 7.37\times10^{4} \text{ J} \end{aligned}$$

Paraphrase

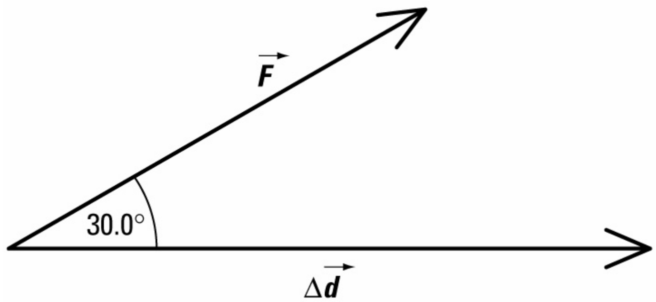
The force of 620 N acting at an angle of 42.0º to the motion of the sled does 7.37×10^4 J of work as it moves the sled 160 m.

2. ***Given***

$\theta = 30.0° \quad W = 9600 \text{ J} \quad \Delta d = 25.0 \text{ m}$

Required

force (F)



Analysis and Solution

Find the magnitude of the force. The direction will be ambiguous. Use the equation for work from the example or from page 293.

$$W = (F\cos\theta)\Delta d$$

$$\begin{aligned} F &= \frac{W}{(\cos\theta)(\Delta d)} \\ &= \frac{9600 \text{ J}}{(\cos 30.0°)(25.0 \text{ m})} \\ &= 443 \text{ N} \end{aligned}$$

Paraphrase

A force of 443 N will be required to do 9600 J of work over 25.0 m if the angle between the force and displacement is 30.0º.

3. ***Given***

$F = 640 \text{ N} \quad W = 1.25\times10^4 \text{ J} \quad \Delta d = 24.0 \text{ m}$

Required

the angle (θ) between the force and displacement

Analysis and Solution

Use the equation for work to calculate the cosine θ then find the arccosine of the answer.

$$W = (F\cos\theta)\Delta d$$

$$\begin{aligned} \cos\theta &= \frac{W}{(F)(\Delta d)} \\ &= \frac{1.25\times10^4 \text{ J}}{(640 \text{ N})(24.0 \text{ m})} \\ &= 0.8138 \\ \theta &= 35.5° \end{aligned}$$

Paraphrase

For a force of 640 N to do 1.25×10^4 J of work while it acts over a displacement of 24.0 m, the angle between the force and the displacement would need to be 35.5º.

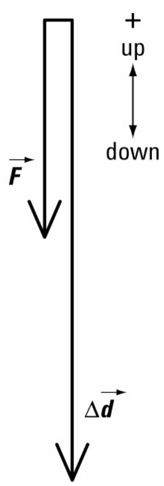
4. **NOTE:** This question acts as a transition between the concept of work and the concept of change in gravitational potential energy. At this time, use the term "work done by gravity" rather than "change in gravitational potential energy". However, many students will realize that these are the same argument.

Given

$m = 60.0 \text{ kg} \quad \Delta d = 20.0 \text{ m}$

$\vec{g} = 9.81 \ \frac{\text{m}}{s^2}[\text{down}]$

Required
work done by the force of gravity while the jumper is in free fall (W)
Analysis and Solution



First, calculate the force of gravity as the product of the jumper's mass and the gravitational acceleration. The direction of free fall and the direction of the force of gravity are parallel so that $\theta = 0°$. Use the equation for work to calculate the work done. [**Note:** Even though the calculation of the force of gravity is a vector calculation only the magnitude of the vector is required for the work. Also, many students may substitute the argument for force of gravity into the equation for work. This technique would produce the answer in a single calculation.]

$$\begin{aligned}\vec{F} &= m\vec{g} \\ &= (60.0\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}[\text{down}]\right) \\ &= 5.89 \times 10^2\ \text{N down}\end{aligned}$$

$$\begin{aligned}W &= (F\cos\theta)\Delta d \\ &= (5.89 \times 10^2\ \text{N})(\cos 0°)(20.0\text{ m}) \\ &= 1.18 \times 10^4\ \text{J}\end{aligned}$$

Paraphrase
If a mass of 60.0 kg falls 20.0 m, gravity does about 1.18×10^4 J

Student Book page 295

Concept Check

In the equation $\Delta E_p = mg\Delta h$ the values of m and g must both be constant over the change in height. Since the force of gravity decreases as the distance from Earth's centre increases, the magnitude of the acceleration due to gravity (g) decreases as well.
The acceleration due to gravity is a constant only over small changes in altitude, so that the equation for change in gravitational potential energy cannot apply over large changes in altitude. [**Note:** This might be a good time to explain that the equation does apply, over

small changes in height, at different altitudes if you know the value of g at those altitudes.]

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Example 6.2 Practice Problems

1. ***Given***

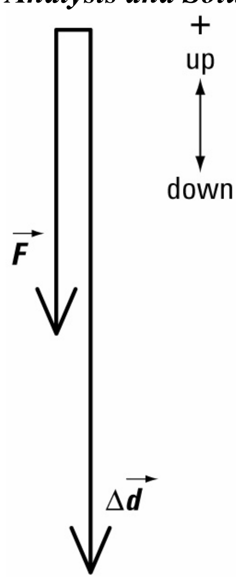
$m_E = 750 \text{ kg} \quad m_A = 65.0 \text{ kg} \quad m_B = 30.0 \text{ kg}$

$m_C = 48.0 \text{ kg} \quad g = 9.81 \ \frac{\text{m}}{\text{s}^2} \quad \Delta h = -21.0 \text{ m}$

Required

change in gravitational potential energy (ΔE_p)

Analysis and Solution



The mass is the sum of the mass of the elevator car plus the masses of the three passengers. Use the equation for change in gravitational potential energy developed on page 295 and used in the example problem.

$$\begin{aligned} m &= m_E + m_A + m_B + m_C \\ &= 750 \text{ kg} + 65.0 \text{ kg} + 30.0 \text{ kg} + 48.0 \text{ kg} \\ &= 893 \text{ kg} \end{aligned}$$

$$\begin{aligned} \Delta E_p &= mg\Delta h \\ &= (893 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(-21.0 \text{ m}) \\ &= -1.84 \times 10^5 \text{ J} \end{aligned}$$

Paraphrase

When the elevator and its passengers are lowered 21.0 m, there is a change in gravitational potential energy of -1.84×10^5 J. The negative sign indicates that the change is a loss of energy.

2. ***Given***

$m = 1.45 \text{ kg} \quad g = 9.81 \ \dfrac{\text{m}}{\text{s}^2} \quad \Delta E_{\text{p}} = 25.0 \text{ J}$

Required

change in height (Δh)

Analysis and Solution

From the equation for change in gravitational potential energy solve for change in height.

$$\begin{aligned}
\Delta E_{\text{p}} &= mg\Delta h \\
\Delta h &= \frac{\Delta E_{\text{p}}}{mg} \\
&= \frac{25.0 \text{ J}}{(1.45 \text{ kg})\left(9.81 \ \dfrac{\text{m}}{\text{s}^2}\right)} \\
&= 1.76 \text{ m}
\end{aligned}$$

Paraphrase

To place it on the shelf, the book was lifted a distance of 1.76 m. Since the movement of the book began on the floor, the shelf must be 1.76 m above the floor.

3. ***Given***

$m = 0.148 \text{ kg} \quad \Delta h = 0.750 \text{ m} \quad \Delta E_{\text{p}} = 0.400 \text{ J}$

Required

value for g on the surface of Mars

Analysis and Solution

Solve the equation for change in gravitational potential energy for the value of the acceleration due to gravity.

$$\begin{aligned}
\Delta E_{\text{p}} &= mg\Delta h \\
g &= \frac{\Delta E_{\text{p}}}{m\Delta h} \\
&= \frac{0.400 \text{ J}}{(0.148 \text{ kg})(0.750 \text{ m})} \\
&= 3.60 \ \frac{\text{m}}{\text{s}^2}
\end{aligned}$$

Paraphrase

On Mars, the acceleration due to gravity is about 3.60 m/s^2.

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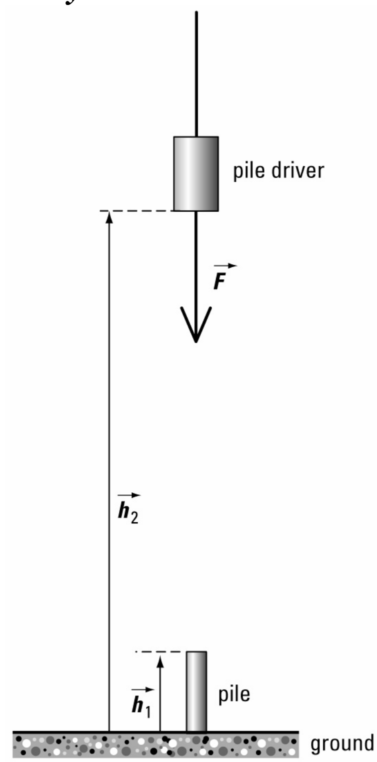
Example 6.3 Practice Problems

1. ***Given***

$m = 550 \text{ kg} \quad g = 9.81 \ \dfrac{\text{m}}{\text{s}^2}$

(a) $h_2 = 12.5 \text{ m}$ (b) $h_1 = 2.30 \text{ m}$

Required
(a) and (b) gravitational potential energies relative to the ground at the highest point (E_{p_f}) and at the lowest point (E_{p_i}). [**Note:** The heights at the highest and lowest points are designated h_2 and h_1, respectively, because in part (c) the mass moves from the lowest to highest point.]
(c) change in energy as the mass is lifted from point 1 to point 2.
Analysis and Solution



(a) and (b) Calculate the potential energies using the equation from page 296 (or in the example).
(a)

$$\begin{aligned} E_{p_2} &= mgh_2 \\ &= (550\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(12.5\ \text{m}) \\ &= 6.74 \times 10^4\ \text{J} \end{aligned}$$

(b)

$$\begin{aligned} E_{p_2} &= mgh_1 \\ &= (550\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(2.30\ \text{m}) \\ &= 1.24 \times 10^4\ \text{J} \end{aligned}$$

(c) Find the difference between the potential energies from parts (a) and (b).

[**Note:** If students wish to combine the arguments into a single one by substituting the arguments for E_p into the argument for ΔE_p, this should be encouraged. It means that the calculation for solution is presented on a single line.]

$$\begin{aligned}\Delta E_p &= E_{p_2} - E_{p_1} \\ &= 6.74\times10^4\ \text{J} - 1.24\times10^4\ \text{J} \\ &= 5.50\times10^4\ \text{J}\end{aligned}$$

Paraphrase

(a) and (b) The gravitational potential energies relative to ground level at the highest and lowest points were 6.74×10^4 J and 1.24×10^4 J, respectively.

(c) The difference of 5.50×10^4 J is the gain in gravitational potential energy as the mass is raised from the lowest to the highest point.

2. ***Given***

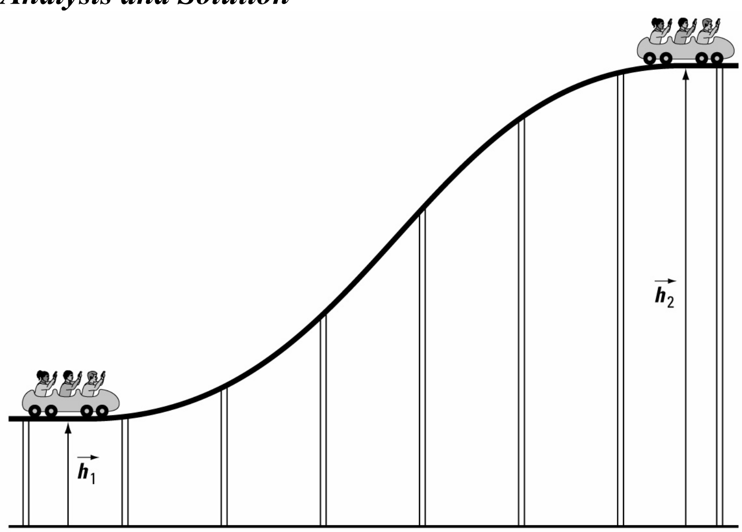
$h_1 = 5.25\ \text{m} \quad \Delta E_p = 4.20\times10^5\ \text{J}$

$m = 875\ \text{kg} \quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Required

height above the ground after the energy increase (h2)

Analysis and Solution



Calculate the initial potential energy; and then add in the energy increase to give the final potential energy. Calculate the final height above the ground from the final potential energy.

$$\begin{aligned}E_{p_1} &= mgh_1 \\ &= (875\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(5.25\ \text{m}) \\ &= 4.51\times10^4\ \text{J}\end{aligned}$$

$$\begin{aligned}E_{p_1} &= E_{p_1} + \Delta E_p \\ &= 4.51\times10^4\ \text{J} + 4.20\times10^5\ \text{J} \\ &= 4.65\times10^5\ \text{J}\end{aligned}$$

$$E_{p_2} = mgh_2$$

$$h_2 = \frac{E_{p_2}}{mg}$$

$$= \frac{4.65 \times 10^5 \text{ J}}{(875 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)}$$

$$= 54.2 \text{ m}$$

Paraphrase

The increase in the potential energy of the trolley increased its height above the ground from 5.25 m to 54.2 m.

3. ***Given***

$m = 250$ kg Δd = 20. m

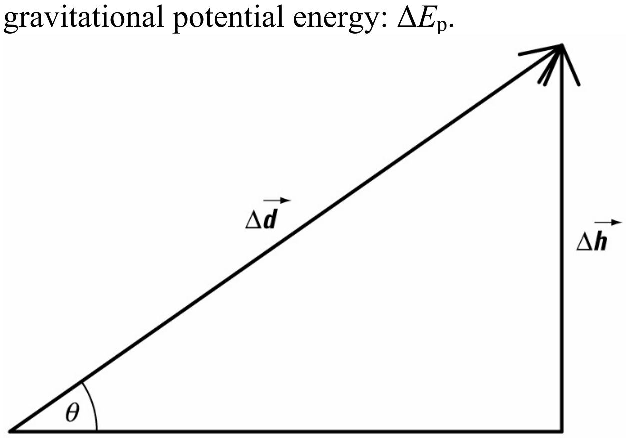
$\theta = 35.0°$ g = $9.81 \ \frac{\text{m}}{\text{s}^2}$

Required

increase in gravitational potential energy for a block pulled up an incline (ΔE_p)

Analysis and Solution

The change in height is the vertical component of the displacement. First, draw a vector diagram of the displacement and use trigonometry to find the change in height produced by pulling the mass up the incline: Δh. Then, calculate the change in gravitational potential energy: ΔE_p.



$$\sin\theta = \frac{\Delta h}{\Delta d}$$

$$\Delta h = (\Delta d)\sin\theta$$

$$= (20.0 \text{ m})(\sin 35.0°)$$

$$= 11.5 \text{ m}$$

$$\Delta E_p = mg\Delta h$$

$$= (250 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(11.5 \text{ m})$$

$$= 2.81 \times 10^4 \text{ J}$$

Paraphrase

The block gains 2.81×10^4 J of energy as the winch pulls it up the plane.

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Concept Check

It is incorrect to use the measurement of the change in the stretch of a spring to find the change in elastic potential energy because the change in elastic potential energy depends on the square of the stretch rather than the stretch itself.

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Example 6.4 Practice Problems

1. ***Given***

(a) $F = 125\text{ N}$ $x_1 = 0.250\text{ m}$ (b) $x_2 = 0.150\text{ m}$

Required

(a) elastic potential energy when the spring is stretched 0.250 m (E_{p_1})

(b) change in elastic potential energy when the stretch is reduced to 0.150 m (ΔE_p)

Analysis and Solution

(a) First find the elastic constant for the spring, then use the equation for elastic potential energy (page 300 or in the example) to calculate the elastic potential energy at 0.250 m.

$$F_1 = kx_1$$

$$\begin{aligned} k &= \frac{F_1}{x_1} \\ &= \frac{125\text{ N}}{0.250\text{ m}} \\ &= 500\ \frac{\text{N}}{\text{m}} \end{aligned}$$

$$\begin{aligned} E_{p_1} &= \tfrac{1}{2}kx_1^{\,2} \\ &= \tfrac{1}{2}\left(500\ \frac{\text{N}}{\text{m}}\right)(0.250\text{ m})^2 \\ &= 15.6\text{ J} \end{aligned}$$

(b) Since the equation for elastic potential energy is not a linear relationship, you must find the elastic potential energy at 0.150 m and then find the change in elastic potential energy by subtracting the initial elastic potential energy from this answer.

$$\begin{aligned} E_{p_2} &= \tfrac{1}{2}kx_2^{\,2} \\ &= \tfrac{1}{2}\left(500\ \frac{\text{N}}{\text{m}}\right)(0.150\text{ m})^2 \\ &= 5.62\text{ J} \end{aligned}$$

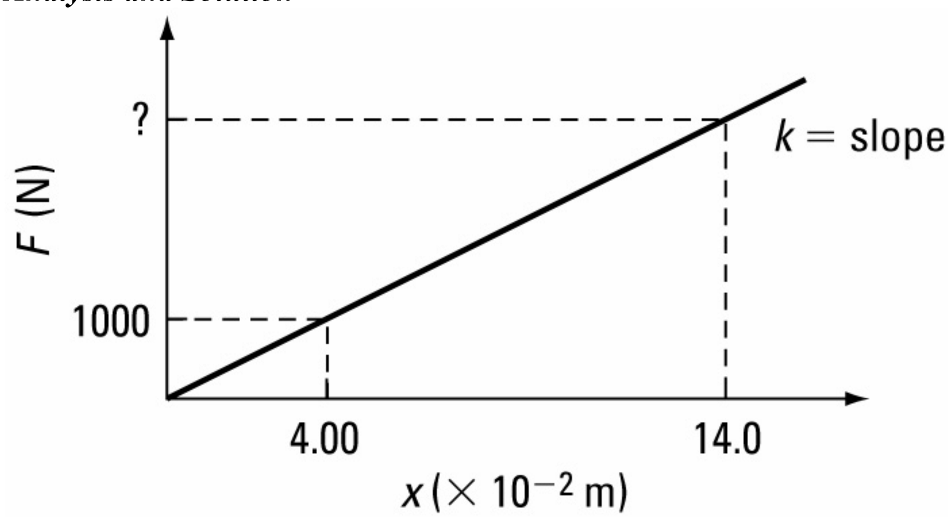
$$\begin{aligned} \Delta E_p &= E_{p_2} - E_{p_1} \\ &= 5.62\text{ J} - 15.6\text{ J} \\ &= -10.0\text{ J} \end{aligned}$$

Paraphrase
The spring stores 15.6 J of elastic potential energy when it is stretched to 0.250 m and only 5.62 J of elastic potential energy when its stretch is reduced to 0.150 m. The change in stretch represents a loss of 10.0 J of elastic potential energy.

2. ***Given***
(a) $x_1 = 4.00 \times 10^{-2}$ m $F_1 = 1.00 \times 10^3$ N (b) $x_2 = 0.140$ m

Required
(a) the elastic constant for the spring
(b) the force required to produce the given compression

Analysis and Solution



Solve for the elastic constant using Hooke's law (k)

$$F_1 = kx_1$$
$$k = \frac{F_1}{x_1}$$
$$= \frac{1.00 \times 10^3 \text{ N}}{4.00 \times 10^{-2} \text{ m}}$$
$$= 2.50 \times 10^4 \ \frac{\text{N}}{\text{m}}$$

(a) Solve for the new compression using the elastic constant (x_2)

$$F_2 = kx_2$$
$$= \left(2.50 \times 10^4 \ \frac{\text{N}}{\text{m}}\right)(0.140 \text{ m})$$
$$= 3.50 \times 10^3 \text{ N}$$

Paraphrase
(a) The elastic constant for the spring is 2.50×10^4 N/m.
(b) The force required to compress this spring 14.0 cm from its original position is 3.50×10^3 N.

3. ***Given***

$k = 750\ \frac{\text{N}}{\text{m}}$ (a) $E_{\text{p}_1} = 45.0\ \text{J}$ (b) $E_{\text{p}_2} = 2E_{\text{p}_1} = 90.0\ \text{J}$

Required

(a) stretch that gives the known elastic potential energy (x_1)

(b) difference in the stretch that doubled the elastic potential energy (Δx)

Analysis and Solution

(a) Solve for stretch in the equation for elastic potential energy.

$$
\begin{aligned}
E_{\text{p}_1} &= \tfrac{1}{2}kx_1^2 \\
x_1 &= \sqrt{\frac{2E_{\text{p}_1}}{k}} \\
&= \sqrt{\frac{2(45.0\ \text{J})}{750\ \frac{\text{N}}{\text{m}}}} \\
&= \sqrt{1.20\times10^{-1}\ \text{m}^2} \\
&= 0.346\ \text{m}
\end{aligned}
$$

(b) Solve for the stretch for the new (doubled) elastic potential energy, and then find the difference in the stretch.

$$
\begin{aligned}
E_{\text{p}_2} &= \tfrac{1}{2}kx_2^2 \\
x_2 &= \sqrt{\frac{2E_{\text{p}_1}}{k}} \\
&= \sqrt{\frac{2(90.0\ \text{J})}{750\ \frac{\text{N}}{\text{m}}}} \\
&= 0.490\ \text{m}
\end{aligned}
$$

$$
\begin{aligned}
\Delta x &= x_2 - x_1 \\
&= 0.490\ \text{m} - 0.346\ \text{m} \\
&= 0.144\ \text{m}
\end{aligned}
$$

Paraphrase

(a) When the spring is stretched 0.346 m the stored elastic potential energy will be 45.0 J.

(b) If the spring is stretched an additional 0.144 m, the elastic potential energy will double.

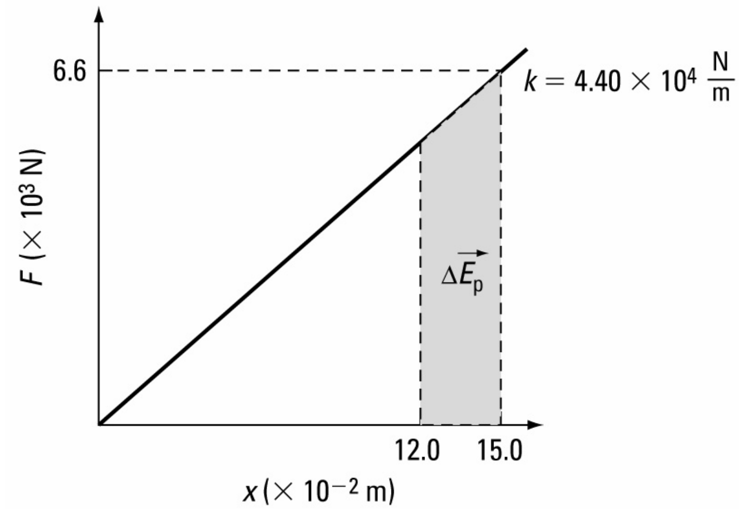
4. ***Given***

$k = 4.40\times10^4\ \frac{\text{N}}{\text{m}}$ $x_1 = 0.125\ \text{m}$ $x_2 = 0.150\ \text{m}$

Required

change in elastic potential energy: ΔE_{p}

Analysis and Solution



Since elastic potential energy is not a linear relationship with stretch, find the elastic potential energy for each of the stretches then subtract. Students who are more confident of their math skills may want to compress the argument by substituting the equations for E_p into the equation for ΔE_p and then doing the calculations on one line.

$$\begin{aligned} E_{p_1} &= \tfrac{1}{2}kx_1^2 \\ &= \tfrac{1}{2}\left(4.40\times10^4\ \frac{\text{N}}{\text{m}}\right)(0.125\ \text{m})^2 \\ &= 344\ \text{J} \end{aligned}$$

$$\begin{aligned} E_{p_2} &= \tfrac{1}{2}kx_2^2 \\ &= \tfrac{1}{2}\left(4.40\times10^4\ \frac{\text{N}}{\text{m}}\right)(0.150\ \text{m})^2 \\ &= 495\ \text{J} \end{aligned}$$

$$\begin{aligned} \Delta E_p &= E_{p_2} - E_{p_1} \\ &= 495\ \text{J} - 344\ \text{J} \\ &= 151\ \text{J} \end{aligned}$$

Paraphrase

When the spring is stretched from 12.5 cm to 15.0 cm, its elastic potential energy increases by 151 J (1.51×10^2 J).

5. ***Given***

$x_1 = 0.400$ m

(a) $E_{p_1} = 5.00 \times 10^2$ J

(b) $F_2 = 1.00 \times 10^3$ N

Required

(a) force that produces the given energy in the spring (F_1)

(b) change in potential energy when the force changes to 1000 N (ΔE_{p_2})

Analysis and Solution

(a) Use the equation for elastic potential energy (E_{p_1}) to find the elastic constant (k) for the spring. Use the elastic constant and the stretch to calculate the force.

$$E_{p_1} = \tfrac{1}{2}kx_1^2$$

$$\begin{aligned} k &= \frac{2E_{p_1}}{x_1^2} \\ &= \frac{2(5.00\times10^2 \text{ J})}{(0.400 \text{ m})^2} \\ &= 6.25\times10^3 \ \frac{\text{N}}{\text{m}} \end{aligned}$$

$$\begin{aligned} F_1 &= kx_1 \\ &= \left(6.25\times10^3 \ \frac{\text{N}}{\cancel{\text{m}}}\right)(0.400 \ \cancel{\text{m}}) \\ &= 2.50\times10^3 \text{ N} \end{aligned}$$

(b) Use the elastic constant and the force (F_2) to calculate the stretch (x_2) then use the elastic constant and the stretch to find the new elastic potential energy (E_{p_2}).

Find the difference in the elastic potential energies: ΔE_p.

$$F_2 = kx_2$$

$$\begin{aligned} x_2 &= \frac{F_2}{k} \\ &= \frac{1.00\times10^3 \ \cancel{\text{N}}}{6.25\times10^3 \ \frac{\cancel{\text{N}}}{\text{m}}} \\ &= 0.160 \text{ m} \end{aligned}$$

$$\begin{aligned} E_{p_2} &= \tfrac{1}{2}kx_2^2 \\ &= \tfrac{1}{2}\left(6.25\times10^3 \ \frac{\text{N}}{\text{m}}\right)(0.160 \text{ m})^2 \\ &= 80.0 \text{ J} \end{aligned}$$

$$\begin{aligned} \Delta E_p &= E_{p_2} - E_{p_1} \\ &= 80.0 \text{ J} - 5.00 \times 10^2 \text{ J} \\ &= -420 \text{ J} \end{aligned}$$

Paraphrase

(a) The force required to store 500 J of potential energy in the spring is 2500 N.

(b) If the force is reduced to 1000 N, then the spring loses 420 J of potential energy, ending up with only 80.0 J of potential energy.

Student Book page 302

Concept Check

Since the kinetic energy varies as the square of the speed, doubling the speed will increase the kinetic energy by four times.

Example 6.5 Practice Problems

1. ***Given***

$m_G = 45.0 \text{ kg} \quad m_B = 16.0 \text{ kg} \quad v = 2.50 \ \frac{\text{m}}{\text{s}}$

Required

the kinetic energy for the combined masses (E_k)

Analysis and Solution

Solve for the kinetic energy using the sum of the masses.

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(m_G + m_B)v^2 \\ &= \frac{1}{2}(45.0 \text{ kg} + 16.0 \text{ kg})\left(2.50 \ \frac{\text{m}}{\text{s}}\right)^2 \\ &= 1.906 \times 10^2 \text{ J} \\ &= 1.91 \times 10^2 \text{ J} \end{aligned}$$

Paraphrase

The combined kinetic energy of the girl and her bicycle is 1.91×10^2 J.

2. ***Given***

$v = 80.0 \ \frac{\text{km}}{\text{h}} = 22.2 \ \frac{\text{m}}{\text{s}} \quad E_k = 4.2 \times 10^5 \text{ J}$

Required

the mass of the car (m)

Analysis and Solution

Solve for mass from the equation for kinetic energy.

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ m &= \frac{2E_k}{v^2} \\ &= \frac{2(4.2 \times 10^5 \text{ J})}{22.2 \frac{\text{m}}{\text{s}}} \\ &= 1.701 \times 10^3 \text{ kg} \\ &= 1.7 \times 10^3 \text{ kg} \end{aligned}$$

Paraphrase

The mass of the car is 1.7×10^3 kg.

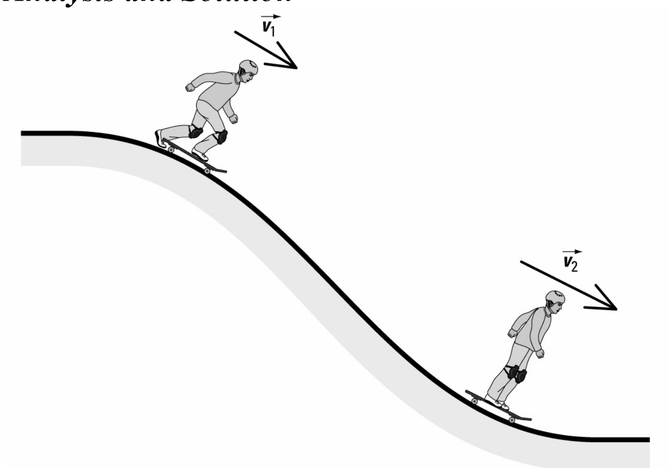
3. ***Given***

$m = 65.0 \text{ kg} \quad v_1 = 1.75 \ \frac{\text{m}}{\text{s}} \quad v_2 = 4.20 \ \frac{\text{m}}{\text{s}}$

Required

change in the kinetic energy (ΔE_k)

Analysis and Solution



Since kinetic energy does not increase linearly with speed, find the two kinetic energies and then calculate the difference.

$$\begin{aligned}
\Delta E_k &= E_{k_2} - E_{k_{1i}} \\
&= \tfrac{1}{2} m v_2{}^2 - \tfrac{1}{2} m v_1{}^2 \\
&= \tfrac{1}{2}(65.0 \text{ kg})\left(4.20 \ \frac{\text{m}}{\text{s}}\right)^2 - \tfrac{1}{2}(65.0 \text{ kg})\left(1.75 \ \frac{\text{m}}{\text{s}}\right)^2 \\
&= 5.73 \times 10^2 \text{ J} - 9.95 \times 10^1 \text{ J} \\
&= 4.74 \times 10^2 \text{ J}
\end{aligned}$$

Paraphrase

The skateboarder gains 474 J of kinetic energy.

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Example 6.6 Practice Problems

1. ***Given***

(a) $k = 2.50 \times 10^3 \ \frac{\text{N}}{\text{m}} \qquad x = 0.540 \text{ m}$

(b) $\text{m} = 95.0 \text{ g} = 9.50 \times 10^{-2} \text{ kg}$

Required

(a) the elastic potential energy stored in the bow (E_p)
(b) the speed of the arrow (v)

Analysis and Solution

(a) Use the equation for elastic potential energy is a spring.

$$E_p = \frac{1}{2}kx^2$$
$$= \frac{1}{2}\left(2.50\times10^3\ \frac{\text{N}}{\text{m}}\right)(0.540\ \text{m})^2$$
$$= 364.5\ \text{J}$$
$$= 365\ \text{J}$$

(b) Calculate the speed of the arrow from its kinetic energy, given that the arrow has a kinetic energy equal to the elastic potential energy in the bow.

$$E_p = E_k$$
$$= \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2(E_p)}{m}}$$
$$= \sqrt{\frac{2(364.5\ \text{J})}{9.50\times10^{-2}\ \text{kg}}}$$
$$= 87.60\ \frac{\text{m}}{\text{s}}$$
$$= 87.6\ \frac{\text{m}}{\text{s}}$$

Paraphrase

If all the potential energy of the bow was transformed into kinetic energy for the arrow the speed of the arrow would be 87.6 m/s.

2. ***Given***

$m_A = 1.5\ \text{kg}\quad v_A = 550\frac{\text{m}}{\text{s}}$

$\frac{m_B}{m_A} = \frac{1}{3}\quad \frac{v_B}{v_A} = \frac{2}{1}$

Required

Compare the kinetic energy of B to that of A (E_{k_B} / E_{k_A})

Analysis and Solution

Calculate the ratio for the kinetic energies of B to A. [**Note:** Students could calculate the actual mass and velocity of B then calculate the actual E_ks for A and B to see which is greater. However, they could just use the ratios, as follows.]

$$\frac{E_{k_B}}{E_{k_A}} = \frac{\frac{1}{2}m_B v_B{}^2}{\frac{1}{2}m_A v_A{}^2}$$

$$= \left(\frac{m_B}{m_A}\right)\left(\frac{v_B}{v_A}\right)^2$$

$$= \left(\frac{1}{3}\right)\left(\frac{2}{1}\right)^2$$

$$= \frac{4}{3}$$

Paraphrase

Since ball B has more energy than ball A, it will do more damage than A.

3. ***Given***

$m = 2.60 \times 10^8 \text{ kg}$

$v = 7.20 \times 10^4 \ \frac{\text{km}}{\text{h}} = 7.20 \times 10^7 \ \frac{\text{m}}{\text{h}}$ $\quad 1.00 \text{ t TNT} \simeq 4.60 \times 10^9 \text{ J}$

Required

(a) kinetic energy of the meteor (E_k)

(b) mass of TNT that would give the kinetic energy

Analysis and Solution

(a) Convert the units for v to m/s then use the equation for kinetic energy from page 302 (or the example problem).

$$v = \left(7.20 \times 10^7 \ \frac{\text{m}}{\cancel{\text{h}}}\right)\left(\frac{1.00 \ \cancel{\text{h}}}{3.60 \times 10^3 \text{ s}}\right)$$

$$= 2.00 \times 10^4 \ \frac{\text{m}}{\text{s}}$$

$$E_k = \tfrac{1}{2}mv^2$$

$$= \tfrac{1}{2}(2.60 \times 10^8 \text{ kg})\left(2.00 \times 10^4 \ \frac{\text{m}}{\text{s}}\right)^2$$

$$= 5.20 \times 10^{16} \text{ J}$$

(b) Use the given conversion factor to calculate the mass of TNT.

$$m_{TNT} = 5.20 \times 10^{16} \ \cancel{\text{J}}\left(\frac{1.00 \text{ t}}{4.60 \times 10^9 \ \cancel{\text{J}}}\right)$$

$$= 1.13 \times 10^7 \text{ t}$$

Paraphrase

The meteor had about 5.20×10^{16} J of kinetic energy. This is equivalent to the energy in 1.13×10^7 t of TNT.

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6.1 Check and Reflect

1. When the force does not act parallel to the displacement, the component of the force parallel to the displacement is reduced. Hence, the quantity of work done by the force is also reduced.

2. Since no component of the force acts parallel to the displacement, when a non-zero force acts at right angles to the displacement it will do no work
3. The gravitational potential energy depends on the object's position as defined relative to a planet. However, the change in gravitational potential energy depends on the difference in the heights of the starting point and the end point. This difference is independent of the reference point used to define the heights.
4. Elastic potential energy is created when an object is twisted, compressed, or stretched out of a shape to which it, when released, will return. Commonly, elastic potential energy is the result of stretching a spring.
5. ***Given***

$\vec{F} = 1.50 \times 10^3 \text{ N [up]} \quad m = 50.0 \text{ kg}$

$\Delta \vec{h} = 24.0 \text{ m [up]} \quad g = 9.81 \ \frac{\text{m}}{\text{s}^2} \text{ [down]}$

Required

(a) work done by the force (W)
(b) change in gravitational potential energy (ΔE_p)
(c) Explanation of why the answers are not equal

Analysis and Solution

(a) Use the equation for work to do this calculation.

$$\begin{aligned} W &= (F\cos\theta)\Delta d \\ &= (1.50 \times 10^3 \text{ N})(\cos 0°)(24.0 \text{ m}) \\ &= 3.60 \times 10^4 \text{ J} \end{aligned}$$

(b) Use the equation for change in gravitational potential energy.

$$\begin{aligned} \Delta E_p &= mg\Delta h \\ &= (50.0 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(24.0 \text{ m}) \\ &= 1.18 \times 10^4 \text{ J} \end{aligned}$$

(c) Explain why the answers differ, if they do differ. The applied force is much greater than the force required to lift the mass at a constant speed. The excess work done by the applied force would become kinetic energy.

Paraphrase

(a) The work done by the applied force is 3.60×10^4 J.
(b) The change in gravitational potential energy is 1.18×10^4 J.
(c) The applied force is much greater than the force required to lift the mass at a constant speed.

6. ***Given***

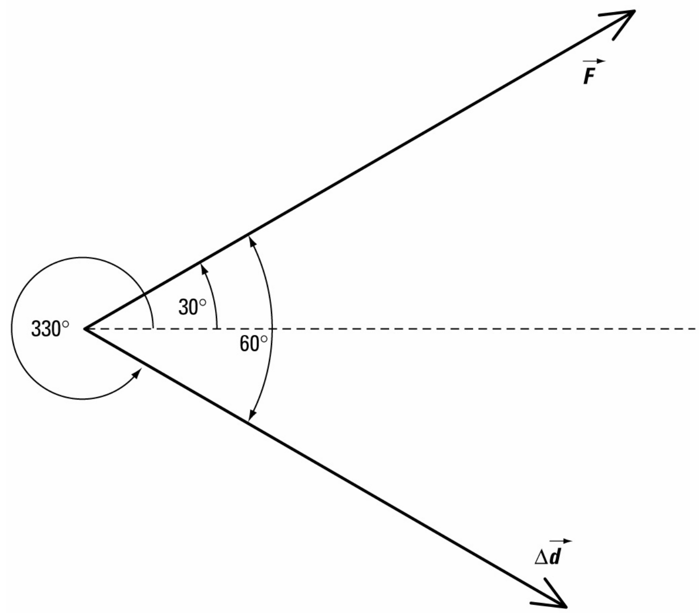
$\vec{F} = 850 \text{ N [30°]} \quad \Delta \vec{d} = 65.0 \text{ m[330°]}$

Required

work done by the force (W)

Analysis and Solution

Make a sketch of the vectors to determine the angle between them.
Use the angle in the equation that calculates work done by a force.



$$\begin{aligned} W &= (F\cos\theta)\Delta d \\ &= (850\ \text{N})(\cos 60°)(65.0\ \text{m}) \\ &= 2.76\times 10^4\ \text{J} \\ &= 2.8\times 10^4\ \text{J} \end{aligned}$$

Paraphrase

The force does 2.8×10^4 J of work.

7. ***Given***

$h_1 = -18.0\ \text{m} \quad h_2 = +22.0\ \text{m}$

$m = 350\ \text{kg} \quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

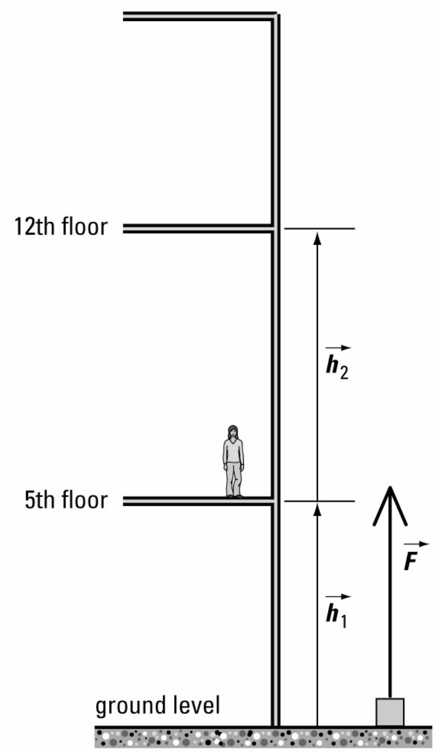
Required

(a) initial gravitational potential energy of the mass (E_{p_1})

(b) final gravitational potential energy of the mass (E_{p_2})

(c) change in gravitational potential energy of the mass (ΔE_p)

Analysis and Solution



(a) Use the initial height to calculate the gravitational potential energy at street level.

$$\begin{aligned} E_{p_1} &= mgh_1 \\ &= (350\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(-18.0\ \text{m}) \\ &= -6.18\times10^4\ \text{J} \end{aligned}$$

(b) Use the final height to calculate the final gravitational potential energy, on the 12th floor.

$$\begin{aligned} E_{p_2} &= mgh_2 \\ &= (350\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(22.0\ \text{m}) \\ &= 7.55\times10^4\ \text{J} \end{aligned}$$

(c) Find the difference between the answers to parts (b) and (a).

$$\begin{aligned} \Delta E_p &= E_{p_2} - E_{p_1} \\ &= \left(7.55\times10^4\ \text{J}\right) - \left(-6.16\times10^4\ \text{J}\right) \\ &= 1.37\times10^5\ \text{J} \end{aligned}$$

Paraphrase

(a) Relative to the 5th floor, the initial gravitational potential energy is -6.18×10^4 J.
(b) Relative to the 5th floor, the final gravitational potential energy is 7.55×10^4 J.
(c) The change in the gravitational potential energy is 1.37×10^5 J.

8. ***Given***

$k = 650\ \dfrac{\text{N}}{\text{m}} \quad x_1 = 0.100\ \text{m}$

$E_{p_2} = 3E_{p_1}$

Required

(a) initial elastic potential energy (E_{p_1})

(b) change in the stretch when the spring is compressed to triple its elastic potential energy (Δx)

Analysis and Solution

(a) Use the equation for elastic potential energy to calculate the initial elastic potential energy.

$$\begin{aligned} E_{p_1} &= \tfrac{1}{2}kx^2 \\ &= \tfrac{1}{2}\left(650\ \frac{\text{N}}{\text{m}}\right)(0.100\ \text{m})^2 \\ &= 3.25\ \text{J} \end{aligned}$$

(b) Triple the answer to part (a) to get the final elastic potential energy, and then find the compression that gives that energy. Find the difference between the initial and final compressions.

$$\begin{aligned} E_{p_1} &= 3\,(3.25\ \text{J}) \\ &= 9.75\ \text{J} \end{aligned}$$

$$\begin{aligned} E_{p_2} &= \tfrac{1}{2}kx_2{}^2 \\ x_2 &= \sqrt{\frac{2E_{p_2}}{k}} \\ &= \sqrt{\frac{2(9.75\ \text{J})}{650\ \frac{\text{N}}{\text{m}}}} \\ &= \sqrt{3.00\times10^{-2}\ \text{m}^2} \\ &= 1.73\times10^{-1}\ \text{m} \end{aligned}$$

$$\begin{aligned} \Delta x &= x_2 - x_1 \\ &= 0.173\ \text{m} - 0.100\ \text{m} \\ &= 0.73\ \text{m} \end{aligned}$$

Paraphrase

(a) When the spring is compressed 0.100 m, its elastic potential energy is 3.25 J.

(b) To triple the elastic potential energy of the spring, it must be compressed to 0.173 m, for a change in compression of 0.073 m.

9. ***Given***

$m_A = m_B = 1.20\times10^3\ \text{kg}$

$v_{i_A} = 12.0\dfrac{\text{m}}{\text{s}}[180°] \quad v_{i_B} = 24.0\ \dfrac{\text{m}}{\text{s}}[180°]$

$\Delta v_A = \Delta v_B = 10.0\ \dfrac{\text{m}}{\text{s}}[180°]$

Required

(a) change in kinetic energy of each car (ΔE_{k_A}, ΔE_{k_B})

(b) explanation of why cars that have equal gains in speed have unequal gains in kinetic energy.

Analysis and Solution

(a) Calculate the initial kinetic energy. Add the change in velocity to the initial velocity to get the final velocity. Calculate the final kinetic energy. Subtract the initial kinetic energy from the final kinetic energy to give the gain in kinetic energy.

$$\begin{aligned} E_{ki_A} &= \tfrac{1}{2}mv_{i_A}{}^2 \\ &= \tfrac{1}{2}\left(1.20\times10^3 \text{ kg}\right)\left(12.0 \ \frac{\text{m}}{\text{s}}\right)^2 \\ &= 8.64\times10^4 \text{ J} \end{aligned}$$

$$\begin{aligned} E_{ki_B} &= \tfrac{1}{2}mv_{i_B}{}^2 \\ &= \tfrac{1}{2}\left(1.20\times10^3 \text{ kg}\right)\left(24.0 \ \frac{\text{m}}{\text{s}}\right)^2 \\ &= 3.46\times10^5 \text{ J} \end{aligned}$$

$$\begin{aligned} v_{f_A} &= v_{i_A} + \Delta v_A \\ &= 12.0 \ \frac{\text{m}}{\text{s}}[180°] + 10.0 \ \frac{\text{m}}{\text{s}}[180°] \\ &= 22.0 \ \frac{\text{m}}{\text{s}}[180°] \end{aligned}$$

$$\begin{aligned} v_{f_B} &= v_{i_B} + \Delta v_B \\ &= 24.0 \ \frac{\text{m}}{\text{s}}[180°] + 10.0 \ \frac{\text{m}}{\text{s}}[180°] \\ &= 34.0 \ \frac{\text{m}}{\text{s}}[180°] \end{aligned}$$

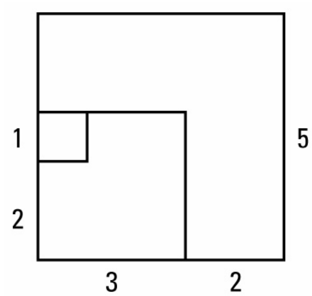
$$\begin{aligned} E_{k_{f_A}} &= \tfrac{1}{2}mv_{f_A}{}^2 \\ &= \tfrac{1}{2}\left(1.20\times10^3 \text{ kg}\right)\left(22.0 \ \frac{\text{m}}{\text{s}}\right)^2 \\ &= 2.90\times10^5 \text{ J} \end{aligned}$$

$$\begin{aligned} E_{k_{f_B}} &= \tfrac{1}{2}mv_{f_B}{}^2 \\ &= \tfrac{1}{2}\left(1.20\times10^3 \text{ kg}\right)\left(34.0 \ \frac{\text{m}}{\text{s}}\right)^2 \\ &= 6.94\times10^5 \text{ J} \end{aligned}$$

$$\begin{aligned} \Delta E_{k_A} &= E_{k_{f_A}} - E_{k_{i_A}} \\ &= 2.90\times10^5 \text{ J} - 8.64\times10^4 \text{ J} \\ &= 2.04\times10^5 \text{ J} \end{aligned}$$

$$\Delta E_{k_B} = E_{k_{fB}} - E_{k_{iB}}$$
$$= 6.94 \times 10^5 \text{ J} - 3.46 \times 10^5 \text{ J}$$
$$= 3.48 \times 10^5 \text{ J}$$

(b) Even though the gain in speed for both masses is equal, the mass that starts with the greater speed gains the most kinetic energy. This is because the kinetic energy varies as the square of the speed.
[**Note**: If students have trouble with this concept, demonstrate it visually by using geometry as shown below.]



Notice that increasing the length of the side of a square from 3 to 5, (+2) increases its area much more than increasing the length of the side from 1 to 3, (also +2). This is because the area of the square is proportional to the square of the length of its sides.

Paraphrase
When each car gains 10.0 m/s of speed, car A, which had an initial speed of 12.0 m/s, gains 2.04×10^5 J of kinetic energy while car B, which had an initial speed of 24.0 m/s, gains 3.49×10^5 J of kinetic energy.

10. ***Given***

$m = 3.00 \text{ kg} \quad g = 9.81 \dfrac{\text{m}}{\text{s}^2}$

$\Delta d = 7.50 \text{ m} \quad \Delta h = 3.75 \text{ m}$

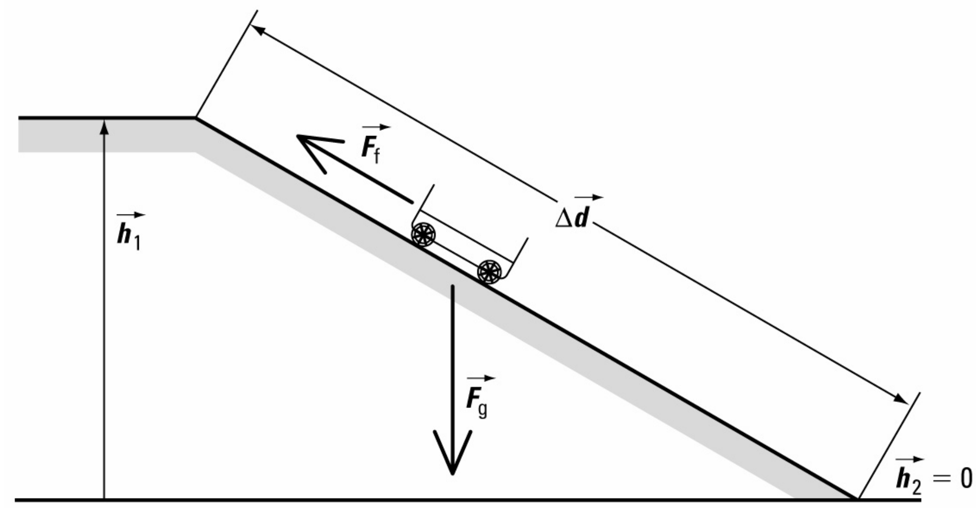
$F_2 = 4.50 \text{ N}$

Required

(a) change in gravitational potential energy (ΔE_p)

(b) work done by friction as the cart rolls down the hill

Analysis and Solution



(a) The change in gravitational potential energy is found by using the change in height; since the object is moving downhill, the change in height is negative.

$$\begin{aligned}\Delta E_p &= mg\Delta h \\ &= (3.00kg)\left(9.81\frac{m}{s^2}\right)(-3.75m) \\ &= -110.4 \text{ J} \\ &= -110 \text{ J}\end{aligned}$$

(b) The work done by the force of friction is calculated using the force of friction and the distance over which it acts. The direction of the force and the direction of the motion are in opposite directions. (θ = 180º). Thus the work done by friction is negative.

$$\begin{aligned}W &= (F\cos\theta)\Delta d \\ &= (4.50\text{ N})(\cos 180°)(7.50\text{ m}) \\ &= -33.75 \text{ J} \\ &= -33.8 \text{ J}\end{aligned}$$

Paraphrase

(a) As the cart rolls downhill it loses 110 J of gravitational potential energy.
(b) Friction does 33.8 J of work on the cart as it rolls downhill. This work removes energy from the cart's kinetic energy.

11. ***Given***

$k = 2.00\times10^3\ \frac{\text{N}}{\text{m}}$ $\quad x =$ 0.400 m

$m = 2.00$ kg $\quad v_1 = 0$

Required

(a) elastic potential energy stored in the spring when it is compressed 0.400 m.
(b) speed of a mass if it was given all the elastic potential energy stored in the spring.

Analysis and Solution

(a) Use the equation for elastic potential energy as a function of position and the elastic constant of the spring.

$$
\begin{aligned}
E_p &= \frac{1}{2}kx^2 \\
&= \frac{1}{2}\left(2.00\times10^3\ \frac{\text{N}}{\text{m}}\right)(0.400\ \text{m})^2 \\
&= 160\ \text{J}
\end{aligned}
$$

(b) Assume that the kinetic energy of the mass is equal to the elastic potential energy of the spring and calculate its speed.

$$
\begin{aligned}
E_k &= E_p \\
&= 160\ \text{J}
\end{aligned}
$$

$$
\begin{aligned}
E_k &= \frac{1}{2}mv^2 \\
v &= \sqrt{\frac{2E_k}{m}} \\
&= \sqrt{\frac{2(160\ \text{J})}{2.00\ \text{kg}}} \\
&= \sqrt{160\frac{\text{m}^2}{\text{s}^2}} \\
&= 12.649\ \frac{\text{m}}{\text{s}} \\
&= 12.6\ \frac{\text{m}}{\text{s}}
\end{aligned}
$$

Paraphrase

(a) When the spring is compressed 0.400 m from its equilibrium position it has 160 J of elastic potential energy stored in it.

(b) The speed of a 2.00 kg mass with 160 J of kinetic energy would be 12.6 m/s.

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Concept Check

Any force from outside the system, which removes mechanical energy from the block as it slides down the incline, would result in the value of the work done on the system being negative. For example, a person lowering the block down the incline using a rope could be considered to be removing mechanical energy from the system.

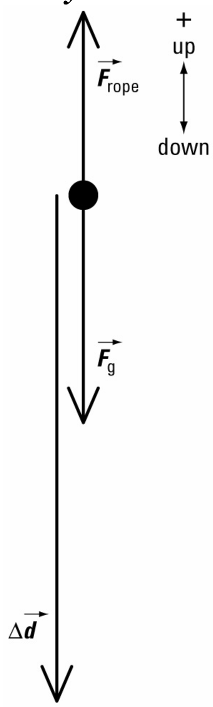
Example 6.7 Practice Problems

1. ***Given***

$\Delta h = -25.0\ \text{m}$ $\quad m = 72.0\ \text{kg}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$v_1 = 0$ $\quad v_2 = 5.00\ \frac{\text{m}}{\text{s}}$

Required
work done on the climber by the rope (W)
Analysis and Solution



According to the work-energy theorem, the work done on the climber is the sum of the change in his kinetic and potential energies.

$$\begin{aligned}
W &= \Delta E_k + \Delta E_p \\
&= \left(E_{k_2} - E_{k_1}\right) + mg\Delta h \\
&= \left(\tfrac{1}{2}mv_2^{\,2} - \tfrac{1}{2}mv_1^{\,2}\right) - mg\Delta h \\
&= \left(\tfrac{1}{2}(72.0 \text{ kg})\left(5.00\ \frac{\text{m}}{\text{s}}\right)^2 - \tfrac{1}{2}(72.0 \text{ kg})\left(0\right)^2\right) \\
&\qquad + (72.0 \text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(-25.0 \text{ m}) \\
&= \left(9.00\times10^2 \text{ J} - 0\right) + \left(-1.77\times10^4 \text{ J}\right) \\
&= -1.68\times10^4 \text{ J}
\end{aligned}$$

Paraphrase
While the climber rappels down the rope, the rope removes 1.68×10^4 J of energy from the climber. [**Note:** At first, students may find it awkward to conceive that the rope is taking energy away from the climber. Point out to them that without the rope the climber would have reached the bottom with a kinetic energy equal to $-\Delta E_p$ or 1.77×10^4 J. In that case the climber would have reached the ground with a speed of about 22 m/s instead of 5 m/s. The rope removed energy so that the climber arrived at the bottom with only 900 J of kinetic energy.]

2. ***Given***

$\vec{F}_{\text{app}} = 150$ N [up] $m = 9.00$ kg $\Delta d = 5.00$ m[up]

$\Delta h = 5.00$ m $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

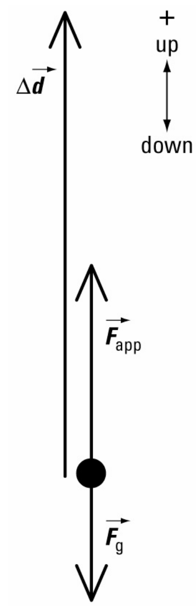
Required

(a) work done (W)

(b) change in gravitational potential energy (ΔE_p)

(c) change in kinetic energy (ΔE_k)

Analysis and Solution



(a) Calculate work using the equation from section 6.1.

$$\begin{aligned} W &= (F_{\text{app}} \cos\theta)\Delta d \\ &= (150\ \text{N} \cos 0°)(5.00\ \text{m}) \\ &= 7.50 \times 10^2\ \text{J} \end{aligned}$$

(b) Calculate the change in gravitational potential energy using the equation from section 6.1.

$$\begin{aligned} \Delta E_p &= mg\Delta h \\ &= (9.00\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(5.00\ \text{m}) \\ &= 4.41 \times 10^2\ \text{J} \end{aligned}$$

(c) Calculate the change in kinetic energy using the work-energy theorem.

$$\begin{aligned} W &= \Delta E_k + \Delta E_p \\ \Delta E_k &= W - \Delta E_p \\ &= 750\ \text{J} - 441\ \text{J} \\ &= 309\ \text{J} \end{aligned}$$

Paraphrase

The force does 750 J of work on the mass, of which 441 J becomes gravitational potential energy and 309 J becomes kinetic energy.

3. ***Given***

Data as in question 2 above, except that the acceleration due to gravity is needed as a vector quantity.

$\vec{g} = 9.81 \ \frac{\text{m}}{\text{s}^2}[\text{down}]$

Required

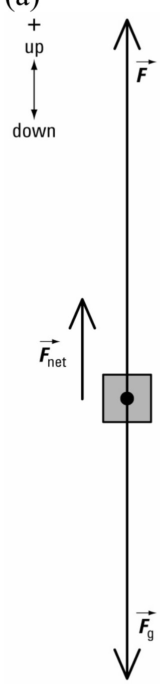
(a) free body diagram of the forces.

(b) net force on the mass ($\vec{F}_{\text{net}}$) and work it does (W_{net})

(c) the relationship of the answer to (b) to the answer to the previous question.

Analysis and Solution

(a)



(b) The sum of the forces gives the net force.

$$\begin{aligned}
\vec{F}_{\text{net}} &= \vec{F}_{\text{app}} + \vec{F}_{\text{g}} \\
&= \vec{F}_{\text{app}} + m\vec{g} \\
&= 150 \text{ N[up]} + (9.00 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}[\text{down}]\right) \\
&= 150 \text{ N[up]} + 88.3 \text{ N[down]} \\
&= 61.7 \text{ N[up]}
\end{aligned}$$

$$\begin{aligned}
W_{\text{net}} &= (F_{\text{net}} \cos\theta)(\Delta d) \\
&= (61.7 \text{ N})(\cos 0°)(5.00 \text{ m}) \\
&= 309 \text{ J}
\end{aligned}$$

(c) The work done by the net force is equivalent to the change in kinetic energy.

$W_{\text{net}} = \Delta E_{\text{k}}$

Paraphrase
(a) and (b) the sum of the forces gives the net force of 61.7 N [up]. [**Note:** This must be completed as a vector calculation.] The net work is equal to 309 J.
(c) According to the work-energy theorem, the work done by the net force is equivalent to the change in kinetic energy. The calculations for questions 2 & 3 confirm this principle for this data.

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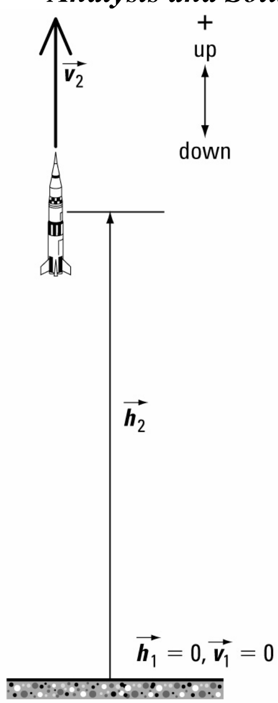
Example 6.8 Practice Problems

1. ***Given***

$h_2 = +5.00 \times 10^3 \text{ m} \quad g = 9.81 \dfrac{\text{m}}{\text{s}^2} \quad v_1 = 0$

$v_2 = 5.40 \times 10^3 \dfrac{\text{km}}{\text{h}} \quad m = 65.0 \times 10^4 \text{ kg}$

Required
mechanical energy of the rocket relative to its original position (E_m)
Analysis and Solution



Define the gravitational potential energy at the launch site to be zero. The mechanical energy relative to its original position is its final mechanical energy. Use the equation for mechanical energy on page 309 (or in the example problem).

First, convert the speed from km/h to m/s.

$$\left(5.40 \times 10^3\ \frac{\text{km}}{\text{h}}\right) = \left(5.40 \times 10^6\ \frac{\text{m}}{\text{h}}\right)\left(\frac{1.00\ \text{h}}{3.60 \times 10^3\ \text{s}}\right)$$

$$= 1.50 \times 10^3\ \frac{\text{m}}{\text{s}}$$

$$E_{\text{m}_2} = E_{\text{k}_2} + E_{\text{p}_2}$$

$$= \tfrac{1}{2}mv_2^{\ 2} + mgh_2$$

$$= \tfrac{1}{2}(6.50 \times 10^4\ \text{kg})\left(1.50 \times 10^3\ \frac{\text{m}}{\text{s}}\right)^2$$

$$+ (6.50 \times 10^4\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(5.00 \times 10^3\ \text{m})$$

$$= 7.31 \times 10^{10}\ \text{J} + 3.19 \times 10^9\ \text{J}$$

$$= 7.63 \times 10^{10}\ \text{J}$$

Paraphrase

Relative to its launch position, the rocket has 7.63×10^{10} J of mechanical energy.

2. ***Given***

$m = 4.50$ kg $\quad E_{\text{T}_2} = 6.27 \times 10^4$ J

$h_2 = 275$ m $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Required

Find the speed when the cannonball is 275 m above its staring point: v_2.

Analysis and Solution

Define $E_\text{p} = 0$ at ground level. Find E_k when the cannonball is 275 m above the ground by finding the difference between the mechanical and the potential energy for that location. Find the speed from the kinetic energy.

$$E_{\text{m}_2} = E_{\text{k}_2} + E_{\text{p}_2}$$

$$E_{\text{k}_2} = E_{\text{m}_2} - E_{\text{p}_2}$$

$$= E_{\text{m}_2} - mgh_2$$

$$= 6.27 \times 10^4\ \text{J}$$

$$- (4.50\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(275\ \text{m})$$

$$= 6.27 \times 10^4\ \text{J} - 1.21 \times 10^4\ \text{J}$$

$$= 5.06 \times 10^4\ \text{J}$$

$$E_{\text{k}_2} = \tfrac{1}{2}mv_2^{\ 2}$$

$$v_2 = \sqrt{\frac{2E_{\text{k}_2}}{m}}$$

$$= \sqrt{\frac{2(5.06 \times 10^4\ \text{J})}{4.50\ \text{kg}}}$$

$$= 150\ \frac{\text{m}}{\text{s}}$$

Paraphrase

When the cannonball is 275 m above the ground, its speed is 150 m/s.

3. ***Given***

$m = 600 \text{ kg} \quad \Delta h = -45.0 \text{ m} \quad v_2 = 30.0 \ \dfrac{\text{m}}{\text{s}}$

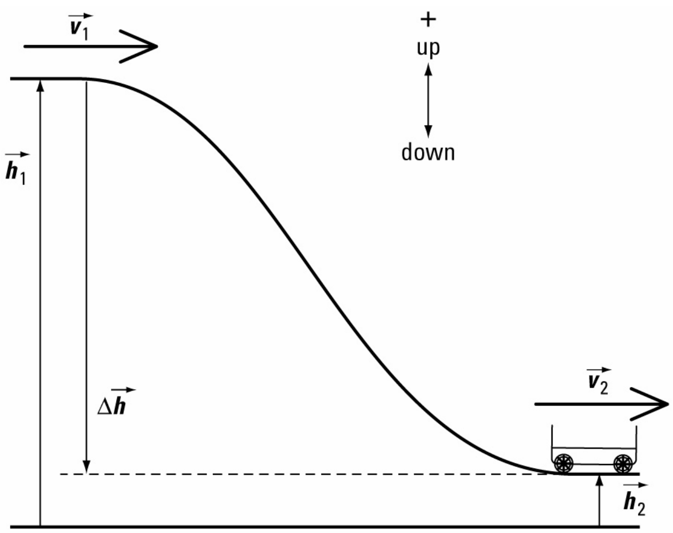
(a) $h_1 = 0$ (b) $h_1 = +51.0 \text{ m} \quad g = 9.81 \ \dfrac{\text{m}}{\text{s}^2}$

Required

(a) final mechanical energy relative to the top of the hill (E_{m_2})

(b) final mechanical energy relative to the ground (E_{m_2})

Analysis and Solution



(a) Define $h = 0$ at the top of the hill and calculate the mechanical energy at the bottom of the hill. This means that $h_1 = 0$. Use the values for h_1 and Δh to identify h_2.

$$\begin{aligned} h_2 &= h_1 + \Delta h \\ &= 0 + -45.0 \text{ m} \\ &= -45.0 \text{ m} \end{aligned}$$

$$\begin{aligned} E_{m_2} &= E_{k_2} + E_{p_2} \\ &= \tfrac{1}{2} m v_2^{\,2} + mgh_2 \\ &= \tfrac{1}{2}(600 \text{ kg})\left(30.0 \ \frac{\text{m}}{\text{s}}\right)^2 \\ &\quad + (600 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(-45.0 \text{ m}) \\ &= 2.70 \times 10^5 \text{ J} - 2.65 \times 10^5 \text{ J} \\ &= 5.13 \times 10^3 \text{ J} \end{aligned}$$

(b) Define $h = 0$ at ground level and calculate the mechanical energy at the bottom of the hill. This means that $h_1 = +51.0$ m. Use the values for h_1 and Δh to identify h_2.

$$\begin{aligned} h_2 &= h_1 + \Delta h \\ &= 51.0 \text{ m} + -45.0 \text{ m} \\ &= 6.00 \text{ m} \end{aligned}$$

$$
\begin{aligned}
E_{m_2} &= E_{k_2} + E_{p_2} \\
&= \tfrac{1}{2} m v_2^{\,2} + m g h_2 \\
&= \tfrac{1}{2}(600\text{ kg})\left(30.0\ \frac{\text{m}}{\text{s}}\right)^2 \\
&\quad + (600\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(6.00\text{ m}) \\
&= 2.70\times10^{5}\text{ J} + 3.53\times10^{4}\text{ J} \\
&= 3.05\times10^{5}\text{ J}
\end{aligned}
$$

Paraphrase

(a) Relative to the top of the hill, when the trolley is at the bottom of the hill, it has 5.13×10^3 J of mechanical energy.

(b) Relative to ground level, when the trolley is at the bottom of the hill, it has 3.05×10^5 J of mechanical energy.

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6.2 Check and Reflect

Knowledge

1. Mechanical energy is made up of kinetic energy, gravitational potential energy, and elastic potential energy.

2. The choice of frame of reference affects the calculation of gravitational potential energy. Since gravitational potential energy is part of mechanical energy, the choice of frame of reference also affects mechanical energy.

3. The work done by the net force is equal to the change in kinetic energy.

4. In a system, the sum of the changes in kinetic and potential energies is equal to the work done on the system by outside forces.

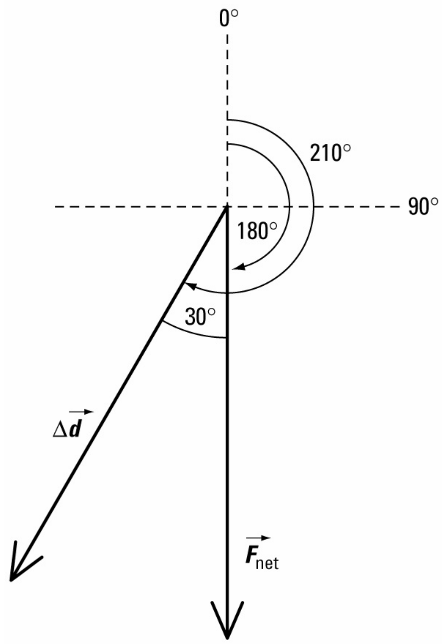
Applications

5. ***Given***

$\vec{F}_{\text{net}} = 5.75\times10^{3}\text{ N[180°]}$ $\quad m = 23.0\text{ kg}$

$\Delta\vec{d} = 360\text{ m[210°]}$

Required



work done by the force (W); prediction of the form of energy into which the work was transformed

Analysis and Solution

Calculate the magnitude of the angle between the vectors by subtracting the angles in which they are directed. Use the equation for work to calculate the answer.

$$\begin{aligned} W &= F_{net}(\cos\theta)\Delta d \\ &= \left(5.75\times10^3 \text{ N}\right)\left(\cos 30°\right)\left(360 \text{ m}\right) \\ &= 1.79\times10^6 \text{ J} \end{aligned}$$

The work done by a net force becomes kinetic energy.

Paraphrase

The net force acts on the mass to give it 1.70×10^6 J of kinetic energy.

6. ***Given***

$h = 75.0 \text{ m} \quad v = 240 \ \frac{\text{m}}{\text{s}}$

$m = 12.0 \text{ kg} \quad g = 9.81 \ \frac{\text{m}}{\text{s}^2}$

Required

mechanical energy (E_m)

Analysis and Solution

Mechanical energy is the sum of the kinetic and the gravitational potential energies.

$$
\begin{aligned}
E_m &= E_k + E_p \\
&= \tfrac{1}{2}mv^2 + mgh \\
&= \tfrac{1}{2}(12.0\text{ kg})\left(240\ \frac{\text{m}}{\text{s}}\right)^2 + (12.0\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(75.0\text{ m}) \\
&= 3.46\times10^5\text{ J} + 8.83\times10^3\text{ J} \\
&= 3.54\times10^5\text{ J}
\end{aligned}
$$

The direction of the velocity does not affect the kinetic energy.

Paraphrase

The cannonball has 3.54×10^5 J of kinetic energy regardless of its direction of travel.

7. ***Given***

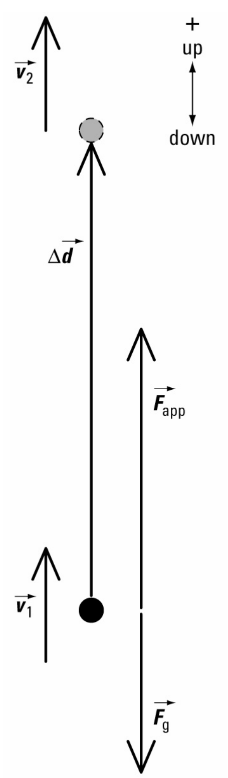
$m = 8.50\text{ kg} \quad \vec{v}_1 = 7.50\ \frac{\text{m}}{\text{s}}[\text{up}] \quad \vec{g} = 9.81\ \frac{\text{m}}{\text{s}^2}[\text{down}]$

$\vec{F} = 340\text{ N[up]} \qquad \Delta\vec{d} = \Delta\vec{h} = 15.0\text{ m[up]}$

Required

(a) work done on the object (W)
(b) change in gravitational potential energy (ΔE_p)
(c) change in kinetic energy (ΔE_k)
(d) final velocity (v_2)

Analysis and Solution



(a) Use the equation for work.

$$W = F(\cos\theta)\Delta d$$
$$= (340\ \text{N})(\cos 0°)(15.0\ \text{m})$$
$$= 5.10 \times 10^3\ \text{J}$$

(b) Use the equation for change in gravitational potential energy.

$$\Delta E_p = mg\Delta h$$
$$= (8.50\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(15.0\ \text{m})$$
$$= 1.25 \times 10^3\ \text{J}$$

(c) Use the work energy theorem to calculate the change in kinetic energy from the work and change in gravitational potential energy.

$$\Delta E_k + \Delta E_p = W$$
$$\Delta E_k = W - \Delta E_p$$
$$= 5.10 \times 10^3\ \text{J} - 1.25 \times 10^3\ \text{J}$$
$$= 3.85 \times 10^3\ \text{J}$$

(d) Calculate the final kinetic energy from the initial kinetic energy and the change in kinetic energy. Then calculate the final speed. Since all actions are upward, the direction of the final velocity is up as well.

$$E_{k_2} = E_{k_1} + \Delta E_k$$
$$= \tfrac{1}{2} m{v_1}^2 + \Delta E_k$$
$$= \tfrac{1}{2}(8.50\ \text{kg})\left(7.50\ \frac{\text{m}}{\text{s}}\right)^2 + (3.85 \times 10^3\ \text{J})$$
$$= 2.39 \times 10^2\ \text{J} + 3.85 \times 10^3\ \text{J}$$
$$= 4.09 \times 10^3\ \text{J}$$
$$E_{k_2} = \tfrac{1}{2} m{v_2}^2$$
$$v_2 = \sqrt{\frac{2E_{k_2}}{m}}$$
$$= \sqrt{\frac{2(4.09 \times 10^3\ \text{J})}{8.50\ \text{kg}}}$$
$$= \sqrt{9.62 \times 10^2\ \frac{\text{m}^2}{\text{s}^2}}$$
$$= 31.0\ \frac{\text{m}}{\text{s}}$$
$$\vec{v}_2 = 31.0\ \frac{\text{m}}{\text{s}}[\text{up}]$$

Paraphrase

(a) The force does 5100 J of work on the mass.
(b) The mass gains 1250 J of potential energy.
(c) The gain in kinetic energy is 3850 J.
(d) The final velocity is 31.0 m/s [up].

Extensions

8. ***Given***

$F = 2.40 \times 10^3$ N $\quad m = 7.00$ kg $\quad x_1 = 0.240$ m

$x_2 = 0.180$ m $\quad v_1 = 0 \quad v_2 = 6.00\ \dfrac{\text{m}}{\text{s}}$

Required

(a) initial mechanical energy (E_{m_1})

(b) final elastic potential energy (E_{p_2})

(c) change in elastic potential energy (ΔE_p)

(d) comparison of the change in elastic potential energy to the change in kinetic energy

Analysis and Solution

(a) Calculate the elastic constant for the spring. Then, the initial mechanical energy is the sum of the initial kinetic and elastic potential energies.

$$
\begin{aligned}
F_1 &= kx_1 \\
k &= \frac{F_1}{x_1} \\
&= \frac{2400\ \text{N}}{0.240\ \text{m}} \\
&= 1.00 \times 10^4\ \frac{\text{N}}{\text{m}}
\end{aligned}
$$

$$
\begin{aligned}
E_{\text{m}_1} &= E_{\text{k}_1} + E_{\text{p}_1} \\
&= \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 \\
&= \tfrac{1}{2}(7.00\ \text{kg})(0)^2 \\
&\quad + \tfrac{1}{2}\left(1.00 \times 10^4\ \frac{\text{N}}{\text{m}}\right)(0.240\ \text{m})^2 \\
&= 0 + 288\ \text{J} \\
&= 288\ \text{J}
\end{aligned}
$$

(b) Use the equation for elastic potential energy.

$$
\begin{aligned}
E_{\text{m}_2} &= E_{\text{k}_2} + E_{\text{p}_2} \\
&= \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2 \\
&= \tfrac{1}{2}(7.00\,\text{kg})\left(6.00\ \frac{\text{m}}{\text{s}}\right)^2 + \tfrac{1}{2}\left(1.00 \times 10^4\ \frac{\text{N}}{\text{m}}\right)(0.180\ \text{m})^2 \\
&= 126\ \text{J} + 162\ \text{J} \\
&= 288\ \text{J}
\end{aligned}
$$

(c) Assume that it is an isolated system so that mechanical energy is conserved. Find the difference between the final and initial elastic potential energies.

$$
\begin{aligned}
\Delta E_\text{p} &= E_{\text{p}_2} - E_{\text{p}_1} \\
&= -(162\ \text{J} - 288\ \text{J}) \\
&= 126\ \text{J}
\end{aligned}
$$

(d) Calculate the change in kinetic energy and compare the answer to that of part (c).

$$\Delta E_k = E_{k_2} - E_{k_1}$$
$$= \tfrac{1}{2}mv_2^{\,2} - \tfrac{1}{2}mv_1^{\,2}$$
$$= \tfrac{1}{2}(7.00\ \text{kg})\left(6.00\ \frac{\text{m}}{\text{s}}\right)^2$$
$$- \tfrac{1}{2}(7.00\ \text{kg})(0)^2$$
$$= 126\ \text{J}$$

The work done on the mass by the spring and the change in kinetic energy are identical.

Paraphrase

(a) The initial mechanical energy in the system is equal to 288 J.
(b) When compressed only 0.180 m, the spring is storing 162 J of energy.
(c) As the spring expands it does 126 J of work on the mass.
(d) The change in kinetic energy of the mass is 126 J, which is the result of the work the spring does on the mass.

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Example 6.9 Practice Problems

1. ***Given***

$E_{k_1} = 250\ \text{J}$ $E_{p_1} = 960\ \text{J}$ $\Delta E_p = -650\ \text{J}$

Required

final kinetic energy (E_{k_2})

Analysis and Solution

This problem can be approached in several ways.

1. Use mechanical energy to isolate initial kinetic energy and change in potential energy, as shown below. Then calculate final kinetic energy.
2. Use $\Delta E_k = -\Delta E_p$, then isolate E_{k_f} from the argument.
3. Find E_{p_2} from E_{p_1} and ΔE_p then isolate E_{k_2} in $E_{m_2} = E_{m_1}$.

$$\Delta E_k = -\Delta E_p$$
$$= -(-650\ \text{J})$$
$$= 650\ \text{J}$$
$$E_{k_2} - E_{k_1} = \Delta E_k$$
$$E_{k_2} = E_{k_1} + \Delta E_k$$
$$= 250\ \text{J} + 650\ \text{J}$$
$$= 900\ \text{J}$$

or

$$\begin{aligned} E_{m_2} &= E_{m_1} \\ E_{k_2} + E_{p_2} &= E_{k_1} + E_{p_1} \\ E_{k_2} &= E_{k_1} + E_{p_1} - E_{p_2} \\ &= E_{k_1} - (E_{p_2} - E_{p_1}) \\ &= E_{k_1} - \Delta E_p \\ &= 250 \text{ J} - (-650 \text{ J}) \\ &= 900 \text{ J} \end{aligned}$$

Paraphrase

If the potential energy decreases by 650 J, the kinetic energy increases by 650 J, to 900 J.

2. ***Given***

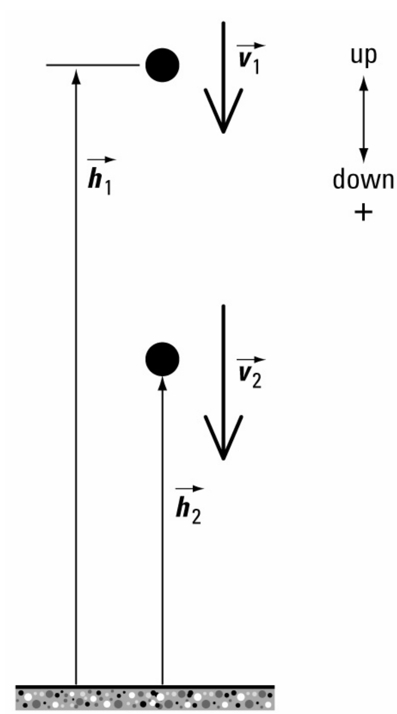
$m = 55.0$ kg $\quad h_1 = +225$ m

$\vec{v}_1 = 36.0\ \dfrac{\text{m}}{\text{s}}[\text{down}] \quad h_2 = +115$ m

Required

velocity when the mass is 115 m above the ground ($\vec{v}_2$)

Analysis and Solution



Find the final kinetic energy using the conservation of mechanical energy, then solve for speed. Since the velocity is requested, the direction, determined from the context of the question, is downward.

$$E_{m_2} = E_{m_1}$$
$$E_{k_2} + E_{p_2} = E_{k_1} + E_{p_1}$$
$$E_{k_2} = E_{k_1} + E_{p_1} - E_{p_2}$$
$$= \tfrac{1}{2}mv_1^{\,2} + mgh_1 - mgh_2$$
$$= \tfrac{1}{2}mv_1^{\,2} + mg(h_1 - h_2)$$
$$= \tfrac{1}{2}(55.0\text{ kg})\left(36.0\ \frac{\text{m}}{\text{s}}\right)^2$$
$$+ (55.0\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(225\text{ m} - 115\text{ m})$$
$$= 3.56\times10^4\text{ J} + 5.94\times10^4\text{ J}$$
$$= 9.50\times10^4\text{ J}$$
$$E_{k_2} = \tfrac{1}{2}mv_2^{\,2}$$
$$v_2 = \sqrt{\frac{2E_{k_2}}{m}}$$
$$= \sqrt{\frac{2(9.50\times10^4\text{ J})}{55.0\text{ kg}}}$$
$$= 58.8\ \frac{\text{m}}{\text{s}}$$
$$\vec{v}_2 = 58.8\ \frac{\text{m}}{\text{s}}[\text{down}]$$

Paraphrase

The object is moving at a velocity of 58.8 m/s [down].

3. ***Given***

$\vec{v}_1 = 21.0\ \frac{\text{m}}{\text{s}}$ [0° up 20°]

$h_1 = 15.0\text{ m} \quad m = 56.0\text{ kg}$

(a) v_2 = the horizontal component of $\vec{v}_1$

(b) $h_2 = 2.00\text{ m}$

Required

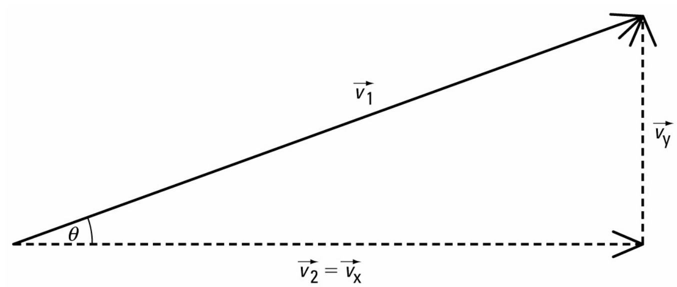
(a) gravitational potential energy at his highest point (E_{p_2})

(b) speed (v_2)

Analysis and Solution

(a) At the highest point the speed of the acrobat is the horizontal component of the original velocity. Find that component, then solve for the potential energy, assuming mechanical energy is conserved. First, calculate the initial mechanical energy since it is used in both parts.

$$
\begin{aligned}
E_{m_1} &= E_{k_1} + E_{p_1} \\
&= \tfrac{1}{2}mv_1^{\,2} + mgh_1 \\
&= \tfrac{1}{2}(56.0\text{ kg})\left(21.0\ \frac{\text{m}}{\text{s}}\right)^2 + (56.0\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(15.0\text{ m}) \\
&= 1.235\times10^4\text{ J} + 8.240\times10^3\text{ J} \\
&= 2.059\times10^4\text{ J} \\
&= 2.06\times10^4\text{ J}
\end{aligned}
$$



$$
\begin{aligned}
v_2 &= v_x \\
\frac{v_2}{v_1} &= (\cos\theta) \\
v_2 &= v_1(\cos\theta) \\
&= \left(21.0\ \frac{\text{m}}{\text{s}}\right)(\cos 20°) \\
&= 19.73\ \frac{\text{m}}{\text{s}} \\
&= 19.7\ \frac{\text{m}}{\text{s}}
\end{aligned}
$$

$$
\begin{aligned}
E_{m_2} &= E_{m_1} \\
E_{k_2} + E_{p_2} &= E_{m_1} \\
E_{p_2} &= E_{m_1} - E_{k_2} \\
&= E_{m_1} - \tfrac{1}{2}mv_2^{\,2} \\
&= 2.059\times10^4\text{ J} - \tfrac{1}{2}(56.0\text{ kg})\left(19.73\ \frac{\text{m}}{\text{s}}\right)^2 \\
&= 2.059\times10^4\text{ J} - 1.090\times10^4\text{ J} \\
&= 9.69\times10^3\text{ J}
\end{aligned}
$$

(b) Assuming that mechanical energy is conserved, find the final kinetic energy. Use the kinetic energy to calculate the final speed.

$$
\begin{aligned}
E_{T_2} &= E_{T_1} \\
E_{k_2} + E_{p_2} &= E_{T_1} \\
E_{k_2} &= E_{T_1} - E_{p_2} \\
&= E_{T_1} - mgh_2 \\
&= 2.059\times10^4\ \text{J} \\
&\quad - (56.0\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(2.00\ \text{m}) \\
&= 2.059\times10^4\ \text{J} - 1.098\times10^3\ \text{J} \\
&= 1.949\times10^4\ \text{J} \\
&= 1.95\times10^4\ \text{J} \\
E_{k_2} &= \tfrac{1}{2}mv_2{}^2 \\
v_2 &= \sqrt{\frac{2E_{k_2}}{m}} \\
&= \sqrt{\frac{2(1.95\times10^4\ \text{J})}{56.0\ \text{kg}}} \\
&= 26.4\ \frac{\text{m}}{\text{s}}
\end{aligned}
$$

Paraphrase

(a) The acrobat's greatest gravitational potential energy is 9.69×10^3 J.
(b) When the acrobat lands in the net, he has a speed of 26.4 m/s.

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Example 6.10 Practice Problems

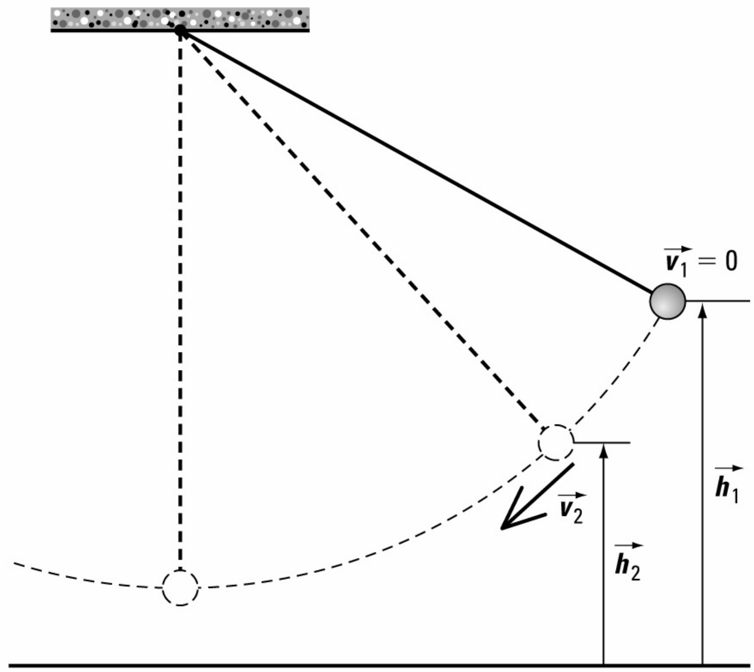
1. ***Given***

$h_1 = 2.50\times10^{-1}\text{m}$ $h_2 = 1.50\times10^{-1}\text{m}$

$m = 0.250$ kg $g = 9.81\ \frac{\text{m}}{\text{s}^2}$ $v_1 = 0$

Required

(a) Find the mechanical energy for the pendulum (E_{m2}) when it is 15.0 cm above the floor.
(b) Find the kinetic potential energy (E_{k2}) at 15.0 cm above the floor.
(c) Find the speed of the pendulum (v_2) when it is 15.0 cm above the floor.



Analysis and Solution

(a) Assume that the mechanical energy is conserved.

$$\begin{aligned}E_{m2} &= E_{m1}\\ &= 0.613\text{ J}\end{aligned}$$

(b) The kinetic energy can be found from the mechanical and the gravitational potential energies at the specified height of 15.0 cm above the floor.

$$\begin{aligned}E_{m2} &= E_{k2} + E_{p2}\\ E_{k2} &= E_{m2} - E_{p2}\\ &= E_{m2} - (mgh_2)\\ &= 0.6131\text{ J} - \left((0.250\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.50\times10^{-1}\text{ m})\right)\\ &= 0.6131\text{ J} - 0.3679\text{ J}\\ &= 0.2452\text{ J}\\ &= 0.245\text{ J}\end{aligned}$$

(c) Find the speed from the kinetic energy.

$$E_{k2} = \frac{1}{2}mv^2$$

$$v_2 = \sqrt{\frac{2E_{k2}}{m}}$$

$$= \sqrt{\frac{2(0.2452 \text{ J})}{0.250 \text{ kg}}}$$

$$= 1.401 \ \frac{\text{m}}{\text{s}}$$

$$= 1.40 \ \frac{\text{m}}{\text{s}}$$

Paraphrase

(a) At 15.0 cm above the floor the mechanical energy is 0.613 J.
(b) At 15.0 cm above the floor, the kinetic energy is 0.245 J.
(c) The speed of the bob is 1.40 m/s.

2. ***Given***

$h_1 = +220 \text{ m}$ $m = 3.00 \text{ kg}$

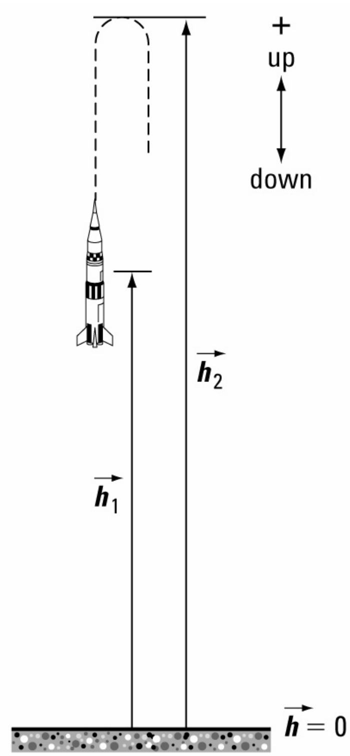
$g = 9.81 \ \frac{\text{m}}{\text{s}^2}$ $v_1 = 165 \ \frac{\text{m}}{\text{s}}$

(a) $v_2 = 0$ (b) $h_2 = 0$

Required

(a) rocket's mechanical energy at 220 m above the ground (E_{m_1})
(b) rocket's gravitational potential energy at its highest point (E_{p_2})
(c) height for the potential energy calculated in part (b) (h_2)
(d) speed when rocket hits the ground (v when $h = 0$)

Analysis and Solution



Assume that the rocket's motion is part of an isolated system.

(a) Calculate the initial mechanical energy from the given data.

$$\begin{aligned}
E_{m_1} &= E_{k_1} + E_{p_1} \\
&= \tfrac{1}{2}mv_1^2 + mgh_1 \\
&= \tfrac{1}{2}(3.00\ \text{kg})\left(165\ \frac{\text{m}}{\text{s}}\right)^2 \\
&\quad + (3.00\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(220\ \text{m}) \\
&= 4.08 \times 10^4\ \text{J} + 6.47 \times 10^3\ \text{J} \\
&= 4.73 \times 10^4\ \text{J}
\end{aligned}$$

(b) Assuming that the mechanical energy at the highest point is equal to the initial mechanical energy, calculate the gravitational energy at that point: E_{p_2} .

$$\begin{aligned}
E_{m_2} &= E_{m_1} \\
E_{p_2} + E_{k_2} &= E_{m_1} \\
E_{p_2} &= E_{m_1} - E_{k_2} \\
&= E_{m_1} - \tfrac{1}{2}mv_2^2 \\
&= 4.73 \times 10^4\ \text{J} - \tfrac{1}{2}(3.00\ \text{kg})(0)^2 \\
&= 4.73 \times 10^4\ \text{J}
\end{aligned}$$

(c) From the gravitational potential energy (in part (b)) calculate the height of the rocket: h_2.

$$E_{p_2} = mgh_2$$
$$h_2 = \frac{E_{p_2}}{mg}$$
$$= \frac{4.73\times10^4\ \text{J}}{(3.00\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$
$$= 1.61\times10^3\ \text{m}$$

(d) Assuming that the mechanical energy at the ground level is equal to the initial mechanical energy, use the kinetic energy at that point to calculate the speed of the rocket: v_2.

$$E_{m_2} = E_{m_1}$$
$$E_{k_2} + E_{p_2} = E_{m_1}$$
$$E_{k_2} = E_{m_1} - E_{p_2}$$
$$= E_{m_1} - mgh_2$$
$$= 4.73\times10^4\ \text{J} - (3.00\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0)$$
$$= 4.73\times10^4\ \text{J}$$
$$E_{k_2} = \tfrac{1}{2}mv_2{}^2$$
$$v_2 = \sqrt{\frac{2E_{k_2}}{m}}$$
$$= \sqrt{\frac{2(4.73\times10^4\ \text{J})}{3.00\ \text{kg}}}$$
$$= 1.78\times10^2\ \frac{\text{m}}{\text{s}}$$

Paraphrase

(a) The initial mechanical energy is 4.73×10^4 J.

(b) At the top of its rise the rocket has no kinetic energy, so that all of the mechanical energy is in the form of gravitational potential energy. Thus it has 4.73×10^4 J of potential energy.

(c) The height of the rocket when it has 4.73×10^4 J of potential energy is 1610 m.

(d) Its speed when it hits the ground will be about 178 m/s.

3. ***Given***

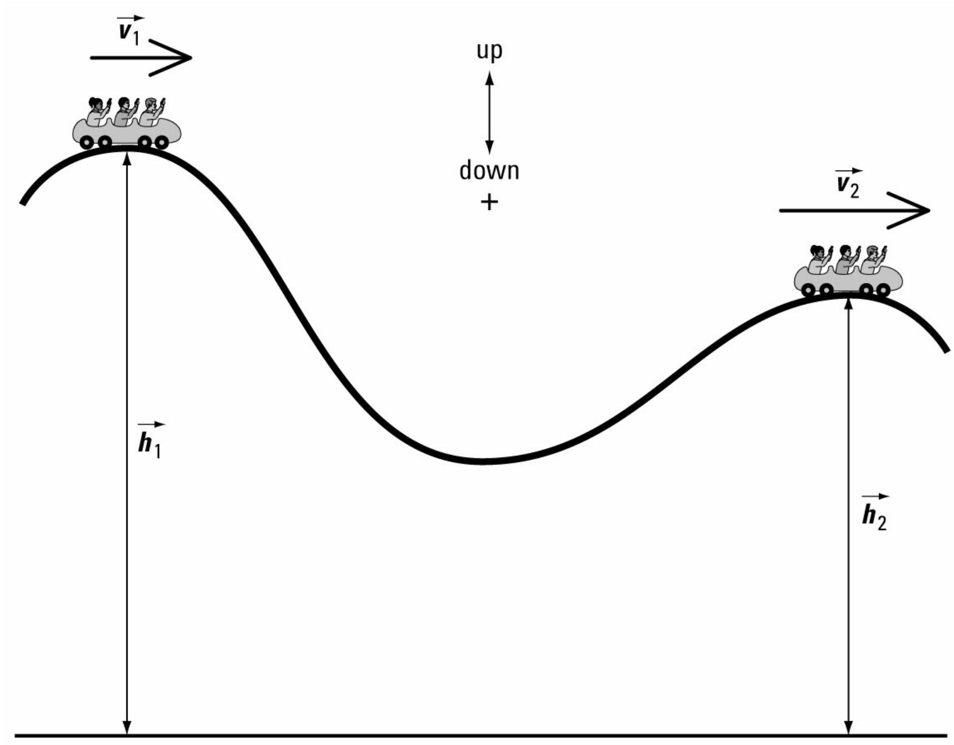
$m = 840\ \text{kg} \quad v_1 = 0.200\ \frac{\text{m}}{\text{s}} \quad h_1 = 85.0\ \text{m}$

$h_2 = 64.0\ \text{m} \quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Required

final speed (v_2)

Analysis and Solution



Calculate the initial mechanical energy. Assume that mechanical energy is conserved, so that at the top of the second hill the kinetic energy of the roller coaster is the difference between the mechanical energy and its gravitational potential energy. Use the calculated kinetic energy to calculate the final speed.

$$\begin{aligned}
E_{m_2} &= E_{m_1} \\
&= E_{k_1} + E_{p_1} \\
&= \tfrac{1}{2} m v_1^2 + mgh_1 \\
&= \tfrac{1}{2}(840\text{ kg})\left(0.200\ \frac{\text{m}}{\text{s}}\right)^2 + (840\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(85.0\text{ m}) \\
&= 16.8\text{ J} + 7.00 \times 10^5\text{ J} \\
&= 7.00 \times 10^5\text{ J}
\end{aligned}$$

$$\begin{aligned}
E_{k_2} + E_{p_2} &= E_{m_2} \\
E_{k_2} &= E_{m_2} - E_{p_2} \\
&= E_{m_2} - mgh_2 \\
&= 7.00 \times 10^5\text{ J} - (840\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(64.0\text{ m}) \\
&= 7.00 \times 10^5\text{ J} - 5.27 \times 10^5\text{ J} \\
&= 1.73 \times 10^5\text{ J}
\end{aligned}$$

Paraphrase
At the top of the second hill the trolley has a kinetic energy of 1.73×10^5 J.
[**Note:** Without friction to slow it down, the trolley would always have greater kinetic energy (and speed) at any point below its release at the top of the highest hill.]

4. ***Given***

$m = 56.0 \text{ kg} \quad g = 9.81 \ \frac{\text{m}}{\text{s}^2}$

$h_1 = 0.850 \text{ m} \quad v_1 = 8.00 \ \frac{\text{m}}{\text{s}}$

$v_2 = 0$

Required
height of the pole-vaulter at the highest point (h_2)

Analysis and Solution
When she is on the ground, the pole-vaulter's centre of mass is 0.850 m above the ground. Calculate the initial mechanical energy and use that to find her potential energy at her greatest height. Use her potential energy to calculate her height at that time. Since she will be in a horizontal position as she goes over the bar, that height is a good estimate of the highest bar that she will be able to clear. Actually, by bending her torso, her centre of mass can pass below the bar as she passes over.

$$\begin{aligned}
E_{\text{m}_1} &= \tfrac{1}{2}mv_1^2 + mgh_1 \\
&= \tfrac{1}{2}(56.0 \text{ kg})\left(8.00 \ \frac{\text{m}}{\text{s}}\right)^2 + (56.0 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(0.850 \text{ m}) \\
&= 1.79 \times 10^3 \text{ J} + 4.67 \times 10^2 \text{ J} \\
&= 2.26 \times 10^3 \text{ J}
\end{aligned}$$

$$\begin{aligned}
E_{\text{m}_2} &= E_{\text{m}_1} \\
E_{\text{k}_2} + E_{\text{p}_2} &= E_{\text{m}_1} \\
E_{\text{p}_2} &= E_{\text{m}_1} - E_{\text{k}_2} \\
mgh_2 &= E_{\text{m}_1} - \tfrac{1}{2}mv_2^2 \\
h_2 &= \frac{E_{\text{m}_1} - \tfrac{1}{2}mv_2^2}{mg} \\
&= \frac{2.26 \times 10^3 \text{ J} - 0}{(56.0 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)} \\
&= 4.11 \text{ m}
\end{aligned}$$

Paraphrase
(a) The greatest height that the pole-vaulter can attain is 4.11 m.
(b) The vaulter first converts chemical energy from her muscles into kinetic energy. When she plants the pole in the vaulting box it begins to bend, transforming some of her kinetic energy into elastic potential energy. At the same time, she begins to transform kinetic energy into gravitational potential energy as she starts her ascent to the bar. Soon her forward motion has stopped and she is rising toward the bar. At that time she has some kinetic energy, some gravitational potential energy, and the pole has some elastic potential energy. As she rises higher, elastic potential

energy and kinetic energy are further transformed into gravitational potential energy until she reaches her highest point, where she has only gravitational potential energy. During the flexing of the pole, as it first bends to accept elastic potential energy then unbends to lose its elastic potential energy, it will transform energy into heat.

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Concept Check

You must assume that the system is isolated, in that mechanical energy is conserved. You assume that only conservative forces act on the objects in the system. Hence, energy may be transformed while it stays within the same object, or transferred between objects.

Student Book page 322

Concept Check

Think of a force or group of forces, equal in magnitude but opposite in direction to that of friction, acting on the block. The work these forces do on the block would replace the energy that friction removes from the system, so that the net change in energy of the block as it moves down the plane would be zero.

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6.3 Check and Reflect

Knowledge

1. An isolated system is one in which no energy can be added or taken out. Mechanical energy is conserved.

2. Even though no energy can enter or exit from an isolated system, work can still be done within the system, by transferring energy between objects within the system, or transforming energy from one form to another.

3. When the acrobat is at the highest point of a jump, all her energy is in the form of gravitational potential energy. If there is no friction, as she falls, gravity converts gravitational potential energy to kinetic energy. At the instant she touches the surface of the trampoline; her kinetic energy is at its greatest. As she depresses the trampoline she is losing kinetic energy and more gravitational potential energy. At the point of maximum stretch, all the gravitational potential energy has been transformed into elastic potential energy in the trampoline. By flexing her knees to jump, she can increase her kinetic energy each time she leaves the surface of the trampoline, so that each successive jump is higher.

4. If a non-conservative force acts on the objects in a system, then mechanical energy cannot be conserved. The system cannot be considered isolated.

5. Initially, the golfer works on the club to give it kinetic energy. When the club contacts the ball, the ball is distorted so that some of the club's kinetic energy is transformed into elastic potential energy, as the force exerted on the ball by the club accelerates the ball to the speed of the club head. As the ball leaves the club face, its elastic potential energy is converted back into kinetic energy so that the ball leaves the club with even greater kinetic energy. As the ball is in flight, initially it is rising so that kinetic energy is being transformed into gravitational potential energy. At the top of the arc, it has minimum kinetic energy and maximum gravitational potential energy. As it falls it increases kinetic energy and loses gravitational potential energy. When it lands on the ground below the cliff, its kinetic energy is at its greatest and its gravitational potential energy is at its least.

6. Since the pendulum clock is only a good approximation of an isolated system, each time it swings frictional forces would transform a tiny bit of kinetic energy into heat. To maintain the mechanical energy of the pendulum at a constant value, on each swing a tiny bit of energy must be added by giving a tiny push.

Applications

7. Yes, this system can be considered isolated.

Given

$m_{\mathrm{A}} = 2.40\ \mathrm{kg} \quad m_{\mathrm{B}} = 1.50\ \mathrm{kg} \quad g = 9.81\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$

$\Delta h_{\mathrm{A}} = -1.40\ \mathrm{m} \quad \Delta h_{\mathrm{B}} = +1.40\ \mathrm{m}$

Required

kinetic energy in the system when mass A hits the table

Analysis and Solution

As mass A falls and loses gravitational potential energy, mass B rises so that it gains gravitational potential energy. The gain in kinetic energy of this system will be equal to the net loss in the gravitational potential energy.

$$\begin{aligned}
\Delta E_{\mathrm{p}} &= \Delta E_{\mathrm{p_A}} + \Delta E_{\mathrm{p_B}} \\
&= m_{\mathrm{A}} g \Delta h_{\mathrm{A}} + m_{\mathrm{B}} g \Delta h_{\mathrm{B}} \\
&= (2.40\ \mathrm{kg})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)(-1.40\ \mathrm{m}) + (1.50\ \mathrm{kg})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)(+1.40\ \mathrm{m}) \\
&= -33.0\ \mathrm{J} + 20.6\ \mathrm{J} \\
&= -12.4\ \mathrm{J}
\end{aligned}$$

$$\begin{aligned}
E_{\mathrm{k_2}} &= \Delta E_{\mathrm{k}} + E_{\mathrm{k_1}} \\
&= -\Delta E_{\mathrm{p}} + E_{\mathrm{k_1}} \\
&= -(-12.4\ \mathrm{J}) + 0 \\
&= 12.4\ \mathrm{J}
\end{aligned}$$

Paraphrase

Since it is an isolated system, it converts 12.4 J of gravitational potential energy into kinetic energy.

8. ***Given***

Masses and relationships as shown in figure, page 323.

Required

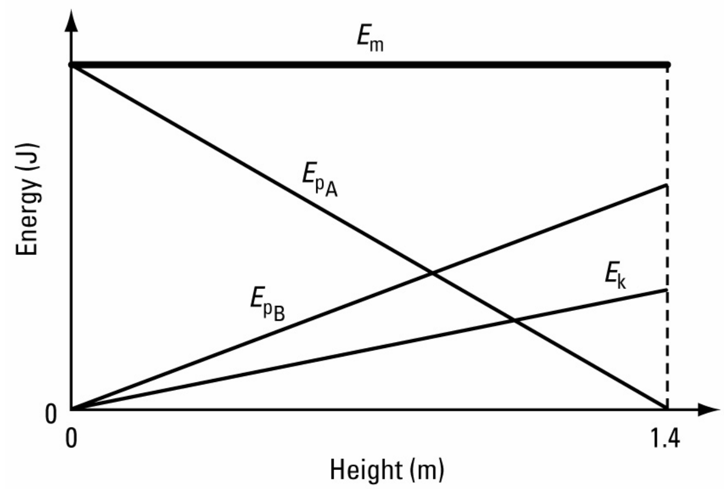
Graphs of gravitational potential, kinetic, and mechanical energy of the system versus the change in the position of mass A for

(a) no friction

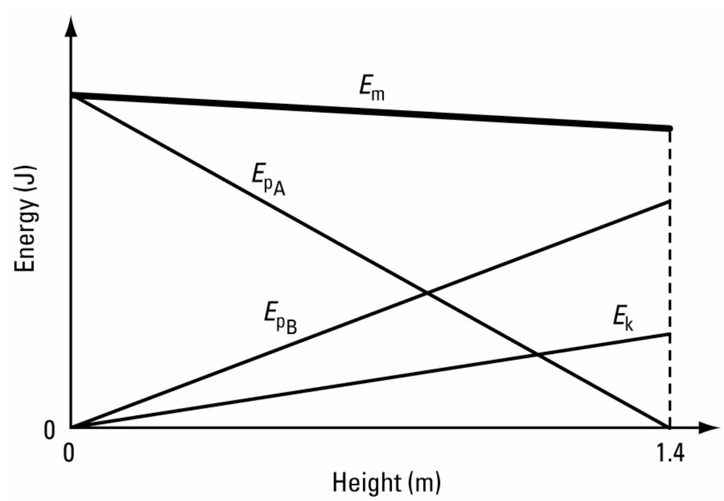
(b) friction, but mass A accelerates

Analysis and Solution

(a) for no friction



(b) For a system with friction. [Note that the gain in kinetic energy is less than in the ideal case (part (a)). This means that the mechanical energy is also reduced; it is reduced by the same quantity as the kinetic energy.] 107



9. (a) The graph shows that the object is gaining gravitational potential energy while it maintains a constant kinetic energy. As a result the mechanical energy increases at the same rate as the gravitational potential energy. Work is being done on this system to increase gravitational potential energy but not kinetic energy. The force is applied to the mass so that it lifts the mass (or pulls it up a frictionless incline) at a constant speed.

(b) The mechanical energy is increasing. Therefore an outside force is adding energy to the system. The gravitational potential energy is decreasing, so that the object is falling. The gain in kinetic energy is the result of the outside work plus the loss in gravitational potential energy. This graph could represent a block being pulled by a force down an inclined plane.

Extensions

10. ***Given***

$L = 3.60 \text{ m} \quad x = 1.80 \text{ m} \quad g = 9.81 \ \dfrac{\text{m}}{\text{s}^2}$

$m = 1.25 \text{ kg} \ \ v_1 = \ 0$

(b) $h_2 = \ 0$

(c) $h_2 = \ 0.250 \text{ m}$

Required

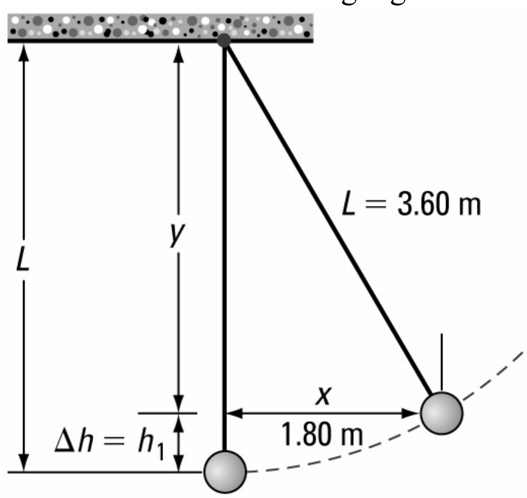
(a) initial height of the pendulum bob relative to its rest position (h_1)

(b) speed of the pendulum bob when it passes through its rest position (v_2)

(c) speed of the bob as it passes through a point 0.250 m above its rest position (v_2)

Analysis and Solution

(a) Make a labelled diagram of the system and use the Pythagorean Theorem to calculate the initial height gain of the pendulum bob.



From the diagram

$$L^2 = x^2 + y^2$$

$$\begin{aligned} y &= \sqrt{L^2 - x^2} \\ &= \sqrt{(3.60 \text{ m})^2 - (1.80 \text{ m})^2} \\ &= 3.118 \text{ m} \\ &= 3.12 \text{ m} \end{aligned}$$

$$\begin{aligned} \Delta h &= L - y \\ &= 3.60 \text{ m} - 3.12 \text{ m} \\ &= 0.482 \text{ m} \end{aligned}$$

$$h_1 = 0.482 \text{ m}$$

(b) Use the equation for conservation of energy to solve for the final kinetic energy from which the final speed can be found.

$$
\begin{aligned}
E_{m_2} &= E_{m_1} \\
E_{k_2} + E_{p_2} &= E_{k_1} + E_{p_1} \\
\tfrac{1}{2}mv_2{}^2 + mgh_2 &= \tfrac{1}{2}mv_1{}^2 + mgh_1 \\
\tfrac{1}{2}mv_2{}^2 + 0 &= 0 + mgh_1 \\
v_2 &= \sqrt{\frac{2mgh_1}{m}} \\
&= \sqrt{2gh_1} \\
&= \sqrt{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.482\ \text{m})} \\
&= 3.08\ \frac{\text{m}}{\text{s}}
\end{aligned}
$$

(c) Use the equation for conservation of energy to solve for the final kinetic energy from which the final speed can be found.

$$
\begin{aligned}
E_{m_2} &= E_{m_1} \\
E_{k_2} + E_{p_2} &= E_{k_1} + E_{p_1} \\
\tfrac{1}{2}mv_2{}^2 + mgh_2 &= \tfrac{1}{2}mv_1{}^2 + mgh_1 \\
\tfrac{1}{2}mv_2{}^2 &= 0 + mgh_1 - mgh_2 \\
v_2 &= \sqrt{\frac{2mgh_1 - mgh_2}{m}} \\
&= \sqrt{2g(h_1 - h_2)} \\
&= \sqrt{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.482\ \text{m} - 0.250\ \text{m})} \\
&= 2.13\ \frac{\text{m}}{\text{s}}
\end{aligned}
$$

Paraphrase

(a) When the bob is pulled sideways a distance of 1.80 m it gains a height of 0.482 m.
(b) At the lowest point of its swing, the bob has a speed of 3.08 m/s.
(c) As the bob passes through the point, which is 0.250 m above its lowest point, its speed is 2.13 m/s.

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Concept Check

In terms of units for mass (kg), length (m) and time(s), the unit of power (W) is

$$1\ \text{W} = 1\frac{\text{kg}\cdot\text{m}^2}{\text{s}^3}$$

Example 6.11 Practice Problems

1. ***Given***

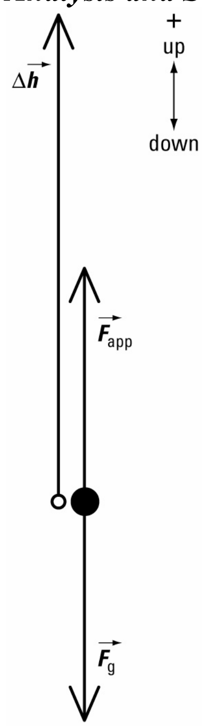
$m = 1.50 \text{ t} = 1.50 \times 10^3 \text{ kg}$

$\Delta h = 65.0 \text{ m} \quad \Delta t = 3.50 \text{ min}\left(\frac{60.0 \text{ s}}{1.00 \text{ min}}\right) = 210 \text{ s}$

Required

power output of the crane (P)

Analysis and Solution



Use the equation for power given in the text on page 324 (or in the example).

$$
\begin{aligned}
P &= \frac{\Delta E}{\Delta t} \\
&= \frac{\Delta Ep}{\Delta t} \\
&= \frac{mg\Delta h}{\Delta t} \\
&= \frac{(1.50 \times 10^3 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(65.0 \text{ m})}{210 \text{ s}} \\
&= \frac{9.56 \times 10^5 \text{ J}}{210 \text{ s}} \\
&= 4.55 \times 10^3 \ \frac{\text{J}}{\text{s}} \\
&= 4.55 \times 10^3 \text{ W or } 4.55 \text{ kW}
\end{aligned}
$$

1 hp = 746 W. Therefore, the power of the crane in hp is $\frac{4.55\times10^{3}}{746}$ hp or 6.10 hp.

Paraphrase

The power output of the crane was 4.55 kW (6.10 hp).

2. ***Given***

$P = 5.60 \text{ kW} = 5.60 \times 10^{3} \text{ W}$

$\Delta t = 20.0 \text{ min} = 1.20 \times 10^{3} \text{ s}$

Required

work done (W)

Analysis and Solution

Solve for work from the equation for power.

$$
\begin{aligned}
P &= \frac{W}{\Delta t} \\
W &= P\Delta t \\
&= (5.60\times10^{3} \text{ W})(1.20\times10^{3} \text{ s}) \\
&= 6.72\times 10^{6} \text{ J}
\end{aligned}
$$

Paraphrase

The motor does 6.72×10^{6} J of work.

3. ***Given***

$F = 7.50\times10^{3} \text{ N} \quad \Delta d = 3.20\times10^{3} \text{ m}$

$P = 25.0 \text{ kW} = 2.50 \times 10^{4} \text{ W}$

Required

time required to do the work (Δt)

Analysis and Solution

Using the equation for power, isolate the variable for time and then solve.

$$
\begin{aligned}
P &= \frac{W}{\Delta t} \\
\Delta t &= \frac{W}{P} \\
&= \frac{F\Delta d}{P} \\
&= \frac{(7.50\times10^{3} \text{ N})(3.20\times10^{3} \text{ m})}{2.50\times10^{4} \text{ W}} \\
&= \frac{2.40\times10^{7} \text{ J}}{2.50\times10^{4} \frac{\text{J}}{\text{s}}} \\
&= 9.60\times10^{2} \text{ s or 16 min.}
\end{aligned}
$$

Paraphrase

It took 960 s (16.0 minutes) to do the work.

Example 6.12 Practice Problems

1. ***Given***

$m = 1.50 \times 10^3 \text{ kg} \quad g = 9.81 \ \frac{\text{m}}{\text{s}^2}$

$v = 0.750 \ \frac{\text{m}}{\text{s}}$

Required

power output (P)

Analysis and Solution

The force required to lift the mass at a constant speed is equal to the force of gravity. The power output will be the product of the force and the speed.

$$\begin{aligned} P &= Fv \\ &= mgv \\ &= (1.50 \times 10^3 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)\left(0.750 \ \frac{\text{m}}{\text{s}}\right) \\ &= 1.10 \times 10^4 \text{ W} \\ &= 11.0 \text{ kW} \end{aligned}$$

Paraphrase

The power output of the motor is 11.0 kW.

2. ***Given***

$P = 150 \text{ kW} = 1.50 \times 10^5 \text{ W} \quad g = 9.81 \ \frac{\text{m}}{\text{s}^2}$

$m = 2.00 \text{ t} = 2.00 \times 10^3 \text{ kg}$

Required

greatest average speed possible to lift the mass (v_{ave})

Analysis and Solution

Find the force required to lift the mass at a constant speed (mg), then solve the power equation for speed.

$$P = Fv_{\text{ave}}$$

$$\begin{aligned} v_{\text{av}} &= \frac{P}{F} \\ &= \frac{P}{mg} \\ &= \frac{1.50 \times 10^5 \text{ W}}{(2.00 \times 10^3 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)} \\ &= 7.65 \ \frac{\text{m}}{\text{s}} \end{aligned}$$

Paraphrase

The average speed of the mass will be 7.65 m/s.

3. ***Given***

$m = 1.25 \times 10^3 \text{ kg} \quad g = 9.81 \ \frac{\text{m}}{\text{s}^2}$

$v_1 = 0 \quad v_2 = 30.0 \ \frac{\text{m}}{\text{s}} \quad \Delta t = 4.00 \text{ s}$

Required

power output of the motor (P)

Analysis and Solution

First, calculate the acceleration to be used to calculate the force acting on the mass. Then calculate the average speed of the mass. Use the force and average speed to calculate the power.

$$\begin{aligned} a &= \frac{v_2 - v_1}{\Delta t} \\ &= \frac{30.0 \ \frac{\text{m}}{\text{s}} - 0}{4.00 \text{ s}} \\ &= 7.50 \ \frac{\text{m}}{\text{s}^2} \end{aligned}$$

$$\begin{aligned} v_{\text{av}} &= \tfrac{1}{2}(v_2 + v_1) \\ &= \tfrac{1}{2}\left(30.0 \ \frac{\text{m}}{\text{s}} + 0\right) \\ &= 15.0 \ \frac{\text{m}}{\text{s}} \end{aligned}$$

$$\begin{aligned} P &= Fv_{\text{av}} \\ &= (ma)v_{\text{ave}} \\ &= \left(1.25 \times 10^3 \text{ kg}\right)\left(7.50 \ \frac{\text{m}}{\text{s}^2}\right)\left(15.0 \ \frac{\text{m}}{\text{s}}\right) \\ &= 1.41 \times 10^5 \text{ W} \\ &= 141 \text{ kW} \end{aligned}$$

Paraphrase

The power output of the motor is 141 kW.

4. ***Given***

$F_{\text{air}}/_{\text{car}} = 6.00 \times 10^2 \text{ N} \quad n = 75 \text{ cars}$

(a) $v_{\text{ave}} = 72.0 \ \frac{\text{km}}{\text{h}}$

(b) eff = 0.150

Required

(a) power output to move the train (P)

(b) actual power the engine must generate

Analysis and Solution

(a) Calculate the total force to move the train by multiplying the number of cars by the force to move each car. Then calculate the power required to exert that force at the given speed.

$$72.0\ \frac{\text{km}}{\text{s}} = \left(72.0\times10^{3}\ \frac{\text{m}}{\cancel{\text{h}}}\right)\left(\frac{1.00\ \cancel{\text{h}}}{3.60\times10^{3}\ \text{s}}\right)$$
$$= 20.0\ \frac{\text{m}}{\text{s}}$$
$$F_{\text{air}} = \frac{F_{\text{air}}}{\text{car}} n$$
$$= \left(\frac{6.00\times10^{2}\ \text{N}}{\text{car}}\right)(75\ \text{cars})$$
$$= 4.50\times10^{4}\ \text{N}$$
$$P = F_{\text{air}} v$$
$$= \left(4.50\times10^{4}\ \text{N}\right)\left(20.0\ \frac{\text{m}}{\text{s}}\right)$$
$$= 9.00\times10^{5}\ \frac{\text{Nm}}{\text{s}}$$
$$= 9.00\times10^{5}\ \text{W}$$

(b) Since the answer to (a) represents 15.0% of the power output of the engine, calculate the real power output of the engine.

$$eff = \frac{P_{\text{out}}}{P_{\text{in}}}$$
$$P_{\text{in}} = \frac{P_{\text{out}}}{eff}$$
$$= \frac{9.00\times10^{5}\ \text{W}}{0.150}$$
$$= 6.00\times10^{6}\ \text{W}$$

Paraphrase

(a) The power used to pull the 75-car train is 9.00×10^{5} W, or 900 kW.

(b) At 15.0% efficiency, the engine would need to produce 6.00×10^{6} W (6.00×10^{3} kW) of power.

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6.4 Check and Reflect

Knowledge

1. The power describes the rate at which the work is done rather than the quantity of work. Given sufficient time, any quantity of work can be done by any power output.

2. The transmission of the truck is geared to produce speed instead of force. The tractor's transmission is set up to exert large forces at low speeds.

3. For a constant force, the power output varies directly as the speed.

Applications

4. ***Given***

$m = 25.0$ kg $\quad \Delta h = 0.800$ m

$g = 9.81\ \frac{\text{m}}{\text{s}^2} \quad \Delta t = 1.20$ s

Required
power (P)
Analysis and Solution
The work done is equal to the change in gravitational potential energy of the mass. Calculate power using the equations for change in gravitational potential energy and power.

$$
\begin{aligned}
P &= \frac{\Delta E_\mathrm{p}}{\Delta t} \\
&= \frac{mg\Delta h}{\Delta t} \\
&= \frac{(25.0\ \mathrm{kg})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)(0.800\ \mathrm{m})}{1.20\ \mathrm{s}} \\
&= 163.5\ \frac{\frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}^2}}{s} \\
&= 164\ \frac{\mathrm{J}}{\mathrm{s}} \\
&= 164\ \mathrm{W}
\end{aligned}
$$

Paraphrase
Your power output is 164 W.

5. ***Given***
$F = 1.20\times10^4\ \mathrm{N} \quad v = 450\ \frac{\mathrm{km}}{\mathrm{h}}$

Required
power output of the engine (P)
Analysis and Solution
Convert the speed to m/s; then use the equation for power.

$$
\begin{aligned}
450\ \frac{\mathrm{km}}{\mathrm{h}} &= \left(4.50\times10^5\ \frac{\mathrm{m}}{\cancel{\mathrm{h}}}\right)\left(\frac{1.00\ \cancel{\mathrm{h}}}{3.60\times10^3\ \mathrm{s}}\right) \\
&= 125\ \frac{\mathrm{m}}{\mathrm{s}}
\end{aligned}
$$

$$
\begin{aligned}
P &= Fv \\
&= \left(1.20\times10^4\ \mathrm{N}\right)\left(125\ \frac{\mathrm{m}}{\mathrm{s}}\right) \\
&= 1.50\times10^6\ \mathrm{W}
\end{aligned}
$$

Paraphrase
To maintain a speed of 450 km/h using a force of 1.20×10^4 N requires a power output of 1.50×10^6 W (1500 kW).

6. ***Given***
$P_\mathrm{in} = 1.50\ \mathrm{kW} \quad \mathrm{eff} = 0.75$

$\Delta t = 3.60\times\ 10^3\ \mathrm{s}$

Required
quantity of work (W)

Analysis and Solution

Calculate the power output at the given efficiency, and then calculate the work that can be done in an hour at that rate.

$$eff = \frac{P_{\text{out}}}{P_{\text{in}}}$$

$$\begin{aligned} P_{\text{out}} &= (eff)P_{\text{in}} \\ &= (0.75)(1.50 \times 10^3 \text{ W}) \\ &= 1.13 \times 10^3 \text{ W} \end{aligned}$$

$$P = \frac{W}{\Delta t}$$

$$\begin{aligned} W &= P\Delta t \\ &= (1125 \text{ W})(3.60 \times 10^3 \text{ s}) \\ &= 4.05 \times 10^6 \text{ J} \end{aligned}$$

Paraphrase

At 75% efficiency, a 1.50 kW motor can do 4.05×10^6 J of work in one hour.

7. ***Given***

$P = 9.50 \text{ kW} = 9.50 \times 10^3 \text{ W}$

$v = 25.0 \ \frac{\text{m}}{\text{s}}$

Required

force of friction on the car (F_{f})

Analysis and Solution

Assume the car is travelling on level ground. Use the equation for power to calculate the force required to generate the given power at the given speed. To keep the car moving at a constant speed, the force applied by the motor must be equal in magnitude to the force of friction.

$$P = Fv$$

$$\begin{aligned} F &= \frac{P}{v} \\ &= \frac{9.50 \times 10^3 \text{ W}}{25.0 \ \frac{\text{m}}{\text{s}}} \\ &= 3.80 \times 10^2 \text{ N} \end{aligned}$$

Paraphrase

The force that the motor must apply to maintain the speed is 380 N. Thus the force of friction on the car must be 380 N.

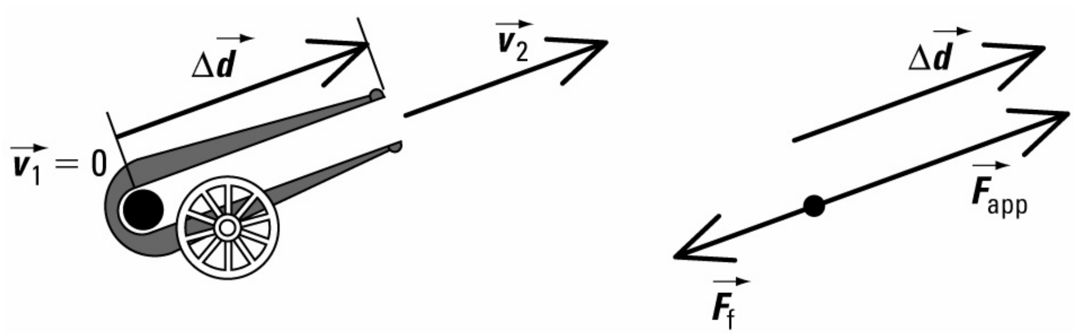
Extension

8. ***Given***

$v_2 = 240 \ \frac{\text{m}}{\text{s}} \quad m = 3.60 \text{ kg} \quad g = 9.81 \ \frac{\text{m}}{\text{s}^2}$

$\Delta d = 1.20 \text{ m} \quad F_{\text{f}} = 650 \text{ N} \quad v_1 = 0$

Required
average power output of the cannon when it fires the ball (P)
Analysis and Solution



You must assume that the cannonball has a uniform acceleration. Calculate the work done on the cannon ball as being equal to its kinetic energy plus the work done to overcome friction. Calculate the time required for the ball to travel the length of the barrel of the cannon using the average speed of the cannon ball over the length of the barrel. Use the equation for power calculated from work and time. [**Note:** There are several valid ways to approach a solution to this problem.]

$$\begin{aligned}
\Delta E_k &= E_{k_2} - E_{k_1} \\
&= \tfrac{1}{2}mv_2{}^2 - \tfrac{1}{2}mv_1{}^2 \\
&= \tfrac{1}{2}(3.60\text{ kg})\left(2.40\times10^2\ \frac{\text{m}}{\text{s}}\right)^2 - \tfrac{1}{2}(3.60\text{ kg})(0)^2 \\
&= 1.0368\times10^5\text{ J} \\
&= 1.037\times10^5\text{ J}
\end{aligned}$$

$$\begin{aligned}
W_f &= F_f\Delta d \\
&= \left(6.50\times10^2\text{ N}\right)(1.20\text{ m}) \\
&= 7.80\times10^2\text{ J}
\end{aligned}$$

$$\begin{aligned}
\Delta E &= W_f + \Delta E_k \\
&= \left(7.80\times10^2\text{ J}\right) + \left(1.037\times10^5\text{ J}\right) \\
&= 1.04\times10^5\text{ J}
\end{aligned}$$

$$\begin{aligned}
v_{ave} &= \tfrac{1}{2}(v_2 + v_1) \\
&= \tfrac{1}{2}\left(2.40\times10^2\ \frac{\text{m}}{\text{s}} + 0\right) \\
&= 1.20\times10^2\ \frac{\text{m}}{\text{s}}
\end{aligned}$$

$$\Delta d = v_{\text{ave}}\Delta t$$

$$\begin{aligned}\Delta t &= \frac{\Delta d}{v_{\text{ave}}}\\ &= \frac{1.20\text{ m}}{1.20\times10^{2}\ \frac{\text{m}}{\text{s}}}\\ &= 1.00\times10^{-2}\ \text{s}\end{aligned}$$

$$\begin{aligned}P &= \frac{\Delta E}{\Delta t}\\ &= \frac{1.04\times10^{5}\ \text{J}}{1.00\times10^{-2}\ \text{s}}\\ &= 1.04\ \times10^{7}\ \text{W}\end{aligned}$$

Paraphrase

The average power output for the cannon to fire this cannonball is 1.04×10^7 W (1.04×10^4 kW).

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Chapter 6 Review

Knowledge

1. (a) The angle between the force and the displacement enables you to calculate the component of force that acts parallel to the displacement, and thus to calculate the work done by the force.

(b) Make a force-displacement graph. The area under the curve is equal to the work done by the force.

(c) Initially, during free fall the bungee jumper is converting gravitational potential energy into kinetic energy. When the cord begins to stretch, gravitational potential energy is converted into elastic potential energy in the cord, as well as kinetic energy. As the cord stretches further, more and more of the gravitational potential energy is converted to elastic potential energy and less into kinetic energy. When the cord is stretched to the point where the restoring force is equal to or greater than the weight of the jumper, the increase in elastic potential energy in the cord is equal to the loss of both gravitational potential energy and kinetic energy.
At maximum stretch, the elastic potential energy stored in the cord is equal to the total loss of gravitational potential energy.

(d) If the students are using different reference points to calculate the gravitational potential energy of the mass then they both may be right.

(e) The lighter mass will have the greater speed but both masses have equal kinetic energy. The energy is the same since the force does the same quantity of work on each of the masses. The smaller mass must have a greater speed if it is to have kinetic energy equal to that of the larger mass.

(f) Yes, running on the treadmill does work, since for each step the runner must exert a force that moves the belt backward the length of the stride.

(g) When you push the cart across the floor, only the force of friction opposes the motion. This means the force exerted on the cart must be equal in magnitude to that force of friction. However, when you push the cart uphill, you must still overcome the effect of friction but also cause the cart to increase its gravitational potential energy. Since when you lift a cart vertically, you just have to increase the gravitational potential energy and not overcome friction this action will require less work (but more force).

(h) All reference points are equally valid to be used to calculate the object's gravitational potential energy. However, it should be noted, that we often like to use our position as a reference point. Thus the floor might be the favoured reference point.

2. (a) Work done by the net force always results in a change in kinetic energy.

(b) If a force acts upward, the work done may result in an increase in gravitational potential energy and an increase in kinetic energy.

(c) Mechanical energy is composed of kinetic energy, gravitational potential energy, and elastic potential energy.

(d) If the gravitational potential energy component of mechanical energy is calculated using different reference points, then the two people would produce different but correct values for the gravitational potential energy and mechanical energy.

3. (a) When the force of gravity acts on a falling object, it will transfer all the gravitational potential energy into other forms of mechanical energy so that the total energy of the object is conserved. Frictional forces convert kinetic energy into thermal energy thus mechanical energy cannot be conserved when friction acts.

(b) Yes, in an ideal spring, since all the work done to extend or compress the spring is stored as elastic potential energy it can later be retrieved. Thus no energy is lost and the elastic force can be considered a conservative force. A mass oscillating up and down on the end of an ideal spring is just converting energy between gravitational potential energy, elastic potential energy and kinetic energy.
No energy is lost during this motion.

(c) By assuming an isolated system, we are able to use the conservation of mechanical energy to solve problems in mechanics.

(d) It does not affect the principle itself but it does affect the accuracy of predictions made using the principle.

(e) To calculate the mechanical energy in any system you must calculate sum of the kinetic energy, the gravitational potential energy and the elastic potential energy of all the objects in the system.

(f) If the mechanical energy is identical at two different times the system may or may not be isolated. While mechanical energy is, by definition, conserved in an isolated system, it may appear to be conserved in a non-isolated system where one force(s) adds the same quantity of energy that another force(s) removes.

4. (a) Yes and No. The amount of work required to lift the elevator is equal to the change in gravitational potential energy plus the work required overcoming the frictional forces. These forces, on the whole, will be independent of the speed of the elevator, so while the rate at which the work is done is increased, the amount of work and the energy to do the work does not increase. However, at a higher speed, the motor may or may not operate as efficiently as it does at lower speed. That may result in a change in the energy consumed by the motor but not in the energy required to lift the elevator.

(b) In a low gear, the work done at the wheels is done at a lower speed and a higher force.

Applications

5. ***Given***

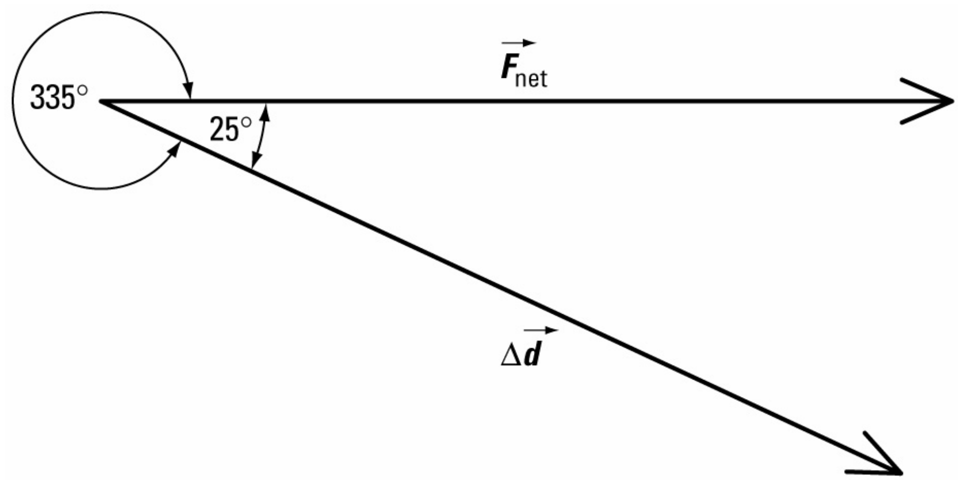
$\vec{F}_{\text{net}} = 3.80 \times 10^3$ N [0°] $\Delta\vec{d} =$ 95.0 m [335°]

Required

change in kinetic energy (ΔE_{k})

Analysis and Solution

The change in kinetic energy is equal to the work done by the net force. Find the angle between the net force and displacement in order to calculate work.



$$\begin{aligned}\Delta E_{\text{k}} &= (F_{\text{net}})(\cos\theta)(\Delta d)\\ &= (3.80 \times 10^3 \text{ N})(\cos 25°)(95.0 \text{ m})\\ &= 3.27 \times 10^5 \text{ J}\end{aligned}$$

Paraphrase

The net force produces a change in kinetic energy of 3.27×10^5 J.

6. (a) Since the elastic potential energy of a spring varies as the stretch squared, when the stretch doubles the energy is increased by four times.

(b) The graph of the gravitational potential energy is a rectangle (the gravitational force is constant) while the graph for elastic potential energy is a triangle with a constant slope (the elastic force increases as the stretch). The area of the rectangle increases in proportion to the base while the area of the rectangle with constant slope increases as the square of the base.

7. *Given*

$m = 0.400\text{ kg} \quad x_1 = 0.250\text{ m} \quad F_1 = 50.0\text{ N}$

$v_1 = 0$

(a) $x_2 = 0.100\text{ m} \quad F_2 = 20.0\text{ N}$

(b) $x_2 = 0 \quad F_2 = 0$

Required

(a) speed of the mass when the spring is stretched 0.100 m (v_2)

(b) speed of the mass when the spring is at its equilibrium position (v_2)

Analysis and Solution

(a) Since the surface is frictionless, assume the conservation of mechanical energy. Calculate the speed from the final kinetic energy, which can be found from the conservation of energy. Data for force at a given stretch is read from the graph.

$$\begin{aligned}
E_{\mathrm{m}_2} &= E_{\mathrm{m}_1} \\
E_{\mathrm{k}_2} + E_{\mathrm{p}_2} &= E_{\mathrm{k}_1} + E_{\mathrm{p}_1} \\
E_{\mathrm{k}_2} &= E_{\mathrm{k}_1} + E_{\mathrm{p}_1} - E_{\mathrm{p}_2} \\
&= \tfrac{1}{2}mv_1{}^2 + \tfrac{1}{2}F_1x_1 - \tfrac{1}{2}F_2x_2 \\
&= \tfrac{1}{2}(0.400\text{ kg})(0)^2 + \tfrac{1}{2}(50.0\text{ N})(0.250\text{ m}) \\
&\qquad - \tfrac{1}{2}(20.0\text{ N})(0.100\text{ m}) \\
&= 0 + 6.25\text{ J} - 1.00\text{ J} \\
&= 5.25\text{ J}
\end{aligned}$$

$$\begin{aligned}
E_{\mathrm{k}_2} &= \tfrac{1}{2}mv_2{}^2 \\
v_2 &= \sqrt{\frac{2E_{\mathrm{k}_2}}{m}} \\
&= \sqrt{\frac{2(5.25\text{ J})}{0.400\text{ kg}}} \\
&= 5.12\ \frac{\text{m}}{\text{s}}
\end{aligned}$$

(b) Since the surface is frictionless, assume the conservation of mechanical energy. Calculate the speed from the final kinetic energy, which can be found from the conservation of energy.

$$\begin{aligned}
E_{\mathrm{m}_2} &= E_{\mathrm{m}_1} \\
E_{\mathrm{k}_2} + E_{\mathrm{p}_2} &= E_{\mathrm{k}_1} + E_{\mathrm{p}_1} \\
E_{\mathrm{k}_2} &= E_{\mathrm{k}_1} + E_{\mathrm{p}_1} - E_{\mathrm{p}_2} \\
&= \tfrac{1}{2}mv_1{}^2 + \tfrac{1}{2}F_1x_1 - \tfrac{1}{2}F_2x_2 \\
&= \tfrac{1}{2}(0.400\text{ kg})(0)^2 + \tfrac{1}{2}(50.0\text{ N})(0.250\text{ m}) \\
&\qquad - \tfrac{1}{2}(20.0\text{ N})(0) \\
&= 0 + 6.25\text{ J} - 0 \\
&= 6.25\text{ J}
\end{aligned}$$

$$E_{k_2} = \tfrac{1}{2}mv_2^2$$

$$\begin{aligned} v_2 &= \sqrt{\frac{2E_{k_2}}{m}} \\ &= \sqrt{\frac{2(6.25\ \text{J})}{0.400\ \text{kg}}} \\ &= 5.59\ \frac{\text{m}}{\text{s}} \end{aligned}$$

Paraphrase

(a) When the spring contracts from a stretch of 0.250 m to 0.100 m, the mass increases its speed from rest to 5.12 m/s.

(b) When the spring has returned to its equilibrium position, the mass has a speed of 5.59 m/s.

8. ***Given***

$m = 65.0\ \text{kg} \quad g = 9.81\ \frac{\text{m}}{\text{s}^2} \quad v_1 = 0 \quad v_2 = 0$

$h_1 = +30.0\ \text{m} \quad h_2 = 0 \quad x_1 = 0 \quad x_2 = 15.0\ \text{m}$

Required

elastic constant for the bungee cord (*k*)

Analysis and Solution

The jumper has no kinetic energy at the beginning and end of the jump. By defining $h = 0$ at the lowest point, the final gravitational potential energy of the jumper must equal 0. Assuming it is an isolated system, the energy lost from gravitational potential energy is gained as elastic potential energy in the cord.

$$\begin{aligned} E_{p_1} &= mgh_1 + \tfrac{1}{2}kx_1^2 \\ &= (65.0\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(+30.0\ \text{m}) + \tfrac{1}{2}k(0)^2 \\ &= 1.91 \times 10^4\ \text{J} \end{aligned}$$

$$\begin{aligned} E_{p_1} &= E_{p_2} \\ &= mgh_2 + \tfrac{1}{2}kx_2^2 \\ &= 0 + \tfrac{1}{2}kx_2^2 \\ &= \tfrac{1}{2}kx_2^2 \\ k &= \frac{2E_{p_1}}{x_2^2} \\ &= \frac{2\left(1.91 \times 10^4\ \text{J}\right)}{(15.0\ \text{m})^2} \\ &= 1.70 \times 10^2\ \frac{\text{N}}{\text{m}} \end{aligned}$$

Paraphrase

The elastic constant for the bungee cord is 1.70×10^2 N/m.

9. ***Given***

$m = 185 \text{ kg} \quad g = 9.81 \ \dfrac{\text{m}}{\text{s}^2} \quad h_1 = 3.20 \text{ m}$

$v_1 = 144 \ \dfrac{\text{km}}{\text{h}} \quad \theta = 20°$

Required

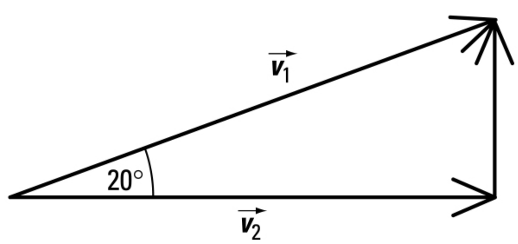
greatest height gained by the cyclist

Analysis and Solution

Assume that mechanical energy is conserved and that the total energy at take-off is equal to the total energy at the highest point. The speed at the highest point is the horizontal component of the speed at take-off. Calculate the gravitational potential energy at the highest point from the total energy and the kinetic energy at the highest point.

$$\begin{aligned}
144 \ \frac{\text{km}}{\text{h}} &= \left(1.44 \times 10^5 \ \frac{\text{m}}{\text{s}}\right)\left(\frac{1.00 \text{ h}}{3.60 \times 10^3 \text{ s}}\right) \\
&= 40.0 \ \frac{\text{m}}{\text{s}}
\end{aligned}$$

$$\begin{aligned}
E_{\text{m}_1} &= E_{\text{k}_1} + E_{\text{p}_1} \\
&= \tfrac{1}{2}mv_1^{\ 2} + mgh_1 \\
&= \tfrac{1}{2}(185 \text{ kg})\left(40.0 \ \frac{\text{m}}{\text{s}}\right)^2 + (185 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(3.20 \text{ m}) \\
&= 1.48 \times 10^5 \text{ J} + 5.81 \times 10^3 \text{ J} \\
&= 1.54 \times 10^5 \text{ J}
\end{aligned}$$



$$\frac{v_2}{v_1} = \cos\theta$$

$$\begin{aligned}
v_2 &= v_1(\cos\theta) \\
&= 40.0 \ \frac{\text{m}}{\text{s}}(\cos 20°) \\
&= 37.6 \ \frac{\text{m}}{\text{s}}
\end{aligned}$$

$$
\begin{aligned}
E_{m_2} &= E_{k_2} + E_{p_2} \\
&= \tfrac{1}{2}mv_2^{\,2} + mgh_2 \\
mgh_2 &= E_{m_2} - \tfrac{1}{2}mv_2^{\,2} \\
h_f &= \frac{E_{m_2} - \tfrac{1}{2}mv_2^{\,2}}{mg} \\
&= \frac{1.54\times10^5 \text{ J} - \tfrac{1}{2}(185 \text{ kg})\left(37.6 \ \dfrac{\text{m}}{\text{s}}\right)^2}{(185 \text{ kg})\left(9.81 \ \dfrac{\text{m}}{\text{s}^2}\right)} \\
&= \frac{2.32\times10^4 \text{ J}}{1.81\times10^3 \text{ N}} \\
&= 12.7 \text{ m}
\end{aligned}
$$

Paraphrase

At its highest point, the motorcycle is 12.7 m above the ground.

10. ***Given***

$m = 65.0 \text{ kg} \quad g = 9.81 \ \dfrac{\text{m}}{\text{s}^2} \quad v = 36.0 \ \dfrac{\text{m}}{\text{s}}$

Required

Calculate the power output of air resistance on the parachutist: P.

Analysis

Since the skydiver is travelling at a constant speed the net force on the diver is zero. Thus the force of air resistance must be equal in magnitude (but opposite in direction) to the force of gravity. The power output of that force will be equal to the product of the force and the speed.

Solution

$$
\begin{aligned}
P_{\text{air}} &= F_{\text{air}}v \\
&= F_g v \\
&= mgv \\
&= 65.0 \text{ kg}\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)\left(36.0 \ \frac{\text{m}}{\text{s}}\right) \\
&= 2.30\times10^4 \text{ W}
\end{aligned}
$$

Paraphrase

Air resistance is removing energy from the skydiver at the rate of 2.30×10^4 W (23.0 kW)

Extension

11. In general, planes are designed to fly low at slow speeds or fly high at high speeds. Jets are obviously in the latter category. The purpose of any flight plan is to get the greatest distance per unit of fuel in order to reduce flight costs. While climbing to such altitudes seems like a waste of energy, it actually saves considerable energy. First, because the atmosphere is thinner, the force of air friction is greatly reduced. Secondly, the navigator tries to make the best use of current wind conditions. Flying from west to east usually entails using the jet stream, which often blows at speed about 200 km/h at altitudes above 9000 m. Flying east to west usually means avoiding the jet stream.

Even for short trips (<400 km) the energy saved by reducing the drag is greater than the cost of lifting the plane to the altitude. Since the plane has such great altitude, the descent to the runway begins much earlier so that the plane runs a considerable distance at greatly reduced power (rate of fuel consumption).

Another advantage of high altitude flying is the fact that at high altitudes the skies are less congested. There are fewer birds, which are a major hazard to planes, especially to their engines, and fewer aircraft.

