Pearson Physics Level 20
Unit I Kinematics: Chapter 2
Solutions

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Skills Practice

Students' answers will vary but may consist of:
(a) scale 1 cm : 1 m; vector will be 5 cm long
(b) scale 1 cm : 5 m; vector will be 4 cm long
(c) scale 1 cm : 10 km; vector will be 3 cm long
(d) scale 1 cm : 50 km; vector will be 3 cm long

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Example 2.1 Practice Problems

1. (a) Δd_{total} = 5.0 m + 10 m + 10 m + 10 m + 20 m + 10 m + 40 m + 10 m
= 115 m

(b) Consider forward to be positive.
$\Delta\vec{d}$ = +5.0 m + (–10 m) + (+10 m) + (–10 m) + (+20 m) + (–10 m) + (+40 m) + (–10 m)
= +35 m
= 35 m [forward]

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Example 2.2 Practice Problems

1. ***Given***

$\vec{d}_1 = 40.0$ m [N]

$\vec{d}_2 = 20.0$ m [N]

$\vec{d}_3 = 100.0$ m [N]

Required

displacement ($\Delta\vec{d}$)

Analysis and Solution

$\Delta\vec{d} = 40.0$ m [N] $+ 20.0$ m [N] $+ 100.0$ m [N]
$= 160.0$ m [N]

Paraphrase

Sprinter's displacement is 160.0 m [N].

2. ***Given***

$\vec{d}_1 = 0.750$ m [right]

$\vec{d}_2 = 3.50$ m [left]

Required
displacement ($\Delta\vec{d}$)
Analysis and Solution
$$\begin{aligned}\Delta\vec{d} &= 0.750 \text{ m [right] } + 3.50 \text{ m [left]}\\ &= -0.750 \text{ m [left] } + 3.50 \text{ m [left]}\\ &= 2.75 \text{ m [left]}\end{aligned}$$
Paraphrase
Player's displacement is 2.75 m [left].

3. ***Given***
$\vec{d}_1 = 0.85$ m [back]
$\vec{d}_2 = -0.85$ m [forth]
Required
distance (Δd)
displacement ($\Delta\vec{d}$)
Analysis and Solution
The bricklayer's hand moves 1.70 m back and forth four times, so $\Delta d = 4(d_1 + d_2)$.
$$\begin{aligned}\Delta d &= 4(0.85 \text{ m} + 0.85 \text{ m})\\ &= 6.80 \text{ m}\end{aligned}$$
Since the player starts and finishes in the same spot, displacement is zero.
$\Delta\vec{d} = 0$ m
Paraphrase
Total distance is 6.80 m. Total displacement is zero.

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2.1 Check and Reflect

Knowledge

1. The vectors have the same magnitude (equal length) but opposite directions.
2. The vector 5 m [N] should be half the length of, and point in the opposite direction to, the vector 10 m [S].
3. $$\frac{5.0 \text{ m [S]}}{6.0 \text{ cm}} = \frac{20 \text{ m [S]}}{x}$$
$$\begin{aligned}x &= \frac{6.0 \text{ cm} \times 20 \text{ m [S]}}{5.0 \text{ m [S]}}\\ &= 24 \text{ cm}\end{aligned}$$
4. $$\frac{5.0 \text{ cm}}{100 \text{ km}} = \frac{1.0 \text{ cm}}{x}$$
$$\begin{aligned}x &= \frac{100 \text{ km} \times 1.0 \text{ cm}}{5.0 \text{ cm}}\\ &= 20 \text{ km}\end{aligned}$$
scale = 1.0 cm : 20 km

Applications

5. ***Given***

scale = 1.0 cm : 520 km

map distance = 4.0 cm

Required

Separation in kilometres (x)

Analysis and Solution

Use equal scale ratios.

$$\frac{x}{4.0\text{ cm}} = \frac{520\text{ km}}{1.0\text{ cm}}$$

$$\begin{aligned} x &= 4.0\text{ cm} \times \frac{520\text{ km}}{1.0\text{ cm}} \\ &= 4.0 \times 520\text{ km} \\ &= 2080\text{ km} \end{aligned}$$

Paraphrase and Verify

The separation is 2080 km. Check: $\frac{2080\text{ km}}{4.0\text{ cm}} = \frac{520\text{ km}}{1.0\text{ cm}}$

6. (a) $\Delta\vec{d} = \vec{d}_1 + (-\vec{d}_1) + \vec{d}_2 + (-\vec{d}_2) + \vec{d}_3 + (-\vec{d}_3) + \vec{d}_4 + (-\vec{d}_4) + \vec{d}_5 + (-\vec{d}_5) + \vec{d}_6 + (-\vec{d}_6) + \vec{d}_7 + (-\vec{d}_7)$
$+\vec{d}_8 + (-\vec{d}_8) + \vec{d}_9 + (-\vec{d}_9) + \vec{d}_{10}$

(b) $\Delta\vec{d}$ = (+10) + (–10) + (+20) + (–20) + (+30) + (–30) + (+40) + (–40) + (+50) + (–50) + (+60) + (–60) + (+70) + (–70) + (+80) + (–80) + (+90) + (–90) + 100
= 100 yards

(c) Δd = 2(10) + 2(20) + 2(30) + 2(40) + 2(50) + 2(60) + 2(70) + 2(80) + 2(90) + 100
= 1000 yards

7. Let x represent each displacement south. Since the car's final position is 50 km [N], its total distance travelled south is 450 km.

$x + (50 + x) + (100 + x) = 450$ km

$3x + 150 = 450$ km

$3x = 300$ km

$x = 100$ km

Therefore, the car's three displacements are 100 km [S], 150 km [S], and 200 km [S].

8. Vectors A and B are equal. They have the same magnitude and point in the same direction.

9. (1) The distance the ball travels is

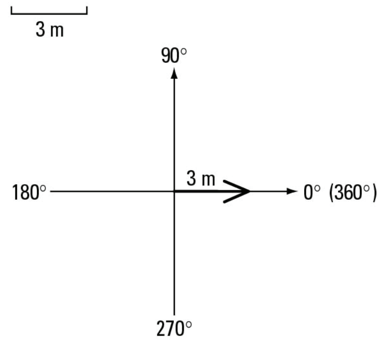
Δd = 10.0 m + 8.0 m + 8.0 m + 4.0 m + 4.0 m + 2.0 m
= 36.0 m

(2) If down is negative, the ball's displacement from its drop point is

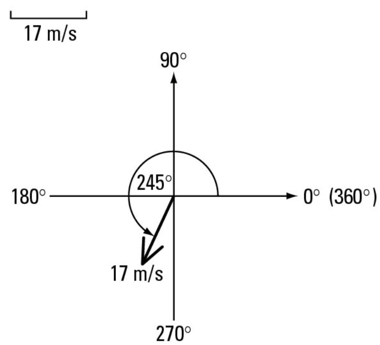
$$\begin{aligned} \Delta\vec{d} &= \vec{d}_\text{f} - \vec{d}_\text{i} \\ &= 2.0\text{ m} - 10.0\text{ m} \\ &= -8.0\text{ m} \\ &= 8.0\text{ m [down]} \end{aligned}$$

Skills Practice

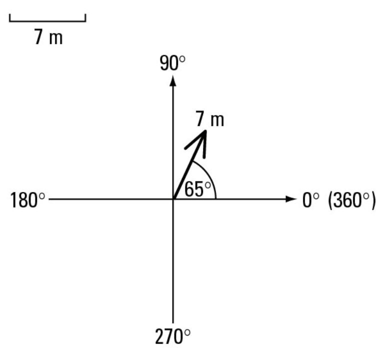
1. (a) Cartesian



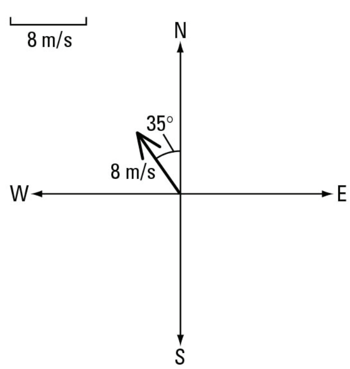
(b) Cartesian



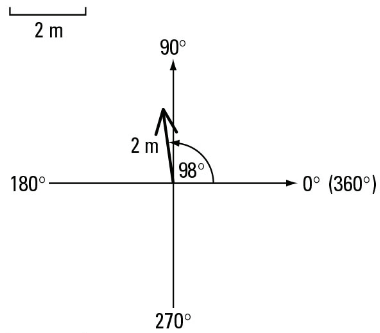
(c) Cartesian



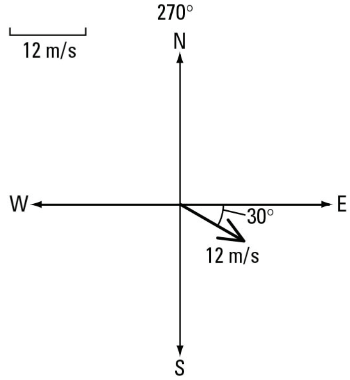
(d) navigator



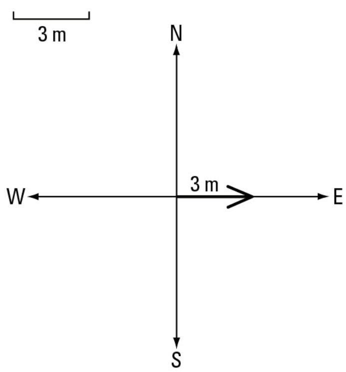
(e) Cartesian



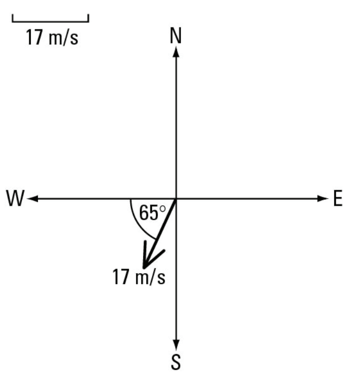
(f) navigator



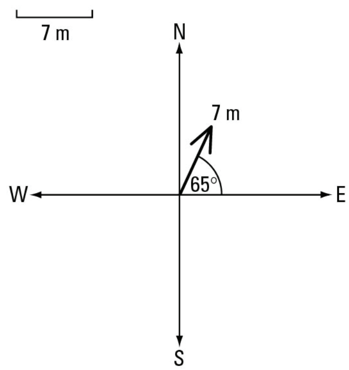
2. (a) 3 m [E]



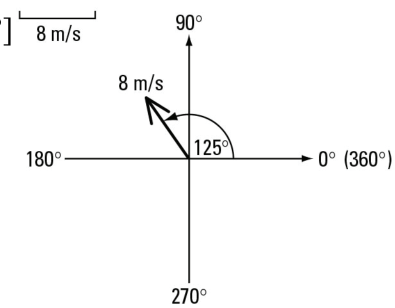
(b) 17 m/s [65° S of W]



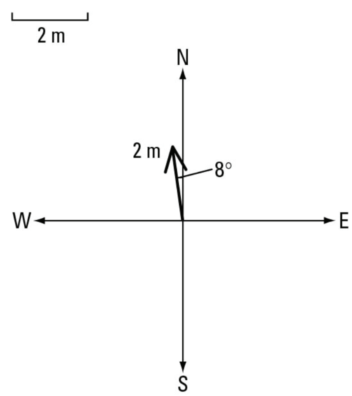
(c) 7 m
[65° N of E]



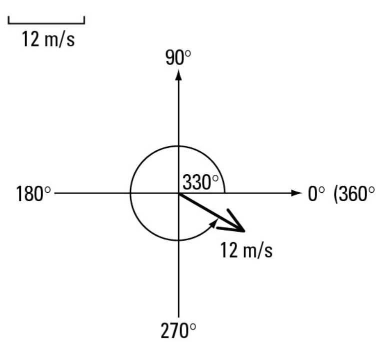
(d) 8 m/s [125°]



(e) 2 m
[8° W of N]



(f) 12 m/s
[330°]



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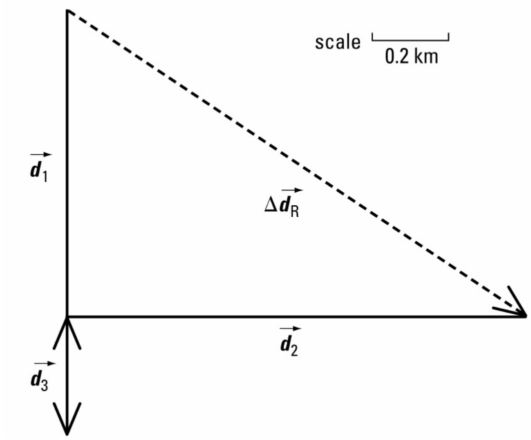
Concept Check

[60° S of W] may also be written [30° W of S].

Example 2.3 Practice Problems

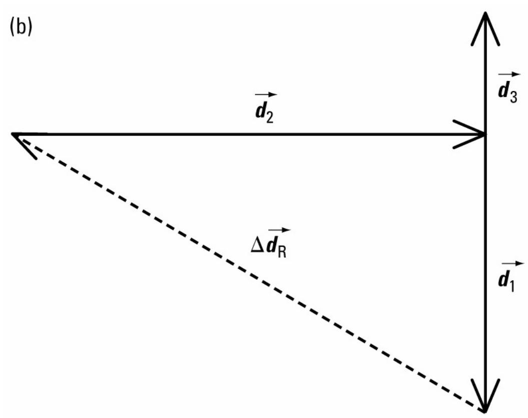
1.

(a)



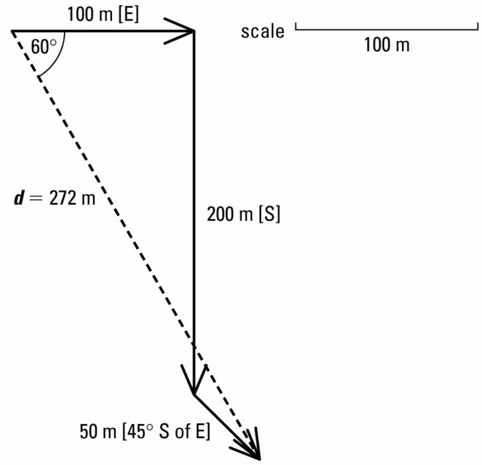
$\vec{d}_1 + \vec{d}_3 + \vec{d}_2 = \Delta\vec{d}_R = 1.30$ km [67° E of S]

(b)



$\vec{d}_2 + \vec{d}_3 + \vec{d}_1 = \Delta\vec{d}_R = 1.30$ km [23° S of E] $=$ 1.30 km [67° E of S]

2.



The student's position relative to her starting point is 272 m [60° S of E].

Skills Practice

	Distance Δd **(m)**	**Final position** $\vec{d}$ **(m) [direction] reference point**	**Displacement** $\Delta\vec{d}$ **(m) [direction]**
AB	56 m	44 m [W]	44 m [W]
BC	63 m	23 m [N]	50 m [27° N of E]
CD	60 m	83 m [N]	60 m [N]
AC	28 m	23 m [N]	23 m [N]
AD	88 m	83 m [N]	83 m [N]

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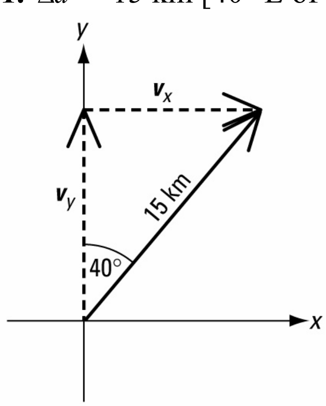
Example 2.4 Practice Problems

1. $\Delta\vec{d}$ = 15 km [40° E of N]

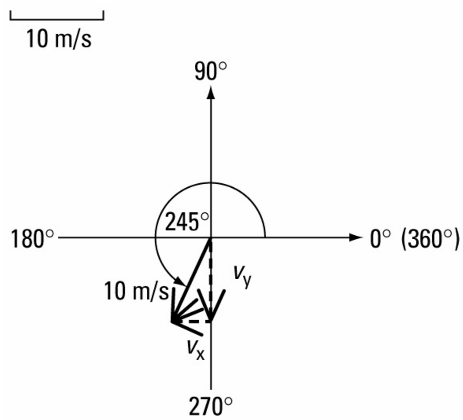


The angle is given with respect to the y-axis (E of N), so use the cosine function to calculate the north component:

$\Delta d_y = (15 \text{ km})(\cos 40°)$

$= 11 \text{ km [N]}$

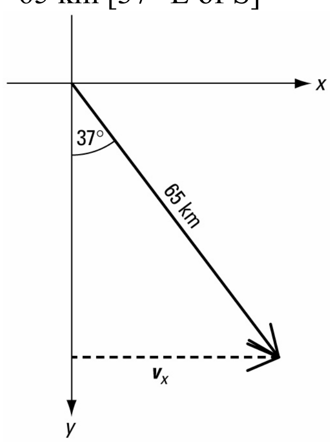
2. $\vec{v} = 10 \text{ m/s [245°]}$



$v_x = (10 \text{ m/s})(\cos 245°)$
$= –4.2 \text{ m/s}$

$v_y = (10 \text{ m/s})(\sin 245°)$
$= –9.1 \text{ m/s}$

3. $\Delta\vec{d} = 65 \text{ km [37° E of S]}$



$\Delta d_x = (65 \text{ km})(\sin 37°) = 39 \text{ km [E]}$

Concept Check

If up and right are designated as positive directions, then R_x in the first quadrant can be negative if it is pointing left and R_y can be negative if it is pointing down. Similarly, R_x and R_y in the other three quadrants can be positive if they point in the positive directions.

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Skills Practice

1. (a) $\vec{R} = 15 \text{ km/h [45° N of W]}$

$$\begin{aligned} R_x &= (15 \text{ km/h})(\cos 45°) \\ &= 11 \text{ km/h} \\ R_y &= (15 \text{ km/h})(\sin 45°) \\ &= 11 \text{ km/h} \end{aligned}$$

(b) $\vec{R} = 200 \text{ km/h [25° E of S]}$

$$\begin{aligned} R_x &= (200 \text{ km/h})(\sin 25°) \\ &= 85 \text{ km/h} \\ R_y &= (200 \text{ km/h})(\cos 25°) \\ &= 181 \text{ km/h} \end{aligned}$$

(c) $\vec{R} = 10 \text{ km/h [N]}$

$$\begin{aligned} R_x &= 0 \\ R_y &= 10 \text{ km/h} \end{aligned}$$

2. (a) $R_x = 12 \text{ m}, \ R_y = 7 \text{ m}$

$$\begin{aligned} R &= \sqrt{(R_x)^2 + (R_y)^2} \\ &= \sqrt{(12 \text{ m})^2 + (7 \text{ m})^2} \\ &= 14 \text{ m} \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{7 \text{ m}}{12 \text{ m}} \\ &= 0.583 \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}(0.583) \\ &= 30° \end{aligned}$$

$\vec{R} = 14 \text{ m [30°]}$

(b) $R_x = 40 \text{ km/h}, \ R_y = 55 \text{ km/h}$

$$
\begin{aligned}
R &= \sqrt{(R_x)^2 + (R_y)^2} \\
&= \sqrt{(40\text{ km/h})^2 + (55\text{ km/h})^2} \\
&= 68\text{ km/h}
\end{aligned}
$$

$$
\begin{aligned}
\tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\
&= \frac{55\text{ km/h}}{40\text{ km/h}} \\
&= 1.375
\end{aligned}
$$

$$
\begin{aligned}
\theta &= \tan^{-1}(1.375) \\
&= 54° \\
\vec{R} &= 68\text{ km/h [54°]}
\end{aligned}
$$

(c) $R_x = 30$ cm, $R_y = 10$ cm

$$
\begin{aligned}
R &= \sqrt{(R_x)^2 + (R_y)^2} \\
&= \sqrt{(30\text{ cm})^2 + (10\text{ cm})^2} \\
&= 32\text{ cm}
\end{aligned}
$$

$$
\begin{aligned}
\tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\
&= \frac{10\text{ cm}}{30\text{ cm}} \\
&= 0.333
\end{aligned}
$$

$$
\begin{aligned}
\theta &= \tan^{-1}(0.333) \\
&= 18° \\
\vec{R} &= 32\text{ cm [18°]}
\end{aligned}
$$

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Example 2.5 Practice Problems

1. ***Given***

   $\vec{d}_1 = 80.0$ m [0°]

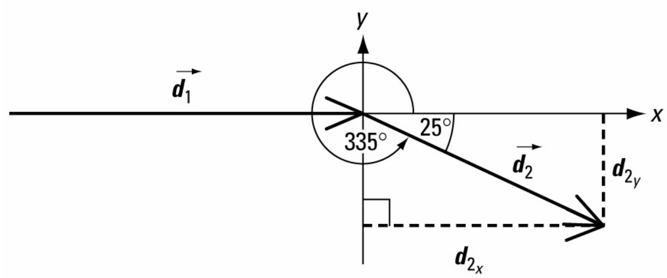
   $\vec{d}_2 = 60.0$ m [335°]

   Required

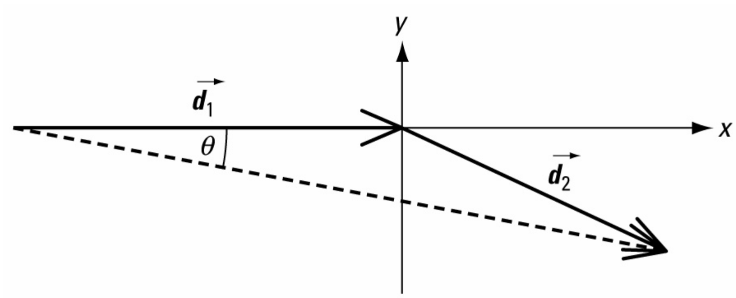
   displacement ($\Delta\vec{d}$)

   Analysis and Solution

   Resolve both vectors in their components and then perform vector addition. Determine the resultant displacement using the Pythagorean theorem.



$$d_{1_x} = 80.0 \text{ m}$$
$$d_{1_y} = 0$$
$$\begin{aligned} d_{2_x} &= d_2 \cos\theta \\ &= (60.0 \text{ m})(\cos 25°) \\ &= 54.38 \text{ m} \end{aligned}$$
$$\begin{aligned} d_{2_y} &= d_2 \sin\theta \\ &= -(60.0 \text{ m})(\sin 25°) \\ &= -25.36 \text{ m} \end{aligned}$$
$$\Delta d_x = d_{1_x} + d_{2_x}$$
$$\begin{aligned} \Delta d_x &= 80.0 \text{ m} + 54.38 \text{ m} \\ &= 134.4 \text{ m} \end{aligned}$$
$$\Delta d_y = d_{1_y} + d_{2_y}$$
$$\begin{aligned} \Delta d_y &= 0 + (-25.36 \text{ m}) \\ &= -25.36 \text{ m} \end{aligned}$$



$$\begin{aligned} \Delta d &= \sqrt{\left(\Delta d_x\right)^2 + \left(\Delta d_y\right)^2} \\ &= \sqrt{(134.4 \text{ m})^2 + (-25.36 \text{ m})^2} \\ &= 136.8 \text{ m} \end{aligned}$$
$$\begin{aligned} \tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{25.36 \text{ m}}{134.4 \text{ m}} \\ &= 0.1887 \end{aligned}$$
$$\begin{aligned} \theta &= \tan^{-1}(0.1887) \\ &= 10.69° \end{aligned}$$

Since the angle is with respect to the positive x-axis in the negative direction, the angle with respect to the positive x-axis in the positive direction is $360° - 10.69° = 349°$.

Paraphrase

The farmer's displacement was 137 m [349°].

2. ***Given***

$\vec{d}_1 = 15 \text{ m } [15° \text{ N of E}]$

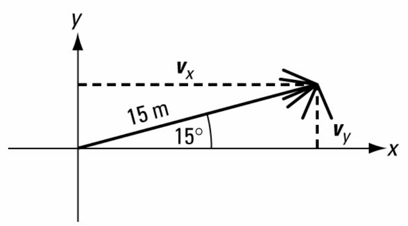
$\vec{d}_2 = 13 \text{ m } [5° \text{ W of N}]$

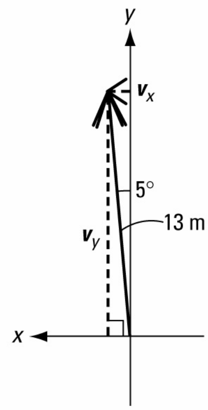
Required

displacement ($\Delta\vec{d}$)

Analysis and Solution

Resolve each position vector into its x and y components and then find the resultant using trigonometry.





$$\begin{aligned} d_{1_x} &= d_1 \cos\theta \\ &= (15 \text{ m})(\cos 15°) \\ &= 14.49 \text{ m} \end{aligned}$$

$$\begin{aligned} d_{1_y} &= d_1 \sin\theta \\ &= (15 \text{ m})(\sin 15°) \\ &= 3.88 \text{ m} \end{aligned}$$

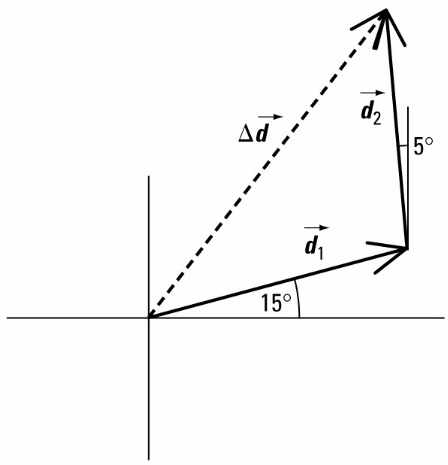
$$\begin{aligned} d_{2_x} &= -d_2 \sin\theta \text{ (since in second quadrant)} \\ &= -(13 \text{ m})(\sin 5°) \\ &= -1.13 \text{ m} \end{aligned}$$

$$\begin{aligned} d_{2_y} &= d_2 \cos\theta \\ &= (13 \text{ m})(\cos 5°) \\ &= 12.95 \text{ m} \end{aligned}$$

$$\Delta d_x = d_{1_x} + d_{2_x}$$

$$\begin{aligned} \Delta d_x &= 14.49 \text{ m} + (-1.13 \text{ m}) \\ &= 13.36 \text{ m} \end{aligned}$$

$$\begin{aligned} \Delta d_y &= d_{1_y} + d_{2_y} \\ &= 3.88 \text{ m} + 12.95 \text{ m} \\ &= 16.83 \text{ m} \end{aligned}$$



$$\begin{aligned}\Delta d &= \sqrt{(\Delta d_x)^2 + (\Delta d_y)^2}\\ &= \sqrt{(13.36\text{ m})^2 + (16.83\text{ m})^2}\\ &= 21.49\text{ m}\end{aligned}$$

$$\begin{aligned}\tan\theta &= \frac{\text{opposite}}{\text{adjacent}}\\ &= \frac{16.83\text{ m}}{13.36\text{ m}}\\ &= 1.260\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}(1.260)\\ &= 52°\end{aligned}$$

Paraphrase

The soccer player's displacement was 21 m [52$^{\text{o}}$ N of E].

3. ***Given***

$\vec{d}_1 = 300$ m [S]

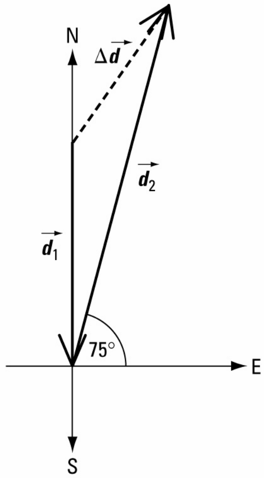
$\vec{d}_2 = 550$ m [75$^{\text{o}}$N of E]

Required

displacement ($\Delta\vec{d}$)

Analysis and Solution

Resolve each position vector into its *x* and *y* components and then add them using trigonometry.



$$d_{1_x} = 0$$

$$d_{1_y} = -300 \text{ m}$$

$$\begin{aligned} d_{2_x} &= d_2 \cos\theta \\ &= (550 \text{ m})(\cos 75°) \\ &= 142.4 \text{ m} \end{aligned}$$

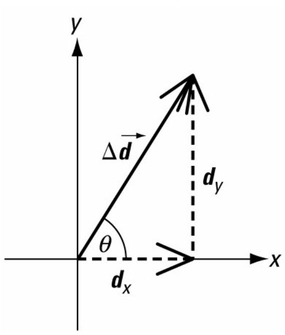
$$\begin{aligned} d_{2_y} &= d_2 \sin\theta \\ &= (550 \text{ m})(\sin 75°) \\ &= 531.3 \text{ m} \end{aligned}$$

$$\Delta d_x = d_{1_x} + d_{2_x}$$

$$\begin{aligned} \Delta d_x &= 0 + 142.4 \text{ m} \\ &= 142.4 \text{ m} \end{aligned}$$

$$\Delta d_y = d_{1_y} + d_{2_y}$$

$$\begin{aligned} \Delta d_y &= -300 \text{ m} + 531.3 \text{ m} \\ &= 231.3 \text{ m} \end{aligned}$$



$$\begin{aligned} \Delta d &= \sqrt{(\Delta d_x)^2 + (\Delta d_y)^2} \\ &= \sqrt{(142.4 \text{ m})^2 + (231.3 \text{ m})^2} \\ &= 2.7 \times 10^2 \text{ m} \end{aligned}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{231.3\text{ m}}{142.4\text{ m}}$$
$$= 1.624$$
$$\theta = \tan^{-1}(1.624)$$
$$= 58°$$

Paraphrase

The wildlife biologist's displacement is 2.7×10^2 m [58° N of E].

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2.2 Check and Reflect

Knowledge

1. The cosine function, $R_x = R\cos\theta$, is used if the angle is with respect to the x-axis. The sine function, $R_x = R\sin\theta$, is used if the angle is with respect to the y-axis.

2. (a) False. The order in which vectors are added is not important because vectors are drawn to scale, and the magnitude and direction of a resultant should be the same regardless of how the vectors are added.

(b) False. Displacement is not always equal to distance because displacement is the straight-line distance between two points, whereas distance is the path travelled between the starting point and the endpoint.

3. The navigator method is used when the compass directions, north, south, east and west, are involved.

Applications

4. The components of this vector are

d_y = (55 cm)(cos 30°)
= 48 cm

d_x = (55 cm)(sin 30°)
= 28 cm

5. (a) ***Given***

$\vec{d}_1$ = 750 m [90°]

$\vec{d}_2$ = 2.20 km [270°] or 2200 m [270°]

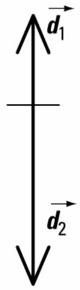
Required

distance (Δd)

displacement ($\Delta\vec{d}$)

Analysis and Solution

To determine distance, add the magnitudes of the position vectors. For displacement, add the position vectors once the units and angles are the same.



$$\Delta d = 750\ \text{m} + 2.20\ \text{km}$$
$$= 0.75\ \text{km} + 2.20\ \text{km}$$
$$= 2.95\ \text{km}$$
$$\Delta\vec{d} = 750\ \text{m}\ [90°]\ + 2.20\ \text{km}\ [270°]$$
$$= -0.75\ \text{km}\ [270°]\ + 2.20\ \text{km}\ [270°]$$
$$= 1.45\ \text{km}\ [270°]$$

Paraphrase

The distance traveled is 2.95 km and the displacement is 1.45 m [270°].

(b) ***Given***

$\vec{d}_1 = 5.0\ \text{km [W]}$

$\vec{d}_2 = 3.0\ \text{km [N]}$

$\vec{d}_3 = 2.0\ \text{km [E]}$

$\vec{d}_4 = 1.5\ \text{km [S]}$

Required

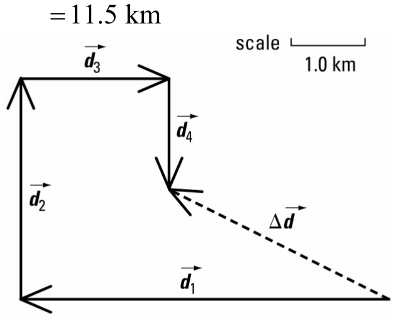
distance (Δd)

displacement ($\Delta\vec{d}$)

Analysis and Solution

For distance, add the magnitudes of the four vectors. For displacement, add the collinear vectors and then use the Pythagorean theorem and the tangent function.

$$\Delta d = 5.0\ \text{km} + 3.0\ \text{km} + 2.0\ \text{km} + 1.5\ \text{km}$$
$$= 11.5\ \text{km}$$



$$\Delta d = 5.0 \text{ km [W]} + 3.0 \text{ km [N]} + 2.0 \text{ km [E]} + 1.5 \text{ km [S]}$$
$$= 5.0 \text{ km [W]} + (-2.0 \text{ km [W]}) + 3.0 \text{ km [N]} + (-1.5 \text{ km [N]})$$
$$= 3.0 \text{ km [W]} + 1.5 \text{ km [N]}$$
$$R = \sqrt{(3.0 \text{ km})^2 + (1.5 \text{ km})^2}$$
$$= 3.4 \text{ km}$$
$$\theta = \tan^{-1}\left(\frac{1.5 \text{ km}}{3.0 \text{ km}}\right)$$
$$= 27°$$
$$\Delta\vec{d} = 3.4 \text{ km [27° N of W]}$$

Paraphrase

The distance is 11.5 km and the displacement is 3.4 km [27° N of W].

(c) ***Given***

$d_1 = 500$ m [N]

$l = 150$ m [E]

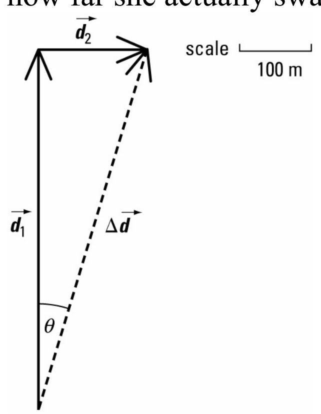
Required

distance (Δd)

displacement ($\Delta\vec{d}$)

Analysis and Solution

Attempting to cross the river, the swimmer is moved off course. Her distance is how far she actually swam in the water.



$$\Delta d = \sqrt{(500 \text{ m})^2 + (150 \text{ m})^2}$$
$$= 522 \text{ m}$$
$$\Delta\vec{d} = 500 \text{ m [N]} + 150 \text{ m [E]}$$

To find the direction for displacement, use of the tangent function.

$$\theta = \tan^{-1}\left(\frac{150 \text{ m}}{500 \text{ m}}\right)$$
$$= 16.7°$$
$$\Delta\vec{d} = 522 \text{ m [16.7° E of N]}$$

Paraphrase

The swimmer swam a distance of 522 m and her displacement was 522 m [16.7° E of N].

6. ***Given***

$\vec{d}_1$ = 5.0 km [45° W of N]

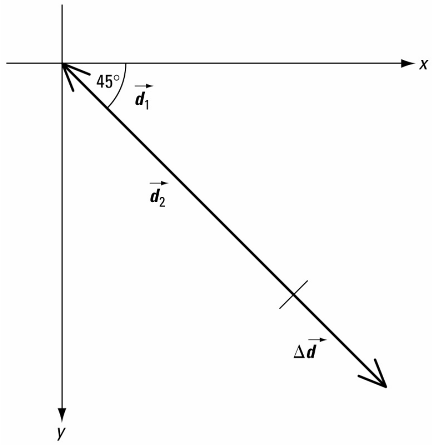
$\vec{d}_2$ = 7.0 km [45° S of E]

Required

displacement ($\Delta\vec{d}$)

Analysis and Solution

The vectors are in opposite directions. So, the magnitude of the displacement of the boat is 2.0 km. The direction is that of the second, longer, position vector, [45° S of E].



Paraphrase

The boat's displacement is 2.0 km [45° S of E].

7. ***Given***

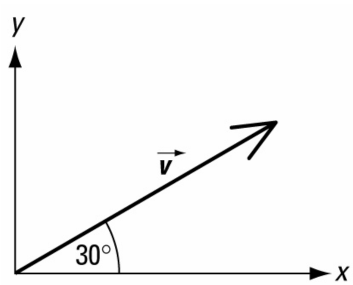
$\vec{v} = 355$ m/s [30°]

Required

vertical velocity component (v_y)

Analysis and Solution

Resolve the vector into its vertical component using $v_y = v\sin\theta$.



$$\begin{aligned} v_y &= v\sin\theta \\ &= (355 \text{ m/s})(\sin 30°) \\ &= 178 \text{ m/s} \end{aligned}$$

Paraphrase

The magnitude of the vertical component of the pellet's velocity is 178 m/s.

8. *Given*

$\vec{d}_1 = 1.20$ km [55° N of E]

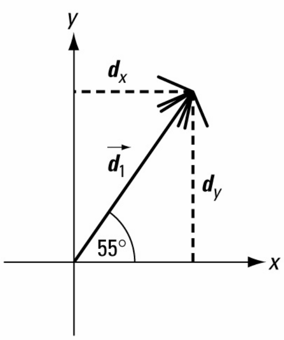
$\vec{d}_2 = 3.15$ km [70° S of E]

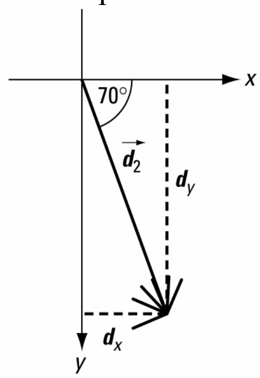
Required

displacement ($\Delta\vec{d}$)

Analysis and Solution

Resolve each vector into its components and perform vector addition.





$$\begin{aligned} d_{1_x} &= d_1 \cos\theta \\ &= (1.20 \text{ km})(\cos 55°) \\ &= 0.6883 \text{ km} \end{aligned}$$

$$\begin{aligned} d_{1_y} &= d_1 \sin\theta \\ &= (1.20 \text{ m})(\sin 55°) \\ &= 0.9830 \text{ km} \end{aligned}$$

$$\begin{aligned} d_{2_x} &= d_2 \sin\theta \\ &= (3.15 \text{ km})(\sin 20°) \\ &= 1.077 \text{ km} \end{aligned}$$

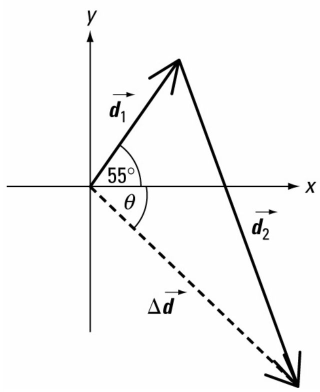
$$\begin{aligned} d_{2_y} &= -d_2 \cos\theta \\ &= -(3.15 \text{ km})(\cos 20°) \\ &= -2.960 \text{ km} \end{aligned}$$

$$\Delta d_x = d_{1_x} + d_{2_x}$$

$$\begin{aligned} \Delta d_x &= 0.6883 \text{ km } + 1.077 \text{ km} \\ &= 1.765 \text{ km} \end{aligned}$$

$$\Delta d_y = d_{1_y} + d_{2_y}$$

$$\begin{aligned} \Delta d_y &= 0.9830 \text{ km} - 2.960 \text{ km} \\ &= -1.977 \text{ km} \end{aligned}$$



$$\Delta d = \sqrt{(\Delta d_x)^2 + (\Delta d_y)^2}$$
$$= \sqrt{(1.765\text{ km})^2 + (-1.977\text{ km})^2}$$
$$= 2.650\text{ km}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{1.977\text{ km}}{1.765\text{ km}}$$
$$= 1.120$$
$$\theta = \tan^{-1}(1.120)$$
$$= 48°$$

From the diagram, this angle is with respect to the positive *x*-axis in the clockwise direction, or S of E.

Paraphrase

The jet ski's displacement is 2.65 km [48° S of E].

9. ***Given***

$\vec{v}_1 = 6.0$ km/h [25° N of W]

$\vec{v}_2 = 4.5$ km/h [65° E of N]

$\Delta t_1 = 35$ min

$\Delta t_2 = 20$ min

Required

displacement ($\Delta\vec{d}$)

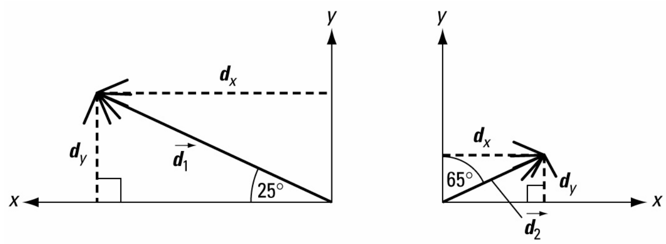
average velocity ($\vec{v}_{ave}$)

Analysis and Solution

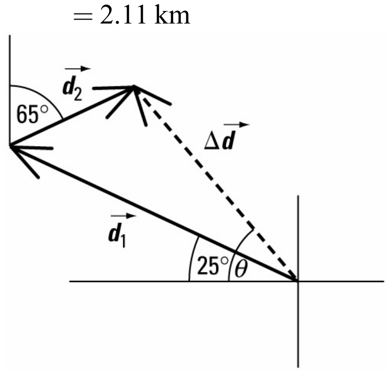
Using the uniform motion equation, determine the displacement in both directions.

$$\Delta d_1 = \left(6.0\ \frac{\text{km}}{\cancel{\text{h}}}\ [25°\text{ N of W}]\right)\left(35\ \cancel{\text{min}} \times \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\right)$$
$$= 3.5\text{ km [25° N of W]}$$
$$\Delta d_2 = \left(4.5\ \frac{\text{km}}{\cancel{\text{h}}}\ [65°\text{ E of N}]\right)\left(20\ \cancel{\text{min}} \times \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\right)$$
$$= 1.5\text{ km [65° E of N]}$$

Resolve each displacement vector into its x and y components and then perform vector addition.



$$\begin{aligned}
\Delta d_{1_x} &= -d_1 \cos\theta \\
&= -(3.5 \text{ km})(\cos 25°) \\
&= -3.17 \text{ km} \\
d_{1_y} &= d_1 \sin\theta \\
&= (3.5 \text{ m})(\sin 25°) \\
&= 1.48 \text{ km} \\
d_{2_x} &= d_2 \sin\theta \\
&= (1.5 \text{ km})(\sin 65°) \\
&= 1.36 \text{ km} \\
d_{2_y} &= d_2 \cos\theta \\
&= (1.5 \text{ km})(\cos 65°) \\
&= 0.634 \text{ km} \\
\Delta d_x &= d_{1_x} + d_{2_x} \\
&= -3.17 \text{ km } + 1.36 \text{ km} \\
&= -1.81 \text{ km} \\
\Delta d_y &= d_{1_y} + d_{2_y} \\
&= 1.48 \text{ km} + 0.634 \text{ km} \\
&= 2.11 \text{ km}
\end{aligned}$$



$$\begin{aligned}\Delta d &= \sqrt{(\Delta d_x)^2 + (\Delta d_y)^2} \\ &= \sqrt{(-1.81 \text{ km})^2 + (2.11 \text{ km})^2} \\ &= 2.78 \text{ km}\end{aligned}$$

$$\begin{aligned}\tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{2.11 \text{ km}}{1.81 \text{ km}} \\ &= 1.17\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}(1.17) \\ &= 49^\circ\end{aligned}$$

From the diagram above, this angle is 49° N of W.

Find average velocity using the equation $\vec{v}_{\text{ave}} = \frac{\Delta\vec{d}}{\Delta t}$.

$$\begin{aligned}\vec{v}_{\text{ave}} &= \frac{\Delta\vec{d}}{\Delta t} \\ &= \frac{2.78 \text{ km } [49^\circ \text{ N of W}]}{(35+20)\ \cancel{\text{min}}} \times \frac{60\ \cancel{\text{min}}}{1 \text{ h}} \\ &= 3.03 \text{ km/h } [49^\circ \text{ N of W}]\end{aligned}$$

Paraphrase

The jogger's total displacement is 2.8 km [49° N of W]. The jogger's average velocity is 3.0 km/h [49° N of W].

10. ***Given***

$\Delta\vec{d} = 38 \text{ m } [90^\circ]$

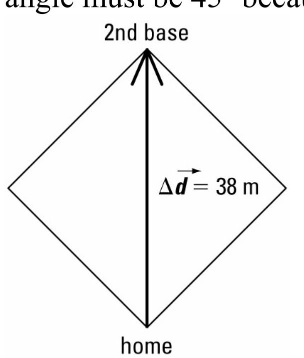
Required

(a) components of displacement (d_x, d_y)

(b) distance to second base (Δd)

Analysis and Solution

Since the baseball diamond is square in shape, the two sides of the triangle formed from home base to first base, and from first base to second base, must be equal. The angle must be 45° because the triangle is isosceles.



Use the sine and cosine functions to determine the components of the player's displacement.

(a) $d_x = d\cos\theta$
$= (38\text{ m})(\cos 45°)$
$= 27\text{ m}$
$d_y = d\sin\theta$
$= (38\text{ m})(\sin 45°)$
$= 27\text{ m}$

(b) The distance is not 38 m because the player does not run in a straight line but to first then to second base, or $27\text{ m} + 27\text{ m} = 54\text{ m}$.

Paraphrase

(a) The components of the baseball player's displacement are 27 m in both the *x* and *y* directions.

(b) The player's distance travelled is 54 m.

11. ***Given***

$\vec{d}_1 = 45.0\text{ m }[310°]$

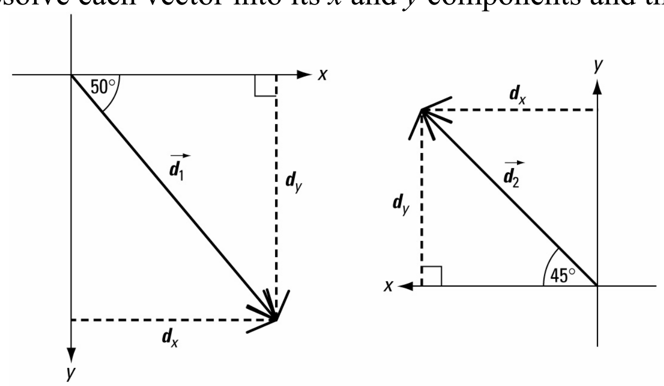
$\vec{d}_2 = 35.0\text{ m }[135°]$

Required

displacement ($\Delta\vec{d}$)

Analysis and Solution

Resolve each vector into its *x* and *y* components and then perform vector addition.



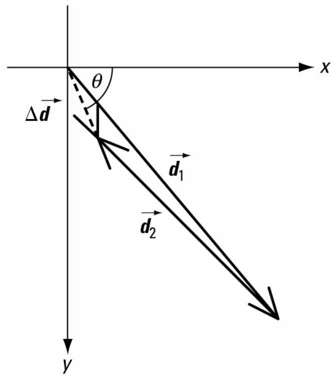
$d_{1_x} = d_1\cos\theta$
$= (45.0\text{ m})(\cos 50°)$
$= 28.925\text{ m}$

$d_{1_y} = d_1\sin\theta$
$= -(45.0\text{ m})(\sin 50°)$
$= -34.47\text{ m}$

$d_{2_x} = d_2\cos\theta$
$= -(35.0\text{ m})(\cos 45°)$
$= -24.75\text{ m}$

$d_{2_y} = d_2\cos\theta$
$= (35.0\text{ m})(\sin 45°)$
$= 24.75\text{ m}$

$$\Delta d_x = d_{1_x} + d_{2_x}$$
$$= 28.925 \text{ m} + (-24.75 \text{ m})$$
$$= 4.175 \text{ m}$$
$$\Delta d_y = d_{1_y} + d_{2_y}$$
$$\Delta d_y = -34.47 \text{ m} + 24.75 \text{ m}$$
$$\Delta d_y = -9.72 \text{ m}$$



$$\Delta d = \sqrt{(\Delta d_x)^2 + (\Delta d_y)^2}$$
$$= \sqrt{(4.175 \text{ m})^2 + (-9.72 \text{ m})^2}$$
$$= 10.6 \text{ m}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{9.72 \text{ m}}{4.175 \text{ m}}$$
$$= 2.328$$
$$\theta = \tan^{-1}(2.328)$$
$$\theta = 66.8°$$

From the diagram, this angle is with respect to the positive *x*-axis in the clockwise direction. Using the Cartesian method, this angle is $360° - 66.8° = 293°$.

Paraphrase

The skateboarder's displacement is 10.6 m [293°].

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Concept Check

Since the observer on the train has the same velocity as the stationary ball and the train (25 m/s [forward]), the observer sees the ball move at a velocity of 25 m/s [forward].
The observer on the ground will see the ball moving at the combined velocities of the ball and train, or 25 m/s [forward] + 25 m/s [forward] = 50 m/s [forward].
If the ball moves 25 m/s [backward], its velocity is the same for an observer on the train. For an observer on the ground, its velocity is 25 m/s [forward] + 25 m/s [backward] = 25 m/s [forward] – 25 m/s [forward] = 0 m/s. Therefore, the ball appears stationary.

Example 2.6 Practice Problems

1. ***Given***

$\vec{v}_{\text{swimmer}} = 1.8 \text{ m/s [N]}$

$\Delta d = 200 \text{ m}$

$\vec{v}_{\text{current}} = 1.2 \text{ m/s [W]}$

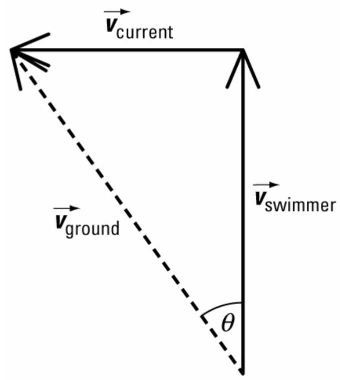
Required

(a) ground velocity ($\vec{v}_{\text{ground}}$)

(b) time (Δt)

Analysis and Solution

(a) $\vec{v}_{\text{ground}} = \vec{v}_{\text{swimmer}} + \vec{v}_{\text{current}}$



$$(v_{\text{ground}})^2 = (v_{\text{swimmer}})^2 + (v_{\text{current}})^2$$

$$\begin{aligned} v_{\text{ground}} &= \sqrt{(1.8 \text{ m/s})^2 + (1.2 \text{ m/s})^2} \\ &= 2.2 \text{ m/s} \end{aligned}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{1.2 \cancel{\text{m/s}}}{1.8 \cancel{\text{m/s}}}\right) \\ &= 34° \end{aligned}$$

From the diagram, this angle is W of N.

(b) Use the equation $\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$ and the swimmer's speed. Use the scalar form of the equation because you are dividing by a vector.

$$\begin{aligned} \Delta t &= \frac{\Delta d}{v} \\ &= \frac{200 \cancel{\text{m}}}{1.8 \frac{\cancel{\text{m}}}{\text{s}}} \\ &= 111 \text{ s} \end{aligned}$$

Paraphrase

(a) The swimmer's ground velocity is 2.2 m/s [34º W of N].

(b) It takes the swimmer 111 s to cross the river.

2. ***Given***

$\vec{v}_{\text{current}} = 1.2$ m/s [W]

$v_{\text{swimmer}} = 1.8$ m/s

direction of ground velocity = N

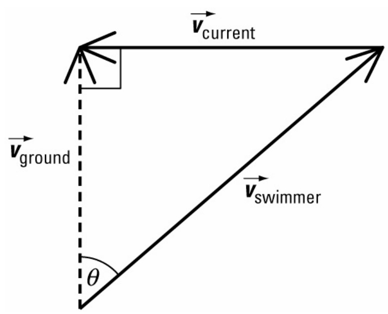
Required

(a) direction

(b) time (Δt)

Analysis and Solution

(a) $\vec{v}_{\text{ground}} = \vec{v}_{\text{swimmer}} + \vec{v}_{\text{current}}$



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\theta = \sin^{-1}\left(\frac{1.2\ \cancel{\text{m}}/\text{s}}{1.8\ \cancel{\text{m}}/\text{s}}\right)$$
$$= 42°$$

From the diagram, this angle is E of N.

(b) Use the equation $\vec{v} = \dfrac{\Delta\vec{d}}{\Delta t}$. Since displacement and velocity must be in the same direction, first find the ground speed.

$$(v_{\text{swimmer}})^2 = (v_{\text{ground}})^2 + (v_{\text{current}})^2$$
$$(v_{\text{ground}})^2 = (v_{\text{swimmer}})^2 - (v_{\text{current}})^2$$
$$v_{\text{ground}} = \sqrt{(1.8\text{ m/s})^2 - (1.2\text{ m/s})^2}$$
$$= 1.34\text{ m/s}$$

Use the scalar form of the equation because you are dividing by a vector.

$$\Delta t = \frac{\Delta d}{v}$$
$$= \frac{200\ \cancel{\text{m}}}{1.34\ \frac{\cancel{\text{m}}}{\text{s}}}$$
$$= 149\text{ s}$$

Paraphrase

(a) The swimmer must direct herself at an angle of [$42°$ E of N].

(b) It takes the swimmer 149 s to cross the river.

Example 2.7 Practice Problems

1. ***Given***

$\vec{v}_{\text{wind}} = 60$ km/h [15° E of N]

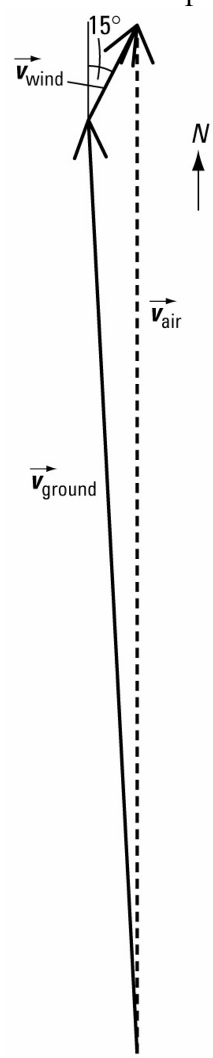
$\vec{v}_{\text{air}} = 750$ km/h [N]

Required

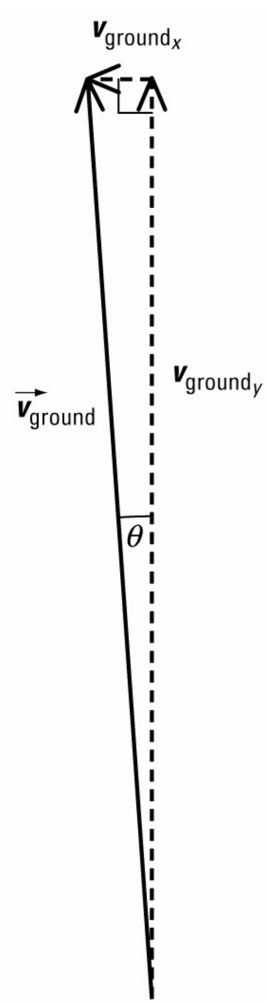
ground velocity ($\vec{v}_{\text{ground}}$)

Analysis and Solution

Determine the ground velocity of the airplane by resolving each vector into its east and north components and then using the Pythagorean theorem and the tangent function. The plane will need to fly W of N to maintain its northerly course.



$$
\begin{aligned}
v_{\text{wind}_x} &= -v_{\text{wind}} \sin\theta \\
&= -\left(60 \text{ km/h}\right)(\sin 15°) \\
&= -15.5 \text{ km/h} \\
v_{\text{wind}_y} &= v_{\text{wind}} \cos\theta \\
&= \left(60 \text{ km/h}\right)(\cos 15°) \\
&= 57.96 \text{ km/h} \\
v_{\text{air}_x} &= 0 \\
v_{\text{air}_y} &= 750 \text{ km/h} \\
\vec{v}_{\text{ground}} &= \vec{v}_{\text{air}} + \vec{v}_{\text{wind}} \\
v_{\text{ground}_x} &= v_{\text{air}_x} + v_{\text{wind}_x} \\
&= 0 + (-15.5 \text{ km/h}) \\
&= -15.5 \text{ km/h}
\end{aligned}
$$



$$v_{\text{ground}_y} = v_{\text{air}_y} + v_{\text{wind}_y}$$
$$= 750 \text{ km/h} + 57.96 \text{ km/h}$$
$$= 808 \text{ km/h}$$

$$v_{\text{ground}} = \sqrt{\left(v_{\text{ground}_x}\right)^2 + \left(v_{\text{ground}_y}\right)^2}$$
$$= \sqrt{(-15.5 \text{ km/h})^2 + (808 \text{ km/h})^2}$$
$$= 808 \text{ km/h}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{15.5 \cancel{\text{ km/h}}}{808 \cancel{\text{ km/h}}}$$
$$= 0.019$$
$$\theta = \tan^{-1}(0.019)$$
$$= 1.1°$$

The angle is therefore 1.1° W of N.

Paraphrase

The airplane's ground velocity must be 808 km/h [1.1° W of N].

2. *Given*

$\vec{v}_{\text{ground}} = 856$ km/h [25.0° W of S]

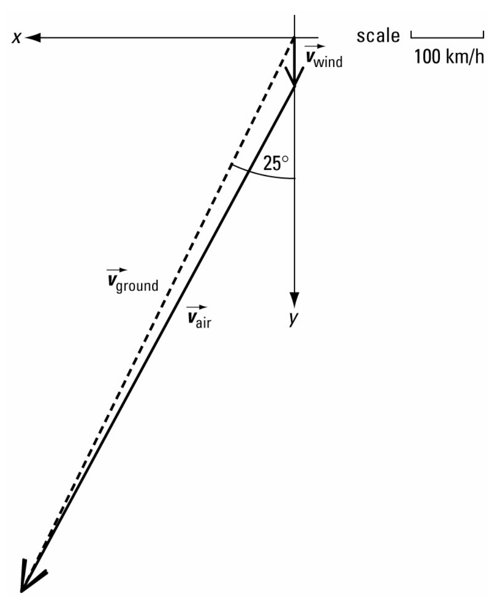
$\vec{v}_{\text{wind}} = 65.0$ km/h [S]

Required

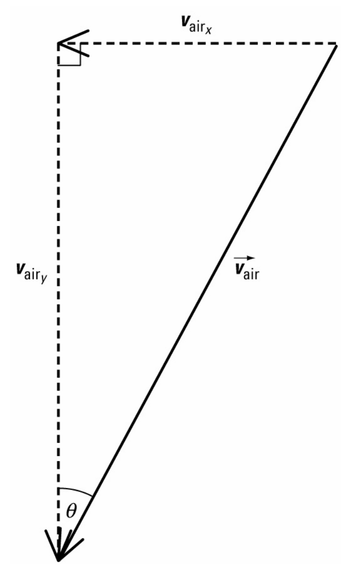
air velocity ($\vec{v}_{\text{air}}$)

Analysis and Solution

Perform vector subtraction to determine the components of the air velocity. Using the Pythagorean theorem, find the magnitude of the air velocity and then use the tangent function to determine the air velocity direction.



$$
\begin{aligned}
v_{\text{ground}_x} &= -v_{\text{ground}} \sin\theta \\
&= -\left(856 \text{ km/h}\right)(\sin 25.0°) \\
&= -361.8 \text{ km/h} \\
v_{\text{ground}_y} &= -v_{\text{ground}} \cos\theta \\
&= -\left(856 \text{ km/h}\right)(\cos 25.0°) \\
&= -775.8 \text{ km/h} \\
v_{\text{wind}_x} &= 0 \\
v_{\text{wind}_y} &= -65.0 \text{ km/h} \\
\vec{v}_{\text{ground}} &= \vec{v}_{\text{air}} + \vec{v}_{\text{wind}} \\
\vec{v}_{\text{air}} &= \vec{v}_{\text{ground}} - \vec{v}_{\text{wind}} \\
v_{\text{air}_x} &= v_{\text{ground}_x} - v_{\text{wind}_x} \\
&= -361.8 \text{ km/h} - 0 \\
&= -361.8 \text{ km/h} \\
v_{\text{air}_y} &= v_{\text{ground}_y} - v_{\text{wind}_y} \\
&= -775.8 \text{ km/h} - (-65.0 \text{ km/h}) \\
&= -710.8 \text{ km/h} \\
v_{\text{air}} &= \sqrt{\left(v_{\text{air}_x}\right)^2 + \left(v_{\text{air}_y}\right)^2} \\
&= \sqrt{\left(-361.8 \text{ km/h}\right)^2 + \left(-710.8 \text{ km/h}\right)^2} \\
&= 798 \text{ km/h}
\end{aligned}
$$



$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{361.8\ \cancel{\text{km/h}}}{710.8\ \cancel{\text{km/h}}}$$
$$= 0.509$$
$$\theta = \tan^{-1}(0.509)$$
$$= 27.0°$$

From the diagram, the angle is 27.0° W of S.

Paraphrase

The air velocity of the jetliner is 798 km/h [27.0° W of S].

3. ***Given***

$\vec{v}_{\text{ground}} = 795$ km/h [25° W of N]

$\Delta\vec{d} = 100$ km [N]

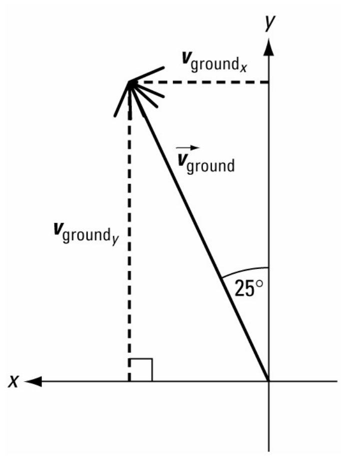
Required

time (Δt)

Analysis and Solution

Calculate the north component of the ground velocity of the plane.

Use $v = \dfrac{\Delta d}{\Delta t}$ to determine the time.



$$v_{\text{ground}_y} = (795\ \text{km/h})(\cos 25°)$$
$$= 720.5\ \text{km/h}$$
$$v_y = \frac{\Delta d_y}{\Delta t}$$
$$\Delta t = \frac{\Delta d_y}{v_y}$$
$$= \frac{100\ \cancel{\text{km}}}{720.5\ \frac{\cancel{\text{km}}}{\text{h}}}$$
$$= 0.139\ \text{h}$$

Paraphrase

The plane takes 0.139 h to travel 100 km north.

Example 2.8 Practice Problems

1. ***Given***

$\vec{v}_{\text{boat}} = 4.50 \text{ m/s [W]}$

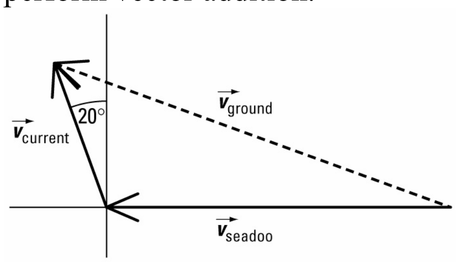
$\vec{v}_{\text{current}} = 2.0 \text{ m/s [20° W of N]}$

Required

ground velocity ($\vec{v}_{\text{ground}}$)

Analysis and Solution

Resolve the boat and current vectors into their west and north components, then perform vector addition.



$$v_{\text{boat}_x} = -4.50 \text{ m/s}$$

$$v_{\text{boat}_y} = 0$$

$$\begin{aligned} v_{\text{current}_x} &= -v_{\text{current}} \sin\theta \\ &= -(2.0 \text{ m/s})(\sin 20°) \\ &= -0.684 \text{ m/s} \end{aligned}$$

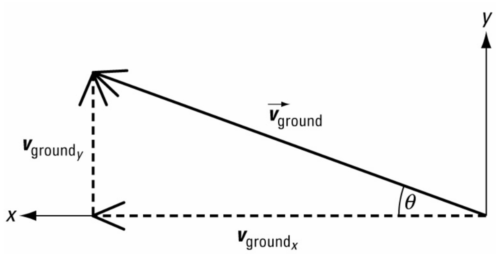
$$\begin{aligned} v_{\text{current}_y} &= v_{\text{current}} \cos\theta \\ &= (2.0 \text{ m/s})(\cos 20°) \\ &= 1.88 \text{ m/s} \end{aligned}$$

$$\vec{v}_{\text{ground}} = \vec{v}_{\text{boat}} + \vec{v}_{\text{current}}$$

$$\begin{aligned} v_{\text{ground}_x} &= v_{\text{boat}_x} + v_{\text{current}_x} \\ &= -4.50 \text{ m/s} + (-0.684 \text{ m/s}) \\ &= -5.18 \text{ m/s} \end{aligned}$$

$$\begin{aligned} v_{\text{ground}_y} &= v_{\text{boat}_y} + v_{\text{current}_y} \\ &= 0 + 1.88 \text{ m/s} \\ &= 1.88 \text{ m/s} \end{aligned}$$

$$\begin{aligned} v_{\text{ground}} &= \sqrt{\left(v_{\text{ground}_x}\right)^2 + \left(v_{\text{ground}_y}\right)^2} \\ &= \sqrt{(-5.18 \text{ m/s})^2 + (1.88 \text{ m/s})^2} \\ &= 5.51 \text{ m/s} \end{aligned}$$



$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{1.88\ \cancel{\text{m/s}}}{5.18\ \cancel{\text{m/s}}}$$
$$= 0.363$$
$$\theta = \tan^{-1}(0.363)$$
$$= 20°$$

From the diagram, the angle is 20° N of W.

Paraphrase

The boat's ground velocity is 5.5 m/s [20° N of W].

2. ***Given***

$\vec{v}_{\text{boat}} = 13$ m/s [N]

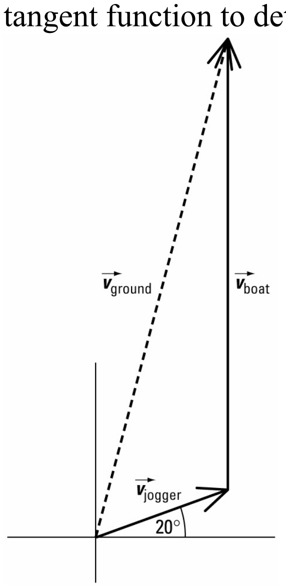
$\vec{v}_{\text{jogger}} = 3.75$ m/s [20° N of E]

Required

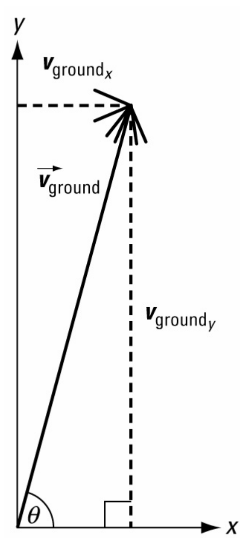
ground velocity ($\vec{v}_{\text{ground}}$)

Analysis and Solution

Determine the north and east components of the jogger's velocity and the ship. Perform vector addition on each component of the resultant velocity and then calculate the magnitude of the resultant using the Pythagorean theorem. Use the tangent function to determine the resultant direction.



$$
\begin{aligned}
v_{\text{boat}_x} &= 0 \\
v_{\text{boat}_y} &= 13 \text{ m/s} \\
v_{\text{jogger}_x} &= v_{\text{jogger}} \cos\theta \\
&= (3.75 \text{ m/s})(\cos 20°) \\
&= 3.52 \text{ m/s} \\
v_{\text{jogger}_y} &= v_{\text{jogger}} \sin\theta \\
&= (3.75 \text{ m/s})(\sin 20°) \\
&= 1.28 \text{ m/s} \\
\vec{v}_{\text{ground}} &= \vec{v}_{\text{boat}} + \vec{v}_{\text{jogger}} \\
v_{\text{ground}_x} &= v_{\text{boat}_x} + v_{\text{jogger}_x} \\
&= 0 + 3.52 \text{ m/s} \\
&= 3.52 \text{ m/s} \\
v_{\text{ground}_y} &= v_{\text{boat}_y} + v_{\text{jogger}_y} \\
&= 13 \text{ m/s} + 1.28 \text{ m/s} \\
&= 14.28 \text{ m/s} \\
v_{\text{ground}} &= \sqrt{\left(v_{\text{ground}_x}\right)^2 + \left(v_{\text{ground}_y}\right)^2} \\
&= \sqrt{(3.52 \text{ m/s})^2 + (14.28 \text{ m/s})^2} \\
&= 14.71 \text{ m/s}
\end{aligned}
$$



$$
\begin{aligned}
\tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\
&= \frac{14.28 \cancel{\text{m/s}}}{3.52 \cancel{\text{m/s}}} \\
&= 4.06 \\
\theta &= \tan^{-1}(4.06) \\
&= 76°
\end{aligned}
$$

From the diagram, this angle is 76° N of E.

Paraphrase

The resultant velocity of the jogger is 15 m/s [76° N of E].

3. ***Given***

$\Delta\vec{d} = 65.0$ km [N]

$\Delta t = 3.0$ h

direction = 55° [W of N]

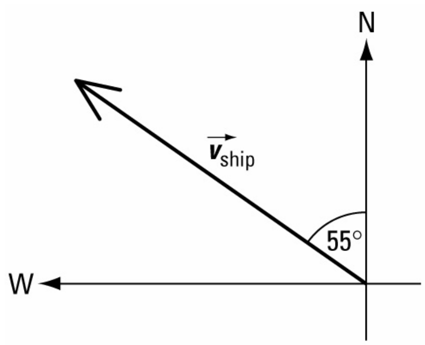
Required

velocity of ship ($\vec{v}_{\text{ship}}$)

Analysis and Solution

Determine the northern component of the ship's velocity using the equation $v = \dfrac{\Delta d}{\Delta t}$.

Use the result to determine the ship's velocity.



$$v_y = \frac{\Delta d_y}{\Delta t} = \frac{65.0\text{ km}}{3.0\text{ h}} = 21.67\text{ km/h}$$

$$v_y = v\cos\theta$$

$$v = \frac{v_y}{\cos\theta} = \frac{21.67\text{ km/h}}{\cos 55^{\circ}} = 38\text{ km/h}$$

Paraphrase

The ship's velocity is 38 km/h [55° W of N].

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Example 2.9 Practice Problems

1. ***Given***

$\vec{v}_{\text{current}} = 3.0$ m/s [W]

$\Delta d = 80$ m

$v_{\text{boat}} = 4.0$ m/s

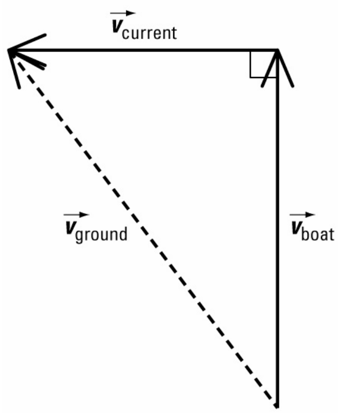
Required

(a) time (Δt) if the boat points directly across

(b) time (Δt) if the boat moves directly across

Analysis and Solution

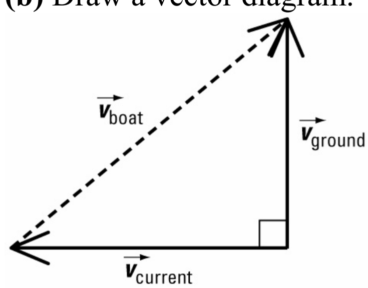
(a) Draw a vector diagram.



Use the equation $\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$ to find the time taken to cross. Since the boat's velocity is parallel to the width of the river, use the boat's velocity to calculate the time. Use the scalar form of the equation because you are dividing by a vector.

$$\begin{aligned}\Delta t &= \frac{\Delta d}{v} \\ &= \frac{80\ \cancel{\text{m}}}{4.0\ \frac{\cancel{\text{m}}}{\text{s}}} \\ &= 20\ \text{s}\end{aligned}$$

(b) Draw a vector diagram.



Since the ground velocity is parallel to the width of the river, first calculate the ground velocity.

$$\begin{aligned}\vec{v}_{\text{ground}} &= \vec{v}_{\text{boat}} + \vec{v}_{\text{current}} \\ (v_{\text{boat}})^2 &= (v_{\text{ground}})^2 + (v_{\text{current}})^2 \\ (v_{\text{ground}})^2 &= (v_{\text{boat}})^2 - (v_{\text{current}})^2 \\ v_{\text{ground}} &= \sqrt{(v_{\text{boat}})^2 - (v_{\text{current}})^2} \\ &= \sqrt{(4.0\ \text{m/s})^2 - (3.0\ \text{m/s})^2} \\ &= 2.65\ \text{m/s}\end{aligned}$$

Use the equation $\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$ to find the time taken to cross. Use the scalar form of the equation because you are dividing by a vector.

$$\begin{aligned}\Delta t &= \frac{\Delta d}{v} \\ &= \frac{80\ \cancel{\text{m}}}{2.65\ \frac{\cancel{\text{m}}}{\text{s}}} \\ &= 30\ \text{s}\end{aligned}$$

Paraphrase

(a) The time taken to cross if the boat points directly across the river is 20 s.
(b) The time taken to cross if the boat moves directly across the river is 30 s.

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2.3 Check and Reflect

Knowledge

1. Wind or current will increase an object's speed when the wind or current is in the same direction as the object's motion. For example, a north wind will increase an object's speed if the object is moving in the northerly direction.

2. Wind or current will change an object's direction only when the components are perpendicular. For example, a river's current that is perpendicular to a swimmer's motion will not alter a swimmer's swimming speed in the water but will alter the swimmer's direction, pushing the swimmer downstream. The swimmer's ground velocity increases, but the time taken to cross the river will remain the same.

3. Zero displacement occurs when the current or wind has the same speed as the moving object but the opposite direction. For example, in a training pool, the current is set at the same speed but in the opposite direction to the swimmer's motion. As a result, the swimmer remains in the same spot.

4. Perpendicular components of motion are independent of one another. An example is a hot-air balloon travelling north. An easterly wind will affect the balloon's direction but not its speed in still air.

Applications

5. ***Given***

$$\begin{aligned}v_{\text{current}} &= 0.60\ \text{m/s} \\ \Delta d &= 106.68\ \text{m} \\ v_{\text{swimmer}} &= 1.35\ \text{m/s}\end{aligned}$$

Required

(a) time (Δt)
(b) time (Δt), distance (Δd)

Analysis and Solution

(a) Since the current is perpendicular to the swimmer's motion, the current does not affect the time taken to cross. Use the uniform motion equation $v = \frac{\Delta d}{\Delta t}$ to solve for time.

$$\Delta t = \frac{\Delta d}{v}$$
$$= \frac{106.68\ \cancel{\text{m}}}{1.35\ \frac{\cancel{\text{m}}}{\text{s}}}$$
$$= 79.0\text{ s}$$

(b) $\Delta d = v\Delta t$
$$= \left(0.60\ \frac{\text{m}}{\cancel{\text{s}}}\right)(79.0\ \cancel{\text{s}})$$
$$= 47\text{ m}$$

Paraphrase

(a) It would take the swimmer 79.0 s to cross the river.

(b) The swimmer will be 47 m downstream if he lets the current carry him. The amount of time to cross the river is the same as in (a).

6. ***Given***

$\vec{v}_{\text{air}} = 265$ km/h [N]

$\vec{v}_{\text{headwind}} = 32.0$ km [S]

$\vec{v}_{\text{tailwind}} = 32.0$ km [N]

$\vec{v}_{\text{crosswind}} = 32.0$ km [W]

Required

ground velocity ($\vec{v}_{\text{ground}}$)

Analysis and Solution

(a) A headwind will decrease the Cessna's velocity. Use vector addition.

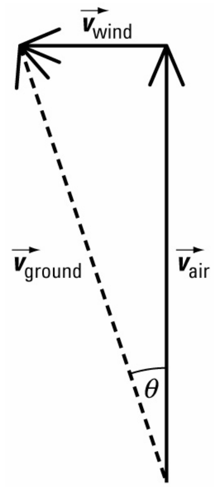
$$\begin{aligned}\vec{v}_{\text{ground}} &= \vec{v}_{\text{air}} + \vec{v}_{\text{wind}}\\ &= 265\text{ km/h [N]} + 32.0\text{ km/h [S]}\\ &= 265\text{ km/h [N]} + (-32.0\text{ km/h [N]})\\ &= 265\text{ km/h [N]} - 32.0\text{ km/h [N]}\\ &= 233\text{ km/h [N]}\end{aligned}$$

(b) A tailwind will increase the Cessna's velocity. Use vector addition.

$$\begin{aligned}\vec{v}_{\text{ground}} &= \vec{v}_{\text{air}} + \vec{v}_{\text{wind}}\\ &= 265\text{ km/h [N]} + 32.0\text{ km/h [N]}\\ &= 297\text{ km/h [N]}\end{aligned}$$

(c) A crosswind will alter the plane's direction. Use the Pythagorean theorem.

$$\vec{v}_{\text{ground}} = \vec{v}_{\text{air}} + \vec{v}_{\text{wind}}$$



$$\begin{aligned} v_{\text{ground}} &= \sqrt{(265 \text{ km/h})^2 + (32.0 \text{ km/h})^2} \\ &= 267 \text{ km/h} \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{32.0 \ \cancel{\text{km/h}}}{265 \ \cancel{\text{km/h}}} \\ \theta &= \tan^{-1}(0.121) \\ &= 6.9° \end{aligned}$$

From the diagram, this angle is $6.9°$ W of N or 83.1$^{\text{o}}$ N of W.

Paraphrase

The Cessna's resultant velocity is

(a) 233 km/h [N]

(b) 297 km/h [N]

(c) 267 km/h [83.1$^{\text{o}}$ N of W]

7. ***Given***

$$\begin{aligned} \Delta d_{\text{upstream}} &= 300 \text{ m} \\ \Delta d_{\text{downstream}} &= 300 \text{ m} \\ v_{\text{current}} &= 1.0 \text{ m/s} \\ v_{\text{swimmer}} &= 2.0 \text{ m/s} \end{aligned}$$

Required

time (Δt)

Analysis and Solution

Calculate the time to go downstream and the time to go upstream using the equation $v = \frac{\Delta d}{\Delta t}$. Then add the two times. The current will increase the swimmer's speed going downstream and will decrease the swimmer's speed going upstream.

$$\begin{aligned} v_{\text{downstream}} &= v_{\text{swimmer}} + v_{\text{current}} \\ &= 2.0 \text{ m/s} + 1.0 \text{ m/s} \\ &= 3.0 \text{ m/s} \end{aligned}$$

$$v = \frac{\Delta d}{\Delta t}$$

$$\begin{aligned}\Delta t_{\text{downstream}} &= \frac{\Delta d_{\text{downstream}}}{v_{\text{downstream}}} \\ &= \frac{300\ \cancel{\text{m}}}{3.0\ \frac{\cancel{\text{m}}}{\text{s}}} \\ &= 100\ \text{s}\end{aligned}$$

$$\begin{aligned}v_{\text{upstream}} &= v_{\text{swimmer}} - v_{\text{current}} \\ &= 2.0\ \text{m/s} - 1.0\ \text{m/s} \\ &= 1.0\ \text{m/s}\end{aligned}$$

$$\begin{aligned}\Delta t_{\text{upstream}} &= \frac{\Delta d_{\text{upstream}}}{v_{\text{upstream}}} \\ &= \frac{300\ \cancel{\text{m}}}{1.0\ \frac{\cancel{\text{m}}}{\text{s}}} \\ &= 300\ \text{s}\end{aligned}$$

$$\begin{aligned}\Delta t &= \Delta t_{\text{downstream}} + \Delta t_{\text{upstream}} \\ &= 100\ \text{s} + 300\ \text{s} \\ &= 400\ \text{s}\end{aligned}$$

Paraphrase

The swimmer takes 400 s to complete the round trip.

8. ***Given***

$\vec{v}_{\text{air}} = 800$ km/h [E]

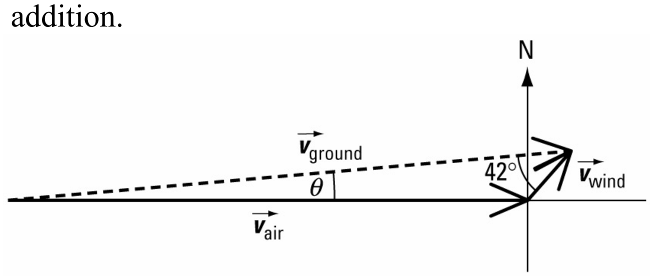
$\vec{v}_{\text{wind}} = 60$ km/h [42° E of N]

Required

ground velocity ($\vec{v}_{\text{ground}}$)

Analysis and Solution

Resolve the air and wind velocities into their components and then perform vector addition.



$$v_{\text{air}_x} = 800\ \text{km/h}$$

$$v_{\text{air}_y} = 0$$

$$\begin{aligned}v_{\text{wind}_x} &= v_{\text{wind}} \sin\theta \\ &= (60\ \text{km/h})(\sin 42°) \\ &= 40.1\ \text{km/h}\end{aligned}$$

$$\begin{aligned} v_{\text{wind}_y} &= v_{\text{wind}} \cos\theta \\ &= (60 \text{ km/h})(\cos 42°) \\ &= 44.6 \text{ km/h} \end{aligned}$$

$$\vec{v}_{\text{ground}} = \vec{v}_{\text{air}} + \vec{v}_{\text{wind}}$$

$$\begin{aligned} v_{\text{ground}_x} &= v_{\text{air}_x} + v_{\text{wind}_x} \\ &= 800 \text{ km/h} + 40.1 \text{ km/h} \\ &= 840.1 \text{ km/h} \end{aligned}$$

$$\begin{aligned} v_{\text{ground}_y} &= v_{\text{air}_y} + v_{\text{wind}_y} \\ &= 0 + 44.6 \text{ km/h} \\ &= 44.6 \text{ km/h} \end{aligned}$$

$$\begin{aligned} v_{\text{ground}} &= \sqrt{\left(v_{\text{ground}_x}\right)^2 + \left(v_{\text{ground}_y}\right)^2} \\ &= \sqrt{(840.1 \text{ km/h})^2 + (44.6 \text{ km/h})^2} \\ &= 8.4 \times 10^2 \text{ km/h} \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{44.6 \cancel{\text{ km/h}}}{840.1 \cancel{\text{ km/h}}} \\ &= 0.0531 \\ \theta &= \tan^{-1}(0.0531) \\ &= 3.0° \end{aligned}$$

From the diagram, this angle is 3.0° N of E.

Paraphrase

The ground velocity of the airplane is 8.4×10^2 km/h [3.0° N of E].

9. ***Given***

$$\begin{aligned} \vec{v}_{\text{swimmer}} &= 3.0 \text{ m/s [E]} \\ \Delta\vec{d} &= 3.20 \text{ km [E]} = 3200 \text{ m [E]} \\ \vec{v}_{\text{current}} &= 1.75 \text{ m/s [25° W of S]} \end{aligned}$$

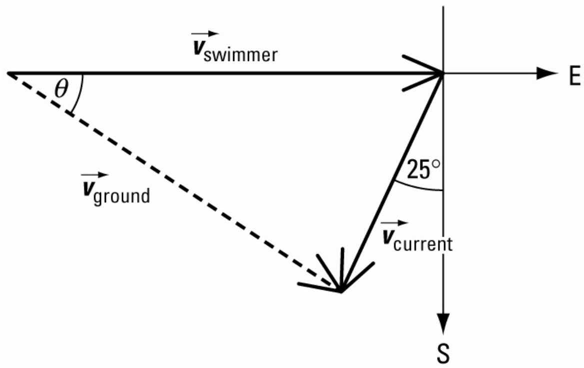
Required

ground velocity ($\vec{v}_{\text{ground}}$)

time (Δt)

Analysis and Solution

Resolve the swimmer's and current's velocities into their *x* and *y* components. Perform vector addition to determine the swimmer's ground velocity. Use the ground speed and the equation $v = \frac{\Delta d}{\Delta t}$ to determine the time taken to cross the river.



$$
\begin{aligned}
v_{swimmer_x} &= 3.0 \text{ m/s} \\
v_{swimmer_y} &= 0 \\
v_{current_x} &= -v_{current} \sin\theta \\
&= -(1.75 \text{ m/s})(\sin 25°) \\
&= -0.740 \text{ m/s} \\
v_{current_y} &= -v_{current} \cos\theta \\
&= -(1.75 \text{ m/s})(\cos 25°) \\
&= -1.59 \text{ m/s} \\
\vec{v}_{ground} &= \vec{v}_{swimmer} + \vec{v}_{current} \\
v_{ground_x} &= v_{swimmer_x} + v_{current_x} \\
&= 3.0 \text{ m/s} + (-0.740 \text{ m/s}) \\
&= 2.26 \text{ m/s} \\
v_{ground_y} &= v_{swimmer_y} + v_{current_y} \\
&= 0 + (-1.59 \text{ m/s}) \\
&= -1.59 \text{ m/s} \\
v_{ground} &= \sqrt{\left(v_{ground_x}\right)^2 + \left(v_{ground_y}\right)^2} \\
&= \sqrt{(2.26 \text{ m/s})^2 + (-1.59 \text{ m/s})^2} \\
&= 2.76 \text{ m/s}
\end{aligned}
$$

$$
\begin{aligned}
\tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\
&= \frac{1.59 \cancel{\text{m/s}}}{2.26 \cancel{\text{m/s}}} \\
&= 0.704 \\
\theta &= \tan^{-1}(0.704) \\
&= 35°
\end{aligned}
$$

From the diagram, this angle is 35° S of E.

$$v = \frac{\Delta d}{\Delta t}$$

$$\begin{aligned}\Delta t &= \frac{\Delta d}{v_{\text{ground}}} \\ &= \frac{3200\ \cancel{\text{m}}}{2.76\ \frac{\cancel{\text{m}}}{\text{s}}} \\ &= 1.2 \times 10^3\ \text{s} \\ &= 19\ \text{min}\end{aligned}$$

Paraphrase

The swimmer's ground velocity is 2.8 m/s [35° S of E]. The swimmer would take 19 minutes to cross the river.

10. ***Given***

$\vec{v}_{\text{air}}$ = 600 km/h [W]

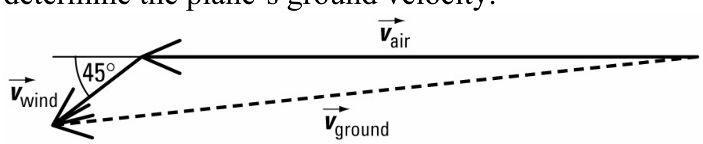
$\vec{v}_{\text{wind}}$ = 100 km/h [45° W of S]

Required

ground velocity ($\vec{v}_{\text{ground}}$)

Analysis and Solution

Resolve the wind velocity into its *x* and *y* components. Perform vector addition to determine the plane's ground velocity.



$$v_{\text{air}_x} = -600\ \text{km/h}$$

$$v_{\text{air}_y} = 0$$

$$\begin{aligned}v_{\text{wind}_x} &= -v_{\text{wind}} \sin\theta \\ &= -(100\ \text{km/h})(\sin 45°) \\ &= -70.7\ \text{km/h}\end{aligned}$$

$$\begin{aligned}v_{\text{wind}_y} &= -v_{\text{wind}} \cos\theta \\ &= -(100\ \text{km/h})(\cos 45°) \\ &= -70.7\ \text{m/s}\end{aligned}$$

$$\vec{v}_{\text{ground}} = \vec{v}_{\text{wind}} + \vec{v}_{\text{air}}$$

$$\begin{aligned} v_{\text{ground}_x} &= v_{\text{wind}_x} + v_{\text{air}_x} \\ &= -70.7 \text{ km/h} + (-600 \text{ km/h}) \\ &= -670.7 \text{ km/h} \end{aligned}$$

$$\begin{aligned} v_{\text{ground}_y} &= v_{\text{wind}_y} + v_{\text{air}_y} \\ &= -70.7 \text{ km/h} + 0 \\ &= -70.7 \text{ km/h} \end{aligned}$$

$$\begin{aligned} v_{\text{ground}} &= \sqrt{\left(v_{\text{ground}_x}\right)^2 + \left(v_{\text{ground}_y}\right)^2} \\ &= \sqrt{(-670.7 \text{ km/h})^2 + (-70.7 \text{ km/h})^2} \\ &= 674.4 \text{ km/h} \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{70.7 \cancel{\text{km/h}}}{670.7 \cancel{\text{km/h}}} \\ &= 0.105 \\ \theta &= \tan^{-1}(0.105) \\ &= 6.0° \end{aligned}$$

From the diagram, this angle is 6.0 ° S of W.

Paraphrase

The plane's ground velocity is 674 km/h [6.0 ° S of W].

11. ***Given***

$\vec{v}_{\text{boat}}$ = 4.0 m/s [N]

$\vec{v}_{\text{current}}$ = 2.5 m/s [W]

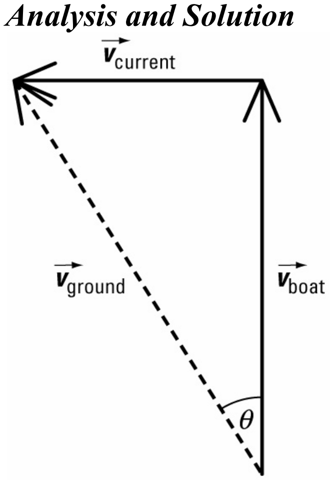
Δd = 0.80 km

Required

(a) ground velocity ($\vec{v}_{\text{ground}}$)

(b) time (Δt)

Analysis and Solution



(a) The vectors are perpendicular, so use the Pythagorean theorem to determine the magnitude of the ground velocity.

$$\vec{v}_{\text{ground}} = \vec{v}_{\text{boat}} + \vec{v}_{\text{current}}$$

$$\begin{aligned} v_{\text{ground}} &= \sqrt{(v_{\text{boat}})^2 + (v_{\text{current}})^2} \\ &= \sqrt{(4.0 \text{ m/s})^2 + (2.5 \text{ m/s})^2} \\ &= 4.7 \text{ m/s} \end{aligned}$$

Use the tangent function to find the angle.

$$\begin{aligned} \tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{2.5 \cancel{\text{m/s}}}{4.0 \cancel{\text{m/s}}} \\ &= 0.625 \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}(0.625) \\ &= 32° \end{aligned}$$

From the diagram, this angle is 32° W of N.

(b) To find the time, use the equation $\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$ and the magnitude of the boat's velocity because it is parallel to the width of the river. Use the scalar form of the equation because you are dividing by a vector.

$$\begin{aligned} \Delta t &= \frac{\Delta d}{v} \\ &= \frac{800 \ \cancel{\text{m}}}{4.0 \ \frac{\cancel{\text{m}}}{\text{s}}} \\ &= 200 \text{ s} \\ &= 3.3 \text{ min} \end{aligned}$$

Paraphrase

(a) The canoe's ground velocity is 4.7 m/s [32° W of N].

(b) It takes the canoe 3.3 minutes to cross the river.

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Concept Check

Horizontal	position	uniform
	velocity	uniform
	acceleration	none
Vertical	position	non-uniform
	velocity	non-uniform
	acceleration	uniform

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Concept Check

(a) The factors of air resistance and spin are being neglected.
(b) The projectile finally stops when it hits the ground. Thus, friction in the horizontal direction brings the object to rest.
(c) The projectile's path would still be parabolic if its initial velocity had a vertical component because its motion is still affected by acceleration due to gravity.

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Example 2.10 Practice Problems

1. ***Given***

$v_x = 30 \text{ cm/s}$

$\Delta d_y = 1.25 \text{ m}$

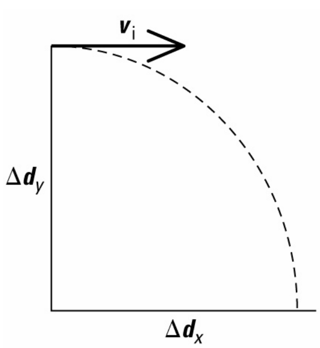
$v_{i_y} = 0 \text{ m/s}$

Required

range (Δd_x)

Analysis and Solution

Choose down to be positive. The coin accelerates due to gravity once it leaves the table. Its initial vertical velocity is zero. Determine the time to fall from a height of 1.25 m using the equation $\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$.



$$\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$\begin{aligned}\Delta t &= \sqrt{\frac{2\Delta d_y}{a_y}}\\ &= \sqrt{\frac{2(1.25\ \cancel{\text{m}})}{9.81\ \frac{\cancel{\text{m}}}{\text{s}^2}}}\\ &= 0.505 \text{ s}\end{aligned}$$

Then determine the range by using the equation $\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$.

$$
\begin{aligned}
\Delta d_x &= v_x \Delta t \\
&= \left(30\ \frac{\text{cm}}{\cancel{\text{s}}}\right)(0.505\ \cancel{\text{s}}) \\
&= 15\ \text{cm}
\end{aligned}
$$

Paraphrase

The coin lands 15 cm from the base of the table.

2. ***Given***

$v_x = 25.0\ \text{m/s}$

$\Delta t = 2.50\ \text{s}$

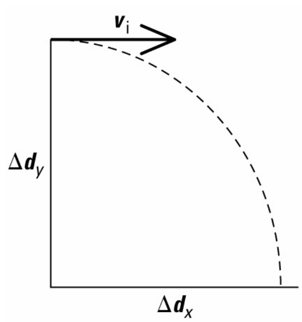
$h = 150.0\ \text{m}$

Required

vertical distance (Δd_y)

Analysis and Solution

Choose down to be positive. The distance dropped is a measure of vertical displacement. Since the object is fired horizontally from the top of the cliff, its initial vertical velocity is zero. Use the equation $\Delta d_y = v_{i_y} \Delta t + \frac{1}{2} a_y (\Delta t)^2$.



$$
\begin{aligned}
\Delta d_y &= v_{i_y} \Delta t + \frac{1}{2} a_y (\Delta t)^2 \\
&= 0 + \frac{1}{2}\left(9.81\ \frac{\text{m}}{\cancel{\text{s}^2}}\right)(2.50\ \cancel{\text{s}})^{\cancel{2}} \\
&= 30.7\ \text{m}
\end{aligned}
$$

Paraphrase

The arrow will fall 30.7 m in 2.50 s.

3. ***Given***

$\Delta d_x = 40.0\ \text{m}$

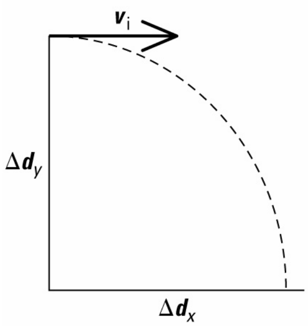
$\Delta d_y = 100\ \text{m}$

Required

horizontal speed (v_x)

Analysis and Solution

Choose down to be positive. Determine the amount of time it takes the object to fall 100 m using the equation $\Delta d_y = v_{i_y} \Delta t + \frac{1}{2} a_y (\Delta t)^2$, where $v_{i_y} = 0$.



$$\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$\Delta t = \sqrt{\frac{2\Delta d_y}{a_y}}$$

$$= \sqrt{\frac{2(100\text{ m})}{9.81\text{ m/s}^2}}$$

$$= 4.515\text{ s}$$

Calculate the horizontal velocity by using the equation $\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$.

$$v_x = \frac{\Delta d_x}{\Delta t}$$

$$= \frac{40.0\text{ m}}{4.515\text{ s}}$$

$$= 8.86\text{ m/s}$$

Paraphrase

The horizontal speed of the object is 8.86 m/s.

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Example 2.11 Practice Problems

1. (a) ***Given***

$v_i = 10.0$ m/s

$\Delta t = 3.0$ s

Required

(a) range (Δd_x)

(b) height (Δd_y)

Analysis and Solution

(a) Since there is no acceleration in the horizontal direction, use the equation $\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$ to find the range.

$$\Delta d_x = v_x \Delta t$$

$$= (10.0\text{ m/s})(3.0\text{ s})$$

$$= 30\text{ m}$$

(b) Use the equation $\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$, where v_{i_y} = 0 because there is no initial vertical velocity component.

$$\Delta d_y = 0 + \frac{1}{2}(9.81\text{ m/s}^2)(3.0\text{ s})^2$$
$$= 44\text{ m}$$

Paraphrase

(a) The ball travels a horizontal distance of 30 m.

(b) The ball was thrown from a height of 44 m.

2. (a) ***Given***

$\vec{v}_i$ = 20.0 m/s [30°]

Δt = 3.0 s

Required

(a) range (Δd_x)

(b) height (Δd_y)

(c) maximum height (Δd_y)

Analysis and Solution

Choose up and forward to be positive.

(a) Find the *x* component of the ball's initial velocity.

$$v_{i_x} = v_i\cos\theta$$
$$= (20.0\text{ m/s})(\cos 30°)$$
$$= 17.3\text{ m/s}$$

Since there is no acceleration in the horizontal direction, use the equation $\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$ to find the range.

$$\Delta d_x = v_x\Delta t$$
$$= (17.3\text{ m/s})(3.0\text{ s})$$
$$= 52\text{ m}$$

(b) Find the *y* component of the ball's initial velocity.

$$v_{i_y} = v_i\sin\theta$$
$$= (20.0\text{ m/s})(\sin 30°)$$
$$= 10.0\text{ m/s}$$

Use the equation $\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$ to find the height from which the ball was thrown.

$$\Delta d_y = (10.0\text{ m/s})(3.0\text{ s}) + \frac{1}{2}(-9.81\text{ m/s}^2)(3.0\text{ s})^2$$
$$= -14\text{ m}$$

The negative sign means that the ball was thrown from a height 14 m above a reference point.

(c) Since the projectile's final vertical velocity component is zero the instant it reaches maximum height, use the equation $v_f^2 = v_i^2 + 2a\Delta d$ to calculate maximum height.

$$v_{f_y}{}^2 = v_{i_y}{}^2 + 2a_y\Delta d_y$$

$$\Delta d_y = \frac{v_{f_y}{}^2 - v_{i_y}{}^2}{2a_y}$$

$$= \frac{0 - (10.0\ \text{m/s})^2}{2(-9.81\ \text{m/s}^2)}$$

$$= 5.1\ \text{m}$$

Paraphrase

(a) The ball travels a horizontal distance of 52 m.

(b) The ball was thrown from a height of 14 m.

(c) The ball's maximum height above its launch point is 5.1 m.

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Example 2.12 Practice Problems

1. ***Given***

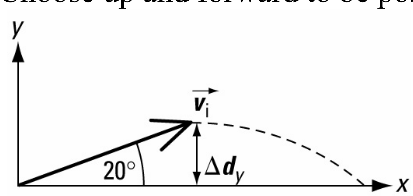
$\vec{v}_i$ = 17.0 m/s [20°]

Required

maximum height (Δd_y)

Analysis and Solution

Choose up and forward to be positive. Calculate the y component of the velocity.



$$v_{i_y} = (17.0\ \text{m/s})(\sin 20°)$$

$$= 5.81\ \text{m/s}$$

Since the projectile's final vertical velocity component is zero the instant it reaches maximum height, use the equation $v_f{}^2 = v_i^2 + 2a\Delta d$ to calculate maximum height.

$$v_{f_y}{}^2 = v_{i_y}{}^2 + 2a_y\Delta d_y$$

$$\Delta d_y = \frac{v_{f_y}{}^2 - v_{i_y}{}^2}{2a_y}$$

$$= \frac{0 - (5.81\ \text{m/s})^2}{2(-9.81\ \text{m/s}^2)}$$

$$= 1.72\ \text{m}$$

Paraphrase

The projectile reaches a maximum height of 1.72 m.

2. ***Given***

$v = 56.0$ km/h

$\Delta d_x = 198$ m

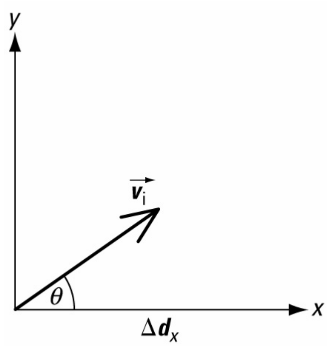
$\Delta t = 30.0$ s

Required

launch angle (θ)

Analysis and Solution

Determine the horizontal velocity component of the fish using the equation $v_x = \frac{\Delta d_x}{\Delta t}$. Then use the cosine function to determine the launch angle.



$$56.0\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}} = 15.56\ \text{m/s}$$

$$\begin{aligned} v_x &= \frac{\Delta d_x}{\Delta t} \\ &= \frac{198\ \text{m}}{30.0\ \text{s}} \\ &= 6.60\ \text{m/s} \end{aligned}$$

$$\begin{aligned} \cos\theta &= \frac{v_x}{v} \\ &= \frac{6.60\ \text{m/s}}{15.56\ \text{m/s}} \\ \theta &= 64.9° \end{aligned}$$

Paraphrase

The launch angle is 64.9°.

3. ***Given***

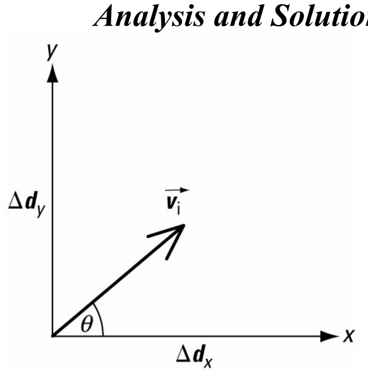
$\Delta d_x = 300\ \text{km [forward]} = 300\ 000\ \text{m [forward]}$

$\Delta d_y = 100\ \text{km [up]} = 100\ 000\ \text{m [up]}$

Required

initial velocity ($\vec{v}_i$)

Analysis and Solution



Choose up and forward to be positive. Determine the amount of time it takes for the rocket to fall from maximum height. Use the equation $\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$, where v_{i_y} (at maximum height) equals zero.

$$\begin{aligned}
\Delta d_y &= v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2 \\
\Delta t &= \sqrt{\frac{2\Delta d_y}{a_y}} \\
&= \sqrt{\frac{2(-100\,000\ \cancel{\text{m}})}{-9.81\ \frac{\cancel{\text{m}}}{\text{s}^2}}} \\
&= 142.8\ \text{s}
\end{aligned}$$

This time is the time taken to fall down from maximum height. The total time of flight is therefore $2 \times 142.8\ \text{s} = 285.6\ \text{s}$.

Using the equation $\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$, divide the range of the rocket by the total time to determine the initial horizontal velocity component.

$$\begin{aligned}
v_{i_x} &= \frac{\Delta d_x}{\Delta t} \\
&= \frac{300\,000\ \text{m}}{285.6\ \text{s}} \\
&= 1050\ \text{m/s}
\end{aligned}$$

Find the initial vertical velocity component using the equation $a = \frac{v_f - v_i}{\Delta t}$, since vertical motion is uniformly accelerated and the final vertical velocity component at maximum height is zero.

y direction:

$$\begin{aligned}
a_y &= \frac{v_{f_y} - v_{i_y}}{\Delta t} \\
v_{i_y} &= v_{f_y} - a_y\Delta t \\
&= 0 - \left(-9.81\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(142.8\ \cancel{\text{s}}) \\
&= 1401\ \text{m/s}
\end{aligned}$$

Use the Pythagorean theorem to determine the magnitude of the initial velocity.

$$\begin{aligned}
v_i &= \sqrt{(v_{i_x})^2 + (v_{i_y})^2} \\
&= \sqrt{(1050\ \text{m/s})^2 + (1401\ \text{m/s})^2} \\
&= 1751\ \text{m/s}
\end{aligned}$$

Use the tangent function to determine the direction of the initial velocity.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{v_{i_y}}{v_{i_x}}$$
$$= \frac{1401\ \cancel{\text{m}}/\text{s}}{1050\ \cancel{\text{m}}/\text{s}}$$
$$\theta = \tan^{-1}(1.334)$$
$$= 53.1°$$

Paraphrase

The rocket had an initial velocity of 1.75×10^3 m/s $[53.1°]$.

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2.4 Check and Reflect

Knowledge

1. Both athletes have vertical and horizontal components to their dives into the water. Platform divers must ensure that they do not hit the diving board on the way down but want to minimize their horizontal movement. Speed swimmers want to get into the pool as far as possible from the starting block, so they minimize their vertical velocity.

2. Range is a measure of horizontal displacement. The horizontal component of motion is dependent on the cosine of the angle, θ.

3. **(a)** The longer the projectile is in the air, the farther it will travel horizontally.

(b) Maximum time is proportional to maximum range since $\Delta d = v\Delta t$.

4. A novice swimmer who jumps straight up risks injury to the spine because he or she will land on the deck of the pool. Most swim instructors teach students to put their toes over the edge of the pool and jump out, giving them horizontal uniform motion, so they land in the water, not on the deck.

Applications

5. ***Given***

$v_x = 6.20$ m/s

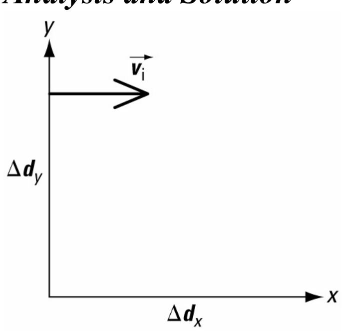
$v_{i_y} = 0$ m/s

$\Delta d_y = 0.50$ m

Required

range (Δd_x)

Analysis and Solution



Choose down to be positive. The cup has an initial horizontal velocity component of 6.20 m/s, whereas its initial vertical velocity component is zero. Use the equation $\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$, where $v_{i_y} = 0$.

y direction:

Find Δt.

$$\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$\begin{aligned}\Delta t &= \sqrt{\frac{2\Delta d_y}{a_y}} \\ &= \sqrt{\frac{2(0.50\ \cancel{\text{m}})}{9.81\ \frac{\cancel{\text{m}}}{\text{s}^2}}} \\ &= 0.319\ \text{s}\end{aligned}$$

The runner must drop the cup in front of the garbage can so that it will land inside the can and not on the ground. The amount of time taken to fall 0.50 m is equal to the amount of time the cup has to travel horizontally at a uniform rate of 6.20 m/s.

x direction:

Calculate the range.

$$\begin{aligned}\Delta d_x &= v_x\Delta t \\ &= \left(6.20\ \frac{\text{m}}{\cancel{\text{s}}}\right)(0.319\ \cancel{\text{s}}) \\ &= 2.0\ \text{m}\end{aligned}$$

Paraphrase

The runner should release his cup when he is 2.0 m from the garbage can.

6. ***Given***

$\vec{v}_i = 27.0\ \text{m/s}\ [35^\circ]$

Required

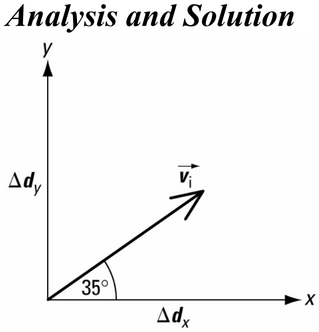
horizontal velocity component (v_{i_x})

vertical velocity component (v_{i_y})

range (Δd_x)

maximum height (Δd_y)

Analysis and Solution



Choose up and forward to be positive. Use $R_x = R\cos\theta$ and $R_y = R\sin\theta$ to determine horizontal and vertical velocity components.

$$\begin{aligned} v_{i_x} &= (27.0 \text{ m/s})(\cos 35°) \\ &= 22.1 \text{ m/s} \\ v_{i_y} &= (27.0 \text{ m/s})(\sin 35°) \\ &= 15.5 \text{ m/s} \end{aligned}$$

Calculate the maximum height using the equation $v_f^2 = v_i^2 + 2a\Delta d$, where $v_f = 0$.

y direction:

$$v_{f_y}{}^2 = v_{i_y}{}^2 + 2a_y \Delta d_y$$

$$\begin{aligned} \Delta d_y &= \frac{v_{f_y}{}^2 - v_{i_y}{}^2}{2a_y} \\ &= \frac{0 - (15.5 \text{ m/s})^2}{2\left(-9.81 \text{ m/s}^2\right)} \\ &= 12.2 \text{ m} \end{aligned}$$

Calculate the time the ball is in the air using the vertical velocity component and the equation $a = \frac{v_f - v_i}{\Delta t}$. The initial and final vertical velocity components have the same magnitude but opposite direction, so

$$a_y = \frac{v_f - v_i}{\Delta t}$$

$$\begin{aligned} \Delta t &= \frac{v_f - v_i}{a_y} \\ &= \frac{-15.5 \text{ m/s} - (15.5 \text{ m/s})}{-9.81 \text{ m/s}^2} \\ &= \frac{-31.0 \ \frac{\cancel{\text{m}}}{\text{s}}}{-9.81 \ \frac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}} \\ &= 3.16 \text{ s} \end{aligned}$$

Then calculate how far the ball will travel using the equation $\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$.

x direction:

The ball's range is therefore

$$\begin{aligned} \Delta d_x &= v_x \Delta t \\ &= \left(22.1 \ \frac{\text{m}}{\cancel{\text{s}}}\right)(3.16 \ \cancel{\text{s}}) \\ &= 69.8 \text{ m} \end{aligned}$$

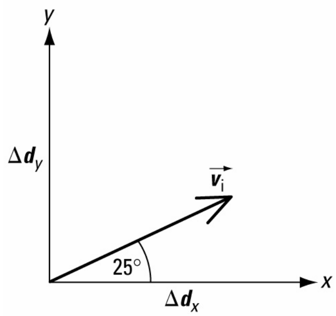
Paraphrase

The initial horizontal velocity component is 22.1 m/s. The initial vertical velocity component is 15.5 m/s. The ball travels up 12.2 m and out 69.8 m.

7. ***Given***

$\vec{v}_i = 17.0 \text{ m/s } [25^\circ]$

Required
receiver's speed (v_{receiver})
Analysis and Solution
Choose up and forward to be positive.



Resolve the football's velocity into its *x* and *y* components.

$$\begin{aligned} v_{i_x} &= v_i \cos\theta \\ &= (17.0 \text{ m/s})(\cos 25°) \\ &= 15.41 \text{ m/s} \end{aligned}$$

$$\begin{aligned} v_{i_y} &= v_i \sin\theta \\ &= (17.0 \text{ m/s})(\sin 25°) \\ &= 7.185 \text{ m/s} \end{aligned}$$

Calculate the amount of time the ball is in the air using the equation $a = \dfrac{v_f - v_i}{\Delta t}$.

Since the football lands at the same height from which it is thrown, the magnitudes of the initial and final vertical velocity components are equal but their directions are opposite. So,

$$a = \frac{v_f - v_i}{\Delta t}$$

$$\begin{aligned} \Delta t &= \frac{v_{f_y} - v_{i_y}}{a} \\ &= \frac{-7.185 \text{ m/s} - (7.185 \text{ m/s})}{-9.81 \text{ m/s}^2} \\ &= \frac{-14.36 \ \dfrac{\cancel{\text{m}}}{\cancel{\text{s}}}}{-9.81 \ \dfrac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}} \\ &= 1.465 \text{ s} \end{aligned}$$

Determine the distance the ball travels using the equation $\vec{v} = \dfrac{\Delta \vec{d}}{\Delta t}$.

The ball is in the air for 1.465 s. During this time, the range of the ball is

$$\begin{aligned} \Delta d_x &= v_x \Delta t \\ &= \left(15.41 \ \frac{\text{m}}{\cancel{\text{s}}}\right)(1.465 \ \cancel{\text{s}}) \\ &= 22.58 \text{ m} \end{aligned}$$

Then subtract the receiver's initial position from this value to determine how far the receiver must run. The receiver must run a distance of

$$\begin{aligned}\Delta d_{\text{run}} &= \Delta d_x - \Delta d_{\text{receiver}}\\ &= 22.58 \text{ m} - 12.0 \text{ m}\\ &= 10.58 \text{ m}\end{aligned}$$

Divide the distance the receiver must run by the time the ball is in flight to determine her velocity.

$$\begin{aligned}v_{\text{receiver}} &= \frac{\Delta d_{\text{run}}}{\Delta t}\\ &= \frac{10.58 \text{ m}}{1.465 \text{ s}}\\ &= 7.222 \text{ m/s}\end{aligned}$$

Paraphrase

The receiver must have a speed of 7.22 m/s to catch the ball.

8. ***Given***

$\theta = 23°$

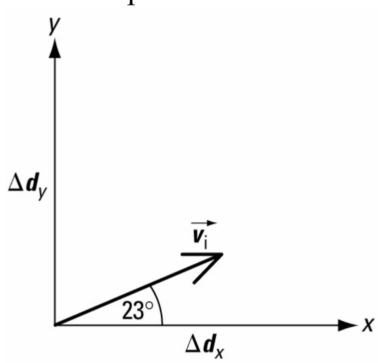
$\Delta d_x = 8.59$ m

Required

speed (v)

Analysis and Solution

Choose up and forward to be positive.



$$v_{\text{i}_x} = v_{\text{i}} \cos\theta$$

$$v_{\text{i}_y} = v_{\text{i}} \sin\theta$$

y direction:

$$a_y = \frac{v_{\text{f}_y} - v_{\text{i}_y}}{\Delta t}$$

$$\begin{aligned}\Delta t &= \frac{v_{\text{f}_y} - v_{\text{i}_y}}{a_y}\\ &= \frac{-v_{\text{i}} \sin 23° - (v_{\text{i}} \sin 23°)}{-9.81 \text{ m/s}^2}\\ &= \frac{-2v_{\text{i}} \sin 23°}{-9.81 \text{ m/s}^2}\\ &= \frac{2v_{\text{i}} \sin 23°}{9.81 \text{ m/s}^2}\end{aligned}$$

x direction:

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$$

$$\Delta t = \frac{\Delta d_x}{v_{i_x}}$$

$$= \frac{8.59 \text{ m}}{v_i \cos 23°}$$

Time travelled in the horizontal direction equals time travelled in the vertical direction.
Solve for takeoff speed:

$$\Delta t_y = \Delta t_x$$

$$\frac{2v_i \sin 23°}{9.81 \text{ m/s}^2} = \frac{8.59 \text{ m}}{v_i \cos 23°}$$

$$v_i^{\ 2} = \frac{8.59 \text{ m}}{\cos 23°} \times \frac{9.81 \text{ m/s}^2}{2 \sin 23°}$$

$$v_i = 11 \text{ m/s}$$

Paraphrase

Phillips' takeoff speed was 11 m/s.

9. ***Given***

$\vec{v}_i = 120 \text{ m/s } [55.0°]$

$\Delta d_y = 50.0 \text{ m}$

Required

(a) time at maximum height ($\Delta t_{\text{max height}}$)

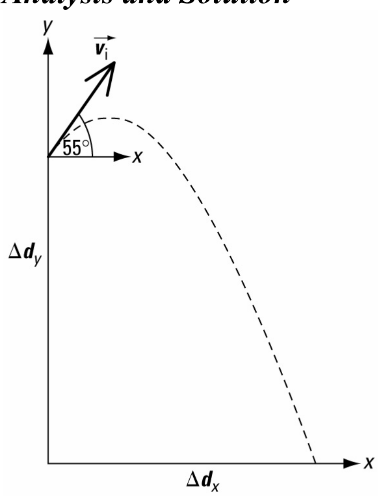
(b) maximum height ($\Delta d_{y_{\text{max}}}$)

(c) time in the air (Δt)

(d) range (Δd_x)

(e) final velocity components (v_{f_x}, v_{f_y})

Analysis and Solution



Choose up and forward to be positive.

(a) Calculate the horizontal and vertical components of the initial velocity.

$$
\begin{aligned}
v_{i_x} &= v_i \cos\theta \\
&= (120 \text{ m/s})(\cos 55.0°) \\
&= 68.83 \text{ m/s} \\
v_{i_y} &= v_i \sin\theta \\
&= (120 \text{ m/s})(\sin 55.0°) \\
&= 98.30 \text{ m/s}
\end{aligned}
$$

At maximum height, vertical velocity is zero. Use the equation $a = \frac{v_f - v_i}{\Delta t}$ to find time.

$$
\begin{aligned}
a &= \frac{v_f - v_i}{\Delta t} \\
\Delta t_{up} &= \frac{v_{f_y} - v_{i_y}}{a_y} \\
&= \frac{0 - \left(98.30 \frac{\cancel{m}}{\cancel{s}}\right)}{-9.81 \frac{\cancel{m}}{s^{\cancel{2}}}} \\
&= 10.02 \text{ s}
\end{aligned}
$$

(b) Calculate the maximum height using the equation $v_f^2 = v_i^2 + 2a\Delta d$, where $v_f = 0$.

$$
\begin{aligned}
\Delta d &= \frac{v_f^2 - v_i^2}{2a} \\
&= \frac{0 - (98.30 \text{ m/s})^2}{2(-9.81 \text{ m/s}^2)} \\
&= \frac{-9662.9 \frac{\text{m}^{\cancel{2}}}{\cancel{s^2}}}{-19.62 \frac{\cancel{m}}{\cancel{s^2}}} \\
&= 492.5 \text{ m}
\end{aligned}
$$

Add the maximum height to the cliff height to determine the vertical displacement.

$$
\begin{aligned}
\Delta d_{y_{max}} &= 492.5 \text{ m} + 50.0 \text{ m} \\
&= 542.5 \text{ m}
\end{aligned}
$$

(c) To find the time taken to fall down, use the equation $\Delta d_y = v_{i_y} \Delta t + \frac{1}{2} a_y (\Delta t)^2$, where v_{i_y} (at maximum height) equals zero and is negative because motion is downward.

$$\Delta t_{\text{down}} = \sqrt{\frac{2\Delta d_y}{a_y}}$$
$$= \sqrt{\frac{2(-542.5\ \cancel{\text{m}})}{-9.81\ \frac{\cancel{\text{m}}}{\text{s}^2}}}$$
$$= 10.52\ \text{s}$$

Total time in the air is the time taken to reach maximum height plus the time taken to fall down to the ground.

$$\Delta t = \Delta t_{\text{up}} + \Delta t_{\text{down}}$$
$$= 10.02\ \text{s} + 10.52\ \text{s}$$
$$= 20.54\ \text{s}$$

(d) The projectile experiences uniform motion in the horizontal direction.

Use the equation $\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$.

$$\Delta d_x = v_x \Delta t$$
$$= \left(68.83\ \frac{\text{m}}{\cancel{\text{s}}}\right)(20.54\ \cancel{\text{s}})$$
$$= 1414\ \text{m}$$

(e) Since motion in the horizontal direction is uniform, the horizontal component of the final velocity is the same as the horizontal component of the initial velocity. For the vertical component of final velocity, use the equation $a = \frac{v_{\text{f}} - v_{\text{i}}}{\Delta t}$.

$$a_y = \frac{v_{\text{f}_y} - v_{\text{i}_y}}{\Delta t}$$
$$v_{\text{f}_y} = v_{\text{i}_y} + a_y \Delta t$$
$$= 98.30\ \frac{\text{m}}{\text{s}} + \left(-9.81\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(20.54\ \cancel{\text{s}})$$
$$= -103\ \text{m/s}$$

Since up and forward are positive, the negative sign indicates that motion is downward.

Paraphrase

(a) The projectile reaches maximum height in 10.0 s.
(b) Maximum height with respect to the ground is 543 m.
(c) The total time in the air 20.5 s.
(d) The range is 1.41 km.
(e) The horizontal component of the final velocity is 68.8 m/s and the vertical component is 103 m/s.

Extension

10.

Angle	v_{i_x}	v_{i_y}	Δd_x	Δd_y
0	20	0	0	0
5	19.9	1.743	7.07	0.1548
10	19.7	3.472	13.945	0.6144
15	19.3	5.176	20.366	1.365
20	18.79	6.84	26.2	2.384
25	18.126	8.45	31.226	3.639
30	17.32	10	35.31	5.096
35	16.38	11.47	38.303	6.705
40	15.32	12.9	39.978	8.35
45	14.14	14.14	40.762	10.19
50	12.9	15.32	40.291	11.96
55	11.47	16.38	38.303	13.675
60	10	17.32	35.31	15.289
65	8.45	18.126	31.226	16.745
70	6.84	18.79	26.2	17.99
75	5.176	19.3	20.366	18.985
80	3.472	19.7	13.873	19.58
85	1.743	19.9	7.07	20.18
90	0	20	0	20.38

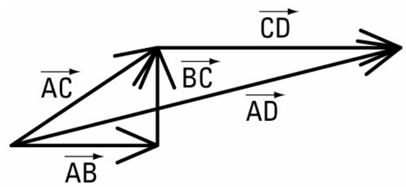
Explanation:

To find v_{i_x} and v_{i_y}, use $v_{i_y} = v_i \sin\theta$ and $v_{i_x} = v_i \cos\theta$, where $v_i = 20$ m/s.
To find Δd_x and Δd_y, first find the time taken to travel the entire range (Δt) using the equation $\Delta d_y = v_{i_y} \Delta t + \frac{1}{2} a_y (\Delta t)^2$, where $\Delta d_y = 0$ and $a_y = 9.81$ m/s^2. Substitute the value of Δt into the equation $\Delta d_x = v_{i_x} \Delta t$ to solve for the range, Δd_x.
To find maximum height, Δd_y, divide the time (Δt) in half and substitute this value into the equation $\Delta d_y = v_{i_y} \Delta t + \frac{1}{2} a_y (\Delta t)^2$.

Chapter 2 Review

Knowledge

1.



	Distance	Final position	Displacement
AB	3.0 km	3.0 km [E]	3.0 km [E]
BC	2.0 km	2.0 km [N]	2.0 km [N]
CD	5.0 km	5.0 km [E]	5.0 km [E]
AC	5.0 km	3.6 km [34°]	3.6 km [34°]
AD	10 km	8.2 km [14°]	8.2 km [14°]

2.
$$\begin{aligned}\Delta d_x &= \Delta d \cos\theta \\ &= (55\text{ m})(\cos 222°) \\ &= -41\text{ m}\end{aligned}$$
$$\begin{aligned}\Delta d_y &= \Delta d \cos\theta \\ &= (55\text{ m})(\sin 222°) \\ &= -37\text{ m}\end{aligned}$$

3. When a projectile reaches maximum height, its vertical velocity component is zero.

4. As a football rises, the vertical component of its velocity decreases. When the football reaches maximum height, its vertical velocity component is zero.

5. The vector should be 10.0 cm long and point north.

6. Since a vector's length represents its magnitude, you can only compare vectors when they are drawn to scale.

7. (a)
$$\begin{aligned}\Delta\vec{d} &= 5.0\text{ m [S]} + 10.0\text{ m [N]} \\ &= 5.0\text{ m [S]} + (-10.0\text{ m [S]}) \\ &= 5.0\text{ m [S]} - 10.0\text{ m [S]} \\ &= -5.0\text{ m [S] or } 5.0\text{ m [N]}\end{aligned}$$

(b)
$$\begin{aligned}\Delta\vec{d} &= 65.0\text{ cm [E]} + 75.0\text{ cm [E]} \\ &= 140\text{ cm [E]}\end{aligned}$$

(c)
$$\begin{aligned}\Delta\vec{d} &= 1.0\text{ km [forward]} + 3.5\text{ km [backward]} \\ &= 1.0\text{ km [forward]} + (-3.5\text{ km [forward]}) \\ &= 1.0\text{ km [forward]} - 3.5\text{ km [forward]} \\ &= -2.5\text{ km [forward] or } 2.5\text{ km [backward]}\end{aligned}$$

(d)
$$\begin{aligned}\Delta\vec{d} &= 35.0\text{ km [right]} - 45.0\text{ km [left]} \\ &= 35.0\text{ km [right]} + 45.0\text{ km [right]} \\ &= 80.0\text{ km [right]}\end{aligned}$$

8. An object thrown vertically upwards has a horizontal velocity component of zero.

Applications

9. ***Given***

$v = 250$ knots

$\Delta t = 50$ min

Required

distance (Δd)

Analysis and Solution

Convert knots to km/h and min to h. Manipulate the velocity equation:

$$v = \frac{\Delta d}{\Delta t}$$

$$\Delta d = v\Delta t$$

$$\Delta d = 250\ \cancel{\text{knots}} \times \frac{1.853\ \frac{\text{km}}{\cancel{\text{h}}} \times 50\ \cancel{\text{min}} \times \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}}{1\ \cancel{\text{knot}}}$$

$$= 3.9 \times 10^2\ \text{km}$$

Paraphrase

The plane travels 3.9×10^2 km.

10. ***Given***

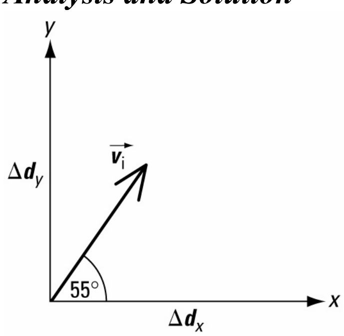
$\vec{v}_\text{i} = 30.0$ m/s [55°]

Required

range (Δd_x)

maximum height (Δd_y)

Analysis and Solution



Choose up and forward to be positive. Calculate the horizontal and vertical components of velocity as an expression of the initial velocity.

$$v_{\text{i}_x} = (30.0\text{ m/s})(\cos 55°)$$
$$= 17.2\text{ m/s}$$
$$v_{\text{i}_y} = (30.0\text{ m/s})(\sin 55°)$$
$$= 24.6\text{ m/s}$$

Determine the amount of time the ball remains in the air using the equation $a = \dfrac{v_\text{f} - v_\text{i}}{\Delta t}$.

$$a_y = \frac{v_{f_y} - v_{i_y}}{\Delta t}$$

$$\begin{aligned}\Delta t &= \frac{v_{f_y} - v_{i_y}}{a_y} \\ &= \frac{-24.6 \text{ m/s} - (24.6 \text{ m/s})}{-9.81 \text{ m/s}^2} \\ &= \frac{-49.2 \ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}{-9.81 \ \frac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}} \\ &= 5.02 \text{ s}\end{aligned}$$

Use the uniform motion equation $\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$ to determine the range.

$$\begin{aligned}\Delta d_x &= v_x \Delta t \\ &= \left(17.2 \ \frac{\text{m}}{\cancel{\text{s}}}\right)(5.02 \ \cancel{\text{s}}) \\ &= 86.3 \text{ m}\end{aligned}$$

The golf ball reaches maximum height in half the time it is in the air, or $\frac{5.02 \text{ s}}{2} = 2.51 \text{ s}$.

Use the equation $\Delta d_y = v_{i_y} \Delta t + \frac{1}{2} a_y (\Delta t)^2$ to find maximum height.

$$\begin{aligned}\Delta d_y &= v_{i_y} \Delta t + \frac{1}{2} a_y (\Delta t)^2 \\ &= \left(24.6 \ \frac{\text{m}}{\cancel{\text{s}}}\right)(2.51 \ \cancel{\text{s}}) + \frac{1}{2}\left(-9.81 \ \frac{\text{m}}{\cancel{\text{s}^2}}\right)(2.51 \ \cancel{\text{s}})^{\cancel{2}} \\ &= 30.8 \text{ m}\end{aligned}$$

Paraphrase

The ball's range is 86.3 m and its maximum height is 30.8 m.

11. ***Given***

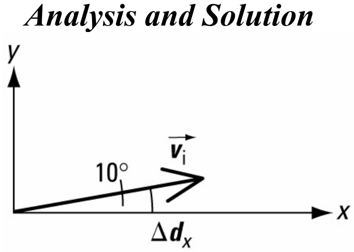
$\vec{v}_i = 80.0 \text{ m/s } [10°]$

Required

range (Δd_x)

time (Δt)

Analysis and Solution



Choose up and forward to be positive.

Calculate the vertical and horizontal components of the initial velocity.

$$v_{i_x} = (80.0 \text{ m/s})(\cos 10°)$$
$$= 78.8 \text{ m/s}$$
$$v_{i_y} = (80.0 \text{ m/s})(\sin 10°)$$
$$= 13.9 \text{ m/s}$$

Determine the time the ball remains airborne using the equation $a = \dfrac{v_f - v_i}{\Delta t}$.

$$a_y = \frac{v_{f_y} - v_{i_y}}{\Delta t}$$

$$\Delta t = \frac{v_{f_y} - v_{i_y}}{a_y}$$
$$= \frac{-13.9 \text{ m/s} - (13.9 \text{ m/s})}{-9.81 \text{ m/s}^2}$$
$$= \frac{-27.8 \ \dfrac{\cancel{\text{m}}}{\cancel{\text{s}}}}{-9.81 \ \dfrac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}}$$
$$= 2.83 \text{ s}$$

Calculate the range using the uniform motion equation $\vec{v} = \dfrac{\Delta \vec{d}}{\Delta t}$.

$$\Delta d_x = v_x \Delta t$$
$$= \left(78.8 \ \frac{\text{m}}{\cancel{\text{s}}}\right)(2.83 \ \cancel{\text{s}})$$
$$= 223 \text{ m}$$

Paraphrase

The golf ball travels 223 m horizontally and is in the air for 2.83 s.

12. ***Given***

$v_{\text{boat}} = 4.0 \text{ m/s}$

$\Delta d = 120 \text{ m}$

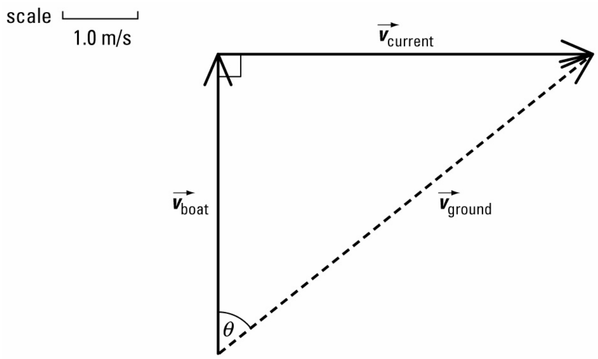
$\vec{v}_{\text{current}} = 5.0 \text{ m/s [E]}$

Required

(a) ground velocity ($\vec{v}_{\text{ground}}$)

(b) time (Δt)

Analysis and Solution



(a) $\vec{v}_{ground} = \vec{v}_{boat} + \vec{v}_{current}$

Use the Pythagorean theorem to calculate the ground velocity.

$$\begin{aligned}(v_{ground})^2 &= (v_{boat})^2 + (v_{current})^2\\ &= (4.0\text{ m/s})^2 + (5.0\text{ m/s})^2\\ v_{ground} &= 6.4\text{ m/s}\end{aligned}$$

Use the tangent function to determine the angle.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{5.0\ \cancel{\text{m/s}}}{4.0\ \cancel{\text{m/s}}}\right)\\ &= 51°\end{aligned}$$

From the diagram, this angle is 51° E of N.

(b) To find the time, substitute distance and the canoe's speed into the uniform motion equation, $\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$. Use the scalar form of the equation because you are dividing by a vector.

$$\begin{aligned}\Delta t &= \frac{\Delta d}{v}\\ &= \frac{120\text{ m}}{4.0\text{ m/s}}\\ &= 30\text{ s}\end{aligned}$$

Paraphrase

(a) The ground velocity of the canoeist is 6.4 m/s [51° E of N].

(b) It takes the canoeist 30 s to cross the river.

13. ***Given***

$v_x = 7.50\text{ m/s}$

$\Delta t = 3.0\text{ s}$

$v_{i_y} = 0\text{ m/s}$

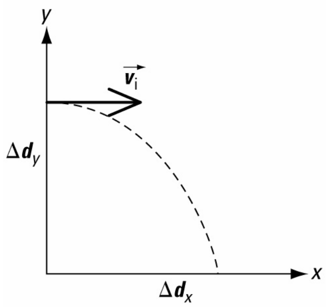
Required

(a) final vertical velocity component (v_{f_y})

(b) range (Δd_x)

(c) horizontal velocity component at $t = 1.50$ s ($v_x(1.50\text{ s})$)

Analysis and Solution



Choose down to be positive.

(a) Use the equation $a = \dfrac{v_f - v_i}{\Delta t}$, where $v_{i_y} = 0$.

$$\begin{aligned} v_{f_y} &= v_{i_y} + a_y \Delta t \\ &= 0 + \left(9.81\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(3.0\ \cancel{\text{s}}) \\ &= 29\text{ m/s} \end{aligned}$$

(b) Use the equation $\vec{v} = \dfrac{\Delta \vec{d}}{\Delta t}$.

$$\begin{aligned} \Delta d_x &= v_x \Delta t \\ &= \left(7.50\ \frac{\text{m}}{\cancel{\text{s}}}\right)(3.0\ \cancel{\text{s}}) \\ &= 23\text{ m} \end{aligned}$$

(c) Because horizontal motion is uniform, the velocity at 1.50 s is the same as the initial horizontal velocity.

$$\begin{aligned} v_x(1.50\text{ s}) &= v_x \\ &= 7.50\text{ m/s} \end{aligned}$$

Paraphrase

(a) The object's vertical velocity component when it reaches the ground is 29 m/s.
(b) The object lands 23 m from the base of the cliff.
(c) The object's horizontal velocity remains constant throughout, at 7.5 m/s.

14. ***Given***

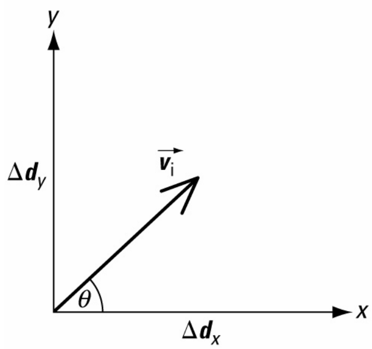
$\Delta d_x = 5.0$ m

$\Delta d_y = 2.0$ m

Required

initial velocity ($\vec{v}_i$)

Analysis and Solution



Choose up and forward to be positive.
Determine the initial vertical velocity component using the equation $v_{f_y}{}^2 = v_{i_y}{}^2 + 2a_y\Delta d_y$, where $v_{f_y} = 0$ (at maximum height).

$$v_{f_y}{}^2 = v_{i_y}{}^2 + 2a_y\Delta d_y$$

$$\begin{aligned} v_{i_y} &= \sqrt{v_{f_y}{}^2 - 2a_y\Delta d_y} \\ &= \sqrt{0 - 2\left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)(2.0\ \text{m})} \\ &= 6.26\ \text{m/s} \end{aligned}$$

Then determine the amount of time it takes to travel 2.0 m vertically using the equation $a = \dfrac{v_f - v_i}{\Delta t}$.

$$\begin{aligned} \Delta t &= \frac{v_{f_y} - v_{i_y}}{a_y} \\ &= \frac{-6.26\ \text{m/s} - (6.26\ \text{m/s})}{-9.81\ \text{m/s}^2} \\ &= \frac{-12.5\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}{-9.81\ \frac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}} \\ &= 1.28\ \text{s} \end{aligned}$$

Use the uniform motion equation $\vec{v} = \dfrac{\Delta \vec{d}}{\Delta t}$ to determine the horizontal speed.

$$\begin{aligned} v_x &= \frac{\Delta d_x}{\Delta t} \\ &= \frac{5.0\ \text{m}}{1.28\ \text{s}} \\ &= 3.91\ \text{m/s} \end{aligned}$$

Use the Pythagorean theorem to determine the magnitude of the initial velocity.

$$\begin{aligned} v_i &= \sqrt{(v_x)^2 + (v_{i_y})^2} \\ &= \sqrt{(3.91\ \text{m/s})^2 + (6.26\ \text{m/s})^2} \\ &= 7.4\ \text{m/s} \end{aligned}$$

Use the tangent function to find the angle.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{6.26\ \cancel{\text{m/s}}}{3.91\ \cancel{\text{m/s}}}$$
$$= 1.60$$
$$\theta = \tan^{-1}(1.60)$$
$$= 58°$$

Paraphrase

The high jumper's initial velocity is 7.4 m/s [58°].

15. ***Given***

Δd = 500 m

$\vec{v}_{\text{current}}$ = 2.0 m/s [E]

$v_{\text{alligator}}$ = 4.0 m/s

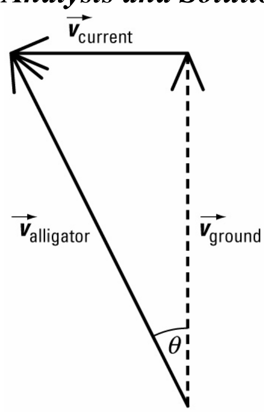
Required

(a) angle (θ)

(b) ground velocity ($\vec{v}_{\text{ground}}$)

(c) time (Δt)

Analysis and Solution



(a) Use the sine function to find the angle.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$= \frac{2.0\text{ m/s}}{4.0\text{ m/s}}$$
$$= 0.5$$
$$\theta = \sin^{-1}(0.5)$$
$$= 30°$$

From the diagram, the angle is 30° W of N.

(b) $\vec{v}_{\text{ground}} = \vec{v}_{\text{alligator}} + \vec{v}_{\text{current}}$

Use the Pythagorean theorem to determine the ground velocity.

$$\left(v_{\text{alligator}}\right)^2 = \left(v_{\text{ground}}\right)^2 + \left(v_{\text{current}}\right)^2$$

$$\begin{aligned}\left(v_{\text{ground}}\right)^2 &= \left(v_{\text{alligator}}\right)^2 - \left(v_{\text{current}}\right)^2 \\ &= (4.0 \text{ m/s})^2 - (2.0 \text{ m/s})^2 \\ v_{\text{ground}} &= 3.5 \text{ m/s}\end{aligned}$$

The direction of the velocity is north (given).

(c) Substitute the width of the river and the magnitude of the ground velocity into the uniform motion equation, $\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$, because the ground velocity is parallel to the width of the river. Use the scalar form of the equation because you are dividing by a vector.

$$\begin{aligned}\Delta t &= \frac{\Delta d}{v} \\ &= \frac{500 \text{ m}}{3.5 \text{ m/s}} \\ &= 1.4 \times 10^2 \text{ s}\end{aligned}$$

Paraphrase

(a) The alligator must head 30° W of N.

(b) The alligator's ground velocity is 3.5 m/s [N].

(c) It takes the alligator 1.4×10^2 s to cross the river.

16. ***Given***

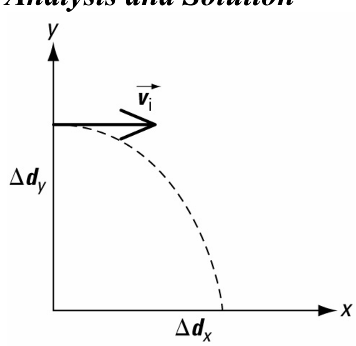
$v_x = 45.0$ m/s

$\Delta d_x = 27.4$ m

Required

vertical distance (Δd_y)

Analysis and Solution



Choose down to be positive.

The time it takes for the ball to travel 27.4 m horizontally equals the time taken for the ball to drop vertically. For horizontal motion, use the equation $v_x = \frac{\Delta d_x}{\Delta t}$.

$$\Delta t = \frac{\Delta d_x}{v_x}$$
$$= \frac{27.4\ \cancel{\text{m}}}{45.0\ \frac{\cancel{\text{m}}}{\text{s}}}$$
$$= 0.6089\ \text{s}$$

For vertical motion, use the equation $\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$, where $v_{i_y} = 0$.

$$\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$
$$= 0 + \frac{1}{2}\left(9.81\ \frac{\text{m}}{\cancel{\text{s}^2}}\right)(0.6089\ \cancel{\text{s}})^{\cancel{2}}$$
$$= 1.82\ \text{m}$$

Paraphrase

The ball will drop 1.82 m.

17. ***Given***

$v = 7.0$ m/s

$\vec{d}_1 = 750$ m [N]

$\vec{d}_2 = 350$ m [E]

Required

time (Δt)

Analysis and Solution

Determine the time for each trip using the equation $v = \frac{\Delta d}{\Delta t}$.

$$\Delta d = d_1 + d_2$$
$$= 750\ \text{m} + 350\ \text{m}$$
$$= 1100\ \text{m}$$

$$\Delta t = \frac{\Delta d}{v}$$
$$= \frac{1100\ \cancel{\text{m}}}{7.0\ \frac{\cancel{\text{m}}}{\text{s}}}$$
$$= 157\ \text{s}$$

Use the Pythagorean theorem to determine the diagonal distance.

$$\Delta\vec{d} = \vec{d}_1 + \vec{d}_2$$
$$= 750 \text{ m [N]} + 350 \text{ m [E]}$$
$$\left|\Delta\vec{d}\right| = \sqrt{(d_1)^2 + (d_2)^2}$$
$$= \sqrt{(750 \text{ m})^2 + (350 \text{ m})^2}$$
$$= 827.6 \text{ m}$$
$$\Delta t = \frac{\left|\Delta\vec{d}\right|}{v}$$
$$= \frac{827.6 \cancel{\text{m}}}{7.0 \frac{\cancel{\text{m}}}{\text{s}}}$$
$$= 118 \text{ s}$$

Subtract the longer time from the shorter time.

$$\Delta t = 157 \text{ s} - 118 \text{ s}$$
$$= 39 \text{ s}$$

Paraphrase

You can save 39 s by travelling diagonally.

18. ***Given***

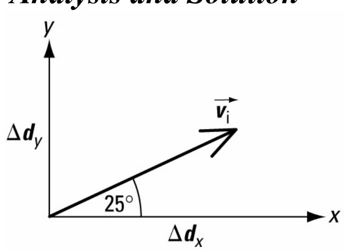
$\theta = 25°$

$\Delta d_x = 50.0 \text{ m}$

Required

time (Δt)

Analysis and Solution



Choose up and forward to be positive.

Both initial vertical and horizontal velocities can be expressed as trigonometric ratios of the initial velocity. The amount of time travelled in the vertical direction equals the time taken to travel in the horizontal direction. Horizontal motion is uniform, so $v_x = \dfrac{\Delta d_x}{\Delta t}$.

x direction:

$$v_x = v\cos\theta$$
$$v_x = \frac{\Delta d_x}{\Delta t}$$
$$v\cos 25° = \frac{50.0 \text{ m}}{\Delta t}$$

Vertical motion is accelerated motion, so $a_y = \dfrac{v_{f_y} - v_{i_y}}{\Delta t}$.

y direction:

$$v_y = v\sin\theta$$

$$a_y = \frac{v_{f_y} - v_{i_y}}{\Delta t}$$

$$\Delta t = \frac{v_{f_y} - v_{i_y}}{a_y}$$

$$= \frac{-v\sin 25° - (v\sin 25°)}{-9.81 \text{ m/s}^2}$$

$$= \frac{-2v\sin 25°}{-9.81 \text{ m/s}^2}$$

$$= \frac{2v\sin 25°}{9.81 \text{ m/s}^2}$$

Substitute for *v*:

$$\Delta t = \frac{2v\sin 25°}{9.81 \text{ m/s}^2}$$

$$= \frac{2\sin 25°}{9.81 \text{ m/s}^2} \times \frac{50.0 \text{ m}}{\cos 25° \Delta t}$$

$$\Delta t^2 = \frac{(2\tan 25°)(50.0 \ \cancel{\text{m}})}{9.81 \ \frac{\cancel{\text{m}}}{\text{s}^2}}$$

$$\Delta t = 2.2 \text{ s}$$

Paraphrase

The arrow is in flight for 2.2 s.

19. ***Given***

v_{air} = 100 km/h

$\vec{v}_{wind}$ = 30 km/h [N]

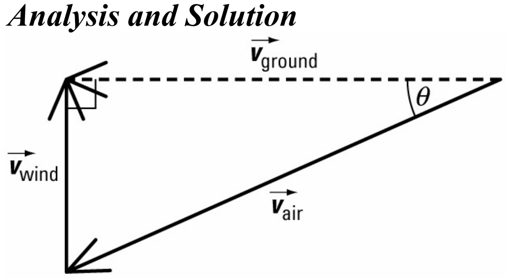
ground direction = W

Required

(a) heading

(b) ground speed (v_{ground})

Analysis and Solution



(a) Use the tangent function to determine the plane's heading.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\theta = \tan^{-1}\left(\frac{30 \ \cancel{\text{km/h}}}{100 \ \cancel{\text{km/h}}}\right)$$

$$= 17°$$

From the diagram, the plane's heading is $17°$ S of W.

(b) Use the Pythagorean theorem to determine the ground speed.

$$\begin{aligned}(v_{\text{air}})^2 &= (v_{\text{ground}})^2 + (v_{\text{wind}})^2\\ (v_{\text{ground}})^2 &= (v_{\text{air}})^2 - (v_{\text{wind}})^2\\ &= (100\text{ km/h})^2 - (30\text{ km/h})^2\\ v_{\text{ground}} &= 95\text{ km/h}\end{aligned}$$

Paraphrase

(a) The plane's heading is 17° S of W.

(b) The plane's ground speed is 95 km/h.

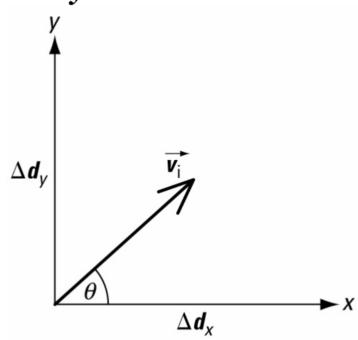
20. ***Given***

$$\begin{aligned}v_{\text{i}} &= 15.7\text{ m/s}\\ \Delta t &= 2.15\text{ s}\\ \Delta d_x &= 25.0\text{ m}\end{aligned}$$

Required

launch angle (θ)

Analysis and Solution



Determine the horizontal velocity component and then apply the cosine function.

$$\begin{aligned}v_x &= \frac{\Delta d_x}{\Delta t}\\ &= \frac{25.0\text{ m}}{2.15\text{ s}}\\ &= 11.63\text{ m/s}\\ v_x &= v\cos\theta\\ \theta &= \cos^{-1}\left(\frac{v_x}{v}\right)\\ &= \cos^{-1}\left(\frac{11.63\text{ m/s}}{15.7\text{ m/s}}\right)\\ &= 42.2°\end{aligned}$$

Paraphrase

The object was thrown at an angle of 42.2°.

21. ***Given***

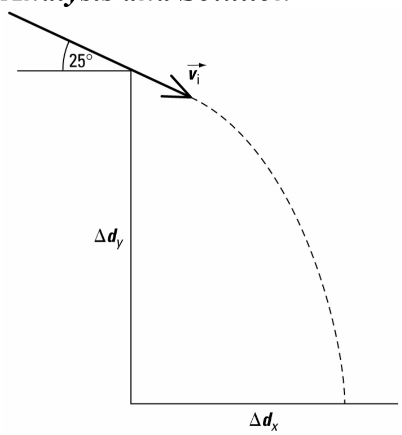
$\vec{v}_i = 30 \text{ m/s } [25.0°]$

$\Delta d_y = 2.5 \text{ m}$

Required

range (Δd_x)

Analysis and Solution



Choose down to be positive. Find the *x* and *y* components of the coin's velocity.

$$\begin{aligned} v_y &= (30 \text{ m/s})(\sin 25.0°) \\ &= 12.7 \text{ m/s} \end{aligned}$$

$$\begin{aligned} v_x &= (30 \text{ m/s})(\cos 25.0°) \\ &= 27.2 \text{ m/s} \end{aligned}$$

First calculate the final vertical velocity component using the equation

$v_f{}^2 = v_i{}^2 + 2a\Delta d$.

$$v_{f_y}{}^2 = v_{i_y}{}^2 + 2a_y \Delta d$$

$$\begin{aligned} v_{f_y} &= \sqrt{v_{i_y}{}^2 + 2a_y \Delta d} \\ &= \sqrt{(12.7 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(2.5 \text{ m})} \\ &= 14.5 \text{ m/s} \end{aligned}$$

Find the time taken to fall using the equation $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$.

$$\begin{aligned} \Delta t &= \frac{v_{f_y} - v_{i_y}}{a_y} \\ &= \frac{14.5 \text{ m/s} - 12.7 \text{ m/s}}{9.81 \text{ m/s}^2} \\ &= 0.183 \text{ s} \end{aligned}$$

The time taken to travel vertically equals the time taken to travel horizontally. Since motion in the horizontal direction is uniform, use the uniform motion equation

$\vec{v} = \dfrac{\Delta \vec{d}}{\Delta t}$ to solve for the range.

$$\begin{aligned}\Delta d_x &= v_x \Delta t \\ &= \left(27.2\ \frac{\text{m}}{\cancel{\text{s}}}\right)(0.183\ \cancel{\text{s}}) \\ &= 5.0\ \text{m}\end{aligned}$$

Paraphrase

The coin lands 5.0 m from the base of the bookcase.

22. Total length is 2(3.96 m + 18.29 m) + 16.46 m = 60.96 m

23. ***Given***

$\vec{v}_{\text{ground}}$ = 151 km/h [11° S of E]

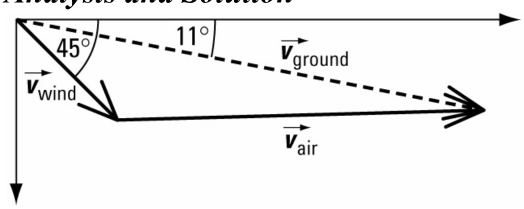
$\vec{v}_{\text{wind}}$ = 40 km/h [45° S of E]

Required

(a) airspeed (v_{air})

(b) heading

Analysis and Solution



(a) Resolve the wind and ground velocities into their x and y components.

$$\begin{aligned}v_{\text{ground}_x} &= (151\ \text{km/h})(\cos 11°) \\ &= 148.2\ \text{km/h}\end{aligned}$$

$$\begin{aligned}v_{\text{ground}_y} &= -(151\ \text{km/h})(\sin 11°) \\ &= -28.8\ \text{km/h}\end{aligned}$$

$$\begin{aligned}v_{\text{wind}_x} &= (40\ \text{km/h})(\cos 45°) \\ &= 28.3\ \text{km/h}\end{aligned}$$

$$\begin{aligned}v_{\text{wind}_y} &= -(40\ \text{km/h})(\sin 45°) \\ &= -28.3\ \text{km/h}\end{aligned}$$

$$\vec{v}_{\text{ground}} = \vec{v}_{\text{air}} + \vec{v}_{\text{wind}}$$

$$\vec{v}_{\text{air}} = \vec{v}_{\text{ground}} - \vec{v}_{\text{wind}}$$

$$\begin{aligned}v_{\text{air}_x} &= v_{\text{ground}_x} - v_{\text{wind}_x} \\ &= 148.2\ \text{km/h} - 28.3\ \text{km/h} \\ &= 119.9\ \text{km/h}\end{aligned}$$

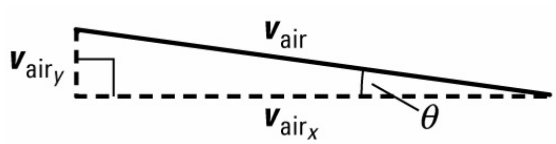
$$\begin{aligned}v_{\text{air}_y} &= v_{\text{ground}_y} - v_{\text{wind}_y} \\ &= -28.8\ \text{km/h} - (-28.3\ \text{km/h}) \\ &= -0.5\ \text{km/h}\end{aligned}$$

Use the Pythagorean theorem to determine the magnitude of the airspeed.

$$(v_{air})^2 = (v_{air_x})^2 + (v_{air_y})^2$$

$$\begin{aligned} v_{air} &= \sqrt{(v_{air_x})^2 + (v_{air_y})^2} \\ &= \sqrt{(119.9 \text{ km/h})^2 + (-0.5 \text{ km/h})^2} \\ &= 120 \text{ km/h} \end{aligned}$$

(b)



Use the tangent function to find the plane's heading.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{0.5 \cancel{\text{ km/h}}}{119.9 \cancel{\text{ km/h}}}\right) \\ &= 0.2° \end{aligned}$$

To the nearest degree, the angle is zero. From the diagram, this direction is east.

Paraphrase

(a) The plane's airspeed is 120 km/h.

(b) The plane's heading is east.

24. ***Given***

$\vec{v} = 25.0$ m/s [35.0°]

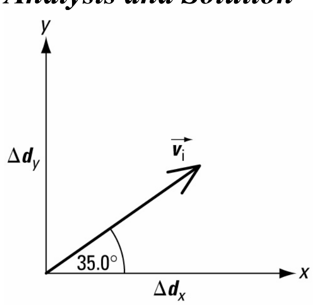
Required

time (Δt)

range (Δd_x)

maximum height (Δd_y)

Analysis and Solution



Choose up and forward to be positive. Calculate the components of the initial velocity using trigonometric ratios.

$$\begin{aligned} v_{i_x} &= (25.0 \text{ m/s})(\cos 35.0°) \\ &= 20.48 \text{ m/s} \end{aligned}$$

$$\begin{aligned} v_{i_y} &= (25.0 \text{ m/s})(\sin 35.0°) \\ &= 14.34 \text{ m/s} \end{aligned}$$

At maximum height, the vertical velocity component is zero.

$$v_{f_y}{}^2 = v_{i_y}{}^2 + 2a_y\Delta d_y$$

$$\Delta d_y = \frac{v_{f_y}{}^2 - v_{i_y}{}^2}{2a_y}$$

$$= \frac{0 - (14.34 \text{ m/s})^2}{2(-9.81 \text{ m/s}^2)}$$

$$= 10.48 \text{ m}$$

Calculate the amount of time it takes the soccer ball to reach maximum height and multiply it by two to find the time the ball is in the air.

From maximum height, $v_{i_y} = 0$.

$$\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$\Delta t_{\text{down}} = \sqrt{\frac{2\Delta d}{a_y}}$$

$$= \sqrt{\frac{2(-10.48 \text{ m})}{-9.81 \text{ m/s}^2}}$$

$$= 1.462 \text{ s}$$

$$\Delta t_{\text{hangtime}} = 2(\Delta t_{\text{down}})$$

$$= 2(1.462 \text{ s})$$

$$= 2.923 \text{ s}$$

Since the amount of time the ball is in the air vertically equals the amount of time the ball travels horizontally, calculate the horizontal distance travelled using the equation $\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$.

$$\Delta d_x = v_{i_x}\Delta t$$

$$= (20.48 \text{ m/s})(2.923 \text{ s})$$

$$= 59.9 \text{ m}$$

Paraphrase

The soccer ball is in the air for 2.92 s. It travels 59.9 m down the field and its maximum height is 10.5 m.

25. ***Given***

$v_x = 4.50$ m/s

$v_y = 3.75$ m/s

Required

angle (θ)

Analysis and Solution

Since you know the components of the velocity, use the tangent function to solve for the angle.

$$\tan\theta = \frac{v_y}{v_x}$$

$$= \frac{3.75\ \cancel{\text{m/s}}}{4.50\ \cancel{\text{m/s}}}$$

$$= 0.833$$

$$\theta = \tan^{-1}(0.833)$$

$$= 39.8°$$

Paraphrase

The launch angle is 39.8°.

Extensions

26. ***Given***

$\vec{v}_{\text{golf ball}} = 50\text{ m/s [35°]}$

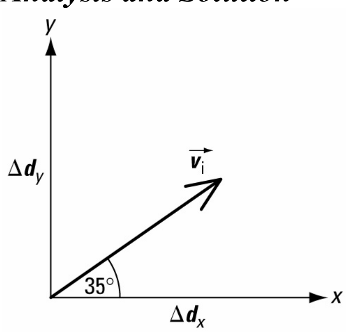
$\vec{g}_{\text{Moon}} = 1.61\text{ m/s}^2\text{ [down]}$

Required

(a) time (Δt)

(b) range (Δd_x)

Analysis and Solution



Choose up and forward to be positive.

(a) Resolve the velocity vector into its components.

$$v_x = (50\text{ m/s})(\cos 35°)$$

$$= 40.96\text{ m/s}$$

$$v_y = (50\text{ m/s})(\sin 35°)$$

$$= 28.68\text{ m/s}$$

Determine the time the golf ball takes to travel up and down using the equation

$$a = \frac{v_f - v_i}{\Delta t}.$$

$$a_y = \frac{v_{f_y} - v_{i_y}}{\Delta t}$$

$$\Delta t = \frac{v_{f_y} - v_{i_y}}{a_y}$$

$$= \frac{-28.68\text{ m/s} - (28.68\text{ m/s})}{-1.61\text{ m/s}^2}$$

$$= 35.63\text{ s}$$

(b) Use the uniform motion equation $\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$ to determine the horizontal distance travelled.

$$\begin{aligned}\Delta d_x &= v_x \Delta t \\ &= \left(40.96 \ \frac{\text{m}}{\cancel{\text{s}}}\right)(35.63 \ \cancel{\text{s}}) \\ &= 1.5 \times 10^3 \ \text{m}\end{aligned}$$

Paraphrase

(a) The golf ball was in the air for 36 s.

(b) The golf ball's range was 1.5 km.

27. ***Given***

$\vec{v}_{\text{air}} = 645$ km/h [forward]

$\vec{v}_{\text{wind}} = -32.2$ km/h [forward]

$\vec{v} = 3.0$ m/s [down]

$\Delta d_y = 914.4$ m

$\Delta d_x = 45.0$ km

Required

plane's range ($\Delta d_{x_{\text{plane}}}$)

Analysis and Solution

Choose forward and down to be positive. The amount of time it takes the plane to descend 914.4 m at a speed of 3.0 m/s is the same amount of time taken to travel horizontally toward the runway.

$$v_y = \frac{\Delta d_y}{\Delta t}$$

$$\begin{aligned}\Delta t &= \frac{\Delta d_y}{v_y} \\ &= \frac{914.4 \ \cancel{\text{m}}}{3.0 \ \frac{\cancel{\text{m}}}{\text{s}}} \\ &= 304.8 \ \cancel{\text{s}} \times \frac{1 \ \text{h}}{3600 \ \cancel{\text{s}}} \\ &= 0.0847 \ \text{h}\end{aligned}$$

Since the plane is travelling against the wind, the plane's ground velocity will be less than 645 km/h [forward].

$$\begin{aligned}\vec{v}_{\text{ground}} &= \vec{v}_{\text{plane}} + \vec{v}_{\text{air}} \\ &= 645 \ \text{km/h [forward]} + (-32.2 \ \text{km/h [forward]}) \\ &= 645 \ \text{km/h [forward]} - 32.2 \ \text{km/h [forward]} \\ &= 612.8 \ \text{km/h [forward]}\end{aligned}$$

$$\begin{aligned}\Delta d_{x_{\text{plane}}} &= v_x \Delta t \\ &= \left(612.8 \ \frac{\text{km}}{\cancel{\text{h}}}\right)\left(0.0847 \ \cancel{\text{h}}\right) \\ &= 51.9 \ \text{km}\end{aligned}$$

Paraphrase
Since the distance travelled horizontally toward the runway is greater than 45.0 km, the pilot must begin her vertical descent earlier in order to land at the beginning of the runway.

